Just starting out: Learning and equilibrium in a new market∗

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Abstract

Deregulation of the frequency response market in the UK allowed electricity firms to compete on price in an otherwise stable environment. We provide an analysis of the evolution of the deregulated market from the date it started. Initial activity was volatile, with some firms exploring different prices, while others made few price changes. This was followed by a period in which prices fell and the variance in the cross-sectional distribution of bids declined markedly. By the end of our study price changes had become relatively rare and small, consistent with convergence to a static Nash equilibrium. We examine how well models of learning do in predicting play during the period prior to convergence but after the initial volatility. Models where perceptions of competitors’ play depend on past play suggest that firms’ weight recent play disproportionately. We also find evidence of statistical learning about the underlying demand.

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parameters conditional on competitors’ play. A model that combines these two features
fits quite well: it is able to explain 37% of the share-weighted variation in prices, even
though none of the model parameters are chosen to fit the pricing behavior.
1 Introduction

Entirely new markets arise frequently, often as a result of innovation or deregulation. Firms entering these markets face considerable uncertainty. This is partly because overall demand is typically uncertain, and partly because the demand for their product depends on the decisions of competitors facing similar uncertainties. This paper explores how this uncertainty manifests itself in the bidding behavior of firms from “day one” of a new market.

An understanding of how firms form strategies in new markets — or indeed in existing markets following a sudden change in conditions — is necessary in any investigation of the implications of changes in market institutions. The empirical industrial organization literature has focused on simulation of new equilibria following counterfactual policy changes (e.g. a merger). But convergence to a new equilibrium is not guaranteed to be swift (or indeed certain), and so having a reliable model of out-of-equilibrium pricing dynamics would be extremely helpful. Moreover, such a model might help narrow down the set of possibilities in cases where there are multiple counterfactual equilibria (see, e.g. Lee and Pakes (2009)).

There has been little empirical work on how new markets (or existing markets with sharp changes in the institutions which govern them) evolve, and consequently little field evidence to discriminate among models of how firms adapt to both the strategic and demand uncertainty presented by changes in the environment. This paper attempts to fill this gap with an analysis of the frequency response market in the UK. Broadly speaking, frequency response is a product required by the system operator in electricity markets to keep the system running smoothly. It is bought from electricity generating firms. Prior to November 2005, generators in the UK were required to provide frequency response to the system operator at a fixed system-wide price. Then the market was deregulated and generators were allowed to bid into an auction market, setting the stage for price competition. We study what happened next.

Though the introduction of the auction essentially started a new market, it is a new market with features which should make it relatively easy to analyze. Prices are set simultaneously each month, as in a standard normal form game. The system operator is required to buy frequency response according to a public and fixed set of regulations, so the rules of the game are reasonably clear. Moreover the same firms had been supplying frequency response in the regulatory period preceding the change, so the bidders had quite a bit of prior knowledge of the vagaries of demand. Finally while the market is somewhat concentrated, there are still
ten big firms, so that tacit collusion is difficult and repeated game considerations may be less relevant.

The paper is primarily an attempt to bring descriptive evidence to the question of how markets adapt to changes in institutions. We show that the FR market reaches a “rest-point” after three and a half years. Generators set substantially different prices in the early stages of the market, and update their prices regularly. But after three and a half years of repeated interaction, price changes became infrequent and small, and the cross-sectional variance in prices had shrunk markedly. We estimate a flexible demand system using all the data, and then back out marginal costs under the assumption that firms play a static Nash equilibrium in the latter part of the data. Our estimated costs are in line with the “cost-reflective” fixed price prior to deregulation, which is evidence in favor of the rest point being a static Nash equilibrium.

When the market starts up we see different firms following different strategies which do not seem to be coordinated in any way. The behavior of some looks to be exploratory, and when the exploration leads to large gains in returns, others seem to follow. There are also firms who hardly change their prices at all. After just over a year competition between firms drives the highest prices down, leading to dramatically lower variance in the cross-sectional bid distribution. We consider a series of learning models that might explain the convergence in this period. Recall that the firms may be uncertain about both the behavior of their rivals and about the demand for their products given rival behavior. In our first set of models we assume away the second source of uncertainty to focus on strategic uncertainty. We allow firms to weight the past play of their competitors differentially for different lagged periods (in the theory literature, this is “fictitious play”, as in Brown (1951)). In our second set of models we allow firms to be uncertain about the demand parameters as well as rival strategies. We model how firms form perceptions about demand parameters by a statistical learning process, assuming firms run regressions given the set of data available to them at each point in time. This approach originates in the macro literature on rational expectations (e.g. Townsend (1983), Marcet and Sargent (1989), Evans and Honkapohja (2001)).

Our criterion for comparing the different models is their ability to make “one-step-ahead” predictions: we compare the predicted bid for time $t$ given data from time $t-1$ to the realized bid at time $t$, where the predictions vary across models. None of the models in which firms know the demand system from the start fit the data particularly well. By contrast, when we allow for statistical learning on the properties of the demand system given the bids of
others, as well as use the fictitious play assumptions for the perceptions of competitors’ bids, we obtain a much better fit. Perhaps not surprisingly, firms seem to optimize given the currently available information and their perceptions evolve over time.

In the context of the fictitious play model we reject a model in which firms think that their rivals are equally likely to take any of their past actions. Instead we find that firms discount the past behavior of their opponents and pay relatively more attention to recent actions; in the extreme this leads to a best response model. A model with firms both re-estimating demand each period and best responding to their rival’s actions fits the data quite well: the (quantity-weighted) \( R^2 \) is 0.37, which is reasonably impressive considering that the demand and cost parameters were estimated in a previous step, and no parameters were chosen to maximize the fit of the learning process per se.

We view the current version of the paper as making two distinct contributions. The first is to carefully document what happens to prices and bidding over time when a new market is started in a fairly stable environment. Since it is common in empirical industrial organization to assume that an equilibrium will be re-asserted following a change in conditions, and yet not clear from the theory on strategic learning that this should be the case, it seems useful to bring empirical evidence to bear on this issue. We find that there is convergence to some sort of rest point, albeit only after three and a half years of monthly strategic interaction. This rest point is consistent with a static Nash equilibrium in bids. To the best of our knowledge, ours is the first paper to demonstrate convergence and describe how it occurs.

A second contribution is to test different learning models against each other by comparing their relative fit during the competitive period. Our results here are preliminary, but suggest that firms more heavily weight recent competitor actions and engage in statistical learning.

**Related literature.** There is a large theory literature on learning in static normal-form games. The organizing feature of this literature has been on deriving conditions under which the canonical models of fictitious play (Brown 1951) and reinforcement learning imply convergence to equilibrium (e.g. Milgrom and Roberts (1991), Fudenberg and Kreps (1993), Börgers and Sarin (1997), Hart and Mas-Colell (2000)); usually in stable and known environments. Experimental economists have pushed this literature further, using lab data to work out which learning models best describe how people actually learn, and proposing new models as a result. Subsequent work has led to more general models, such as experience-weighted attractive learning (Camerer and Ho 1999); or models with sophisticated learners,
who try to influence how other players learn (Camerer, Ho, and Chong 2002). There are now a number of meta-studies, combining data from multiple lab experiments: Boylan and El-Gamal (1993) finds that a fictitious play model performs better than adaptive best response; Cheung and Friedman (1997) find evidence of heterogeneity across players in the discount rate in fictitious play models; Erev and Roth (1998) show that a simple one-parameter reinforcement learning model can fit their data quite well. But recently Salmon (2001) has argued using Monte Carlo simulations that in fact there is an “identification failure”: different models are hard to tell apart statistically because they make similar predictions.

A second, distinct, theoretical literature considers behavior when there is uncertainty about the state of nature. There is a long literature in applied mathematics and statistics analyzing bandit problems, in which forward-looking agents trade off “exploration” versus “exploitation”. Economists have contributed to this literature by analyzing what happens when multiple agents compete in such an environment, noting informational free-riding incentives (e.g. Bolton and Harris (1999)) and incentives to “signal jam” (e.g. Mirman, Samuelson, and Urbano (1993)).

There has been no prominent empirical work on either of these topics (i.e. convergence to equilibrium, or estimating learning models), but there are some papers on related topics. Hortaçsu and Puller (2008) show that in the newly created spot market for electricity in Texas, big firms made bids that were best responses to rival play, but small firms failed to optimize fully (although their behavior improved over time). Goldfarb and Xiao (2011) show that managers with different experience and education levels make different entry decisions in local US telephone markets after deregulation. They rationalize this with a cognitive hierarchy model, in which more experienced managers think more steps ahead.

Surprisingly, there is also little empirical work on how agents learn about demand for their product from the observation of their and their rivals’ sales. In the IO and marketing literature, a number of papers have examined how agents may learn their demand for experience goods from their own experimentation (Erdem and Keane 1996, Ackerberg 2003, Dickstein 2013). There is also a small empirical literature on observational learning, where agents see the choice of others but not outcomes (e.g. Zhang (2010) on patient decisions to accept a kidney offer, and Newberry (2013) on music downloads). Social learning has been more widely studied in other contexts (see e.g. Griliches (1957) on hybrid corn, Conley and Udry (2010) on fertilizer in Ghana, and Covert (2013) on fracking in the Bakken Shale).
Structure of paper. The paper proceeds as follows. In the next two sections we describe the frequency response market, our data on it, and some descriptive evidence on how it evolved over time. The following section outlines our estimation strategy for recovering the supply and demand primitives. We then estimate a number of learning models and compare their comparative fit, before concluding. Additional information on the construction of the data and the estimation procedures are to be found in the appendix.

2 Overview of the UK electricity market

The UK electricity market is a network of generators and distributors, connected by a transmission grid. This grid is owned and operated by a company called National Grid plc (NG). NG is responsible for the transmission of electricity from the generators to the distributors, as well as the balancing of supply and demand in real time. Figure “overview-pic” summarizes the UK electricity market.

The unit of exchange in this market is a given amount of power supplied for a half-hour (measured in megawatt hours (MWh)). About 98% of electricity is sold through bilateral forward contracts between generators and distributors. These contracts can be formed months or even years in advance. There are also shorter term contracts (both day ahead and day of) which are often traded on power exchanges. One hour prior to the settlement period, both generators and distributors must submit their contracted positions to NG, who then holds an auction to equate supply and demand as expected over the settlement period. This multi-unit discriminatory auction is called the balancing mechanism (BM), and it accounts for the remaining 2% of electricity sales. The generators bidding in the BM and are called BM units. A power station typically consists of multiple BM units, and multiple stations may be owned by the same firm. The BM units belonging to the same station tend to be identical.

Frequency response. NG is obligated by government regulation to maintain a system frequency within a one-percent band of 50 Hertz (Hz, the number of cycles per second). System frequency is determined in real time by imbalances between the supply and demand of electricity. The higher demand is relative to supply, the lower the system frequency is, and vice versa. Imbalances occur due to shocks that cannot be corrected in advance through
the BM. To balance the supply and demand in real time, NG instructs one or more BM units into frequency response (FR) mode. Once in this mode, NG can rapidly adjust the energy production of the BM unit using so-called governor controls.

NG is required by government regulation to hold a certain amount of FR capacity at all times.¹ This response requirement is based on risk-response curves that assess the likelihood and magnitude of possible shocks given the total amount of electricity demanded. As the

¹There are in fact three types of FR. Primary response is additional energy from a BM unit that is available ten seconds after an event and can be sustained for a further twenty seconds. Secondary response is additional energy that is available within thirty seconds for up to thirty minutes. High response is a reduction of energy within thirty seconds. These responses are technologically constrained and correspond to dilating the steam valve (primary), increasing the supply of fuel (secondary), and decreasing the supply of fuel (high). For historical reasons, BM units are instructed into FR mode in the combinations primary-high and primary-secondary-high. To simplify the presentation and analysis, we aggregate the three types of FR; see the data appendix for details.
total amount of electricity demanded evolves, NG instructs BM units in and out of FR mode to satisfy its response requirement. To the best of our knowledge, the response requirement remained unchanged over the sample period.

FR services are thus a second product, distinct from electricity, that BM units can sell to NG, and the FR market is distinct from the main market (comprised of the BM and bilateral forward contracts). Providing FR is costly: a BM unit in FR mode incurs additional wear and tear as it may have to make rapid adjustments to its energy production in response to supply and demand shocks. It also runs less efficiently, with a degraded heat rate. The BM unit is compensated by NG by a holding payment and an energy response payment. The holding payment is per unit of FR capacity and paid for the time that it is called into FR mode regardless of whether the BM unit has to adjust its energy production in response to supply and demand shocks. The energy response payment compensates the BM unit for actual adjustments to its energy production. The energy response payment is considered by industry insiders to be a relatively small source of profit and, in contrast to the holding payment, remained constant over the sample period.

Deregulation. Our interest in FR stems from a change in the way the holding payment is determined. This changed occurred with the enactment of an amendment to the Connection and Use of System Code called CAP047 and “went live” on November 1, 2005. Pre CAP047, providing FR was mandatory, and the holding payment was at an administered price which had been fairly constant over time (see Figure 2). CAP047 replaced the mandatory provision of FR with a market. In this market, the holding payment is determined by an auction.

Post CAP047, a BM unit tenders a bid each month for providing FR. The bid for the next month is submitted before the 20th of the current month, well in advance of electricity production, and consists of a price per unit of FR capacity (measured in £/MWh). If called upon by NG, the BM unit is paid a holding payment equal to its bid per unit of FR capacity for time spent in FR mode (i.e., it gets “paid-as-it-bids”). Its bid commits the BM unit to offer FR at a fixed price over the next month. The quantity that the BM unit delivers

\[ \text{Quantity} = \text{Bid} \times \text{Time} \]

2We have checked the publicly available minutes of all meetings of the Balancing Services Standing Group (comprising representatives of the generators and NG) and found no discussion of a change in the response requirement.

3If the BM unit produces more energy than it was initially contracted to in the BM, NG pays it 125% of the current market price per additional unit of energy; if the BM unit produces less energy, it pays NG 75% of the current market price.
Figure 2: Holding payment for high response by day pre and post CAP047. Source: National Grid.

if instructed into FR mode varies with its current operating position and system deviation according to a unit-specific contract between the generator and NG that is generally fixed over the sample period.\textsuperscript{4}

NG can choose to call upon any BM unit at any time, and often does not choose the lowest bidders to provide FR. Instead, it simultaneously accepts bids in the BM and instructs BM units into FR mode to equate supply and demand and maintain the mandated amount of FR capacity in the most cost-effective way. In practice, this cost minimization problem is solved in real time by a proprietary linear program running on a supercomputer. NG may not choose the lowest bids for at least two reasons. First, BM units differ in the precision of their governor controls, and NG may prefer to call upon more expensive but more precise BM units. Second, because the FR capacity of a BM unit depends on its operating position, NG may prefer to call upon a BM unit operating in the middle of its range, and thus with plenty of FR capacity, rather than a BM unit operating at the extremes of its range. Indeed, NG may first alter the operating position of the BM unit by taking over part of its obligations in the BM before instructing the BM unit into FR mode.\textsuperscript{5}

\textsuperscript{4}This contract takes the form of a $5 \times 3$ matrix for each type of FR (see footnote \textsuperscript{1}) that specifies the quantity delivered at five deload points (operating positions) and three system deviations ($0.2\text{Hz}$, $0.5\text{Hz}$, and $0.8\text{Hz}$ away from $50\text{Hz}$). At other deload points and deviations, the quantity is determined by linear interpolation. The matrices are proprietary information, but selected entries are published by NG in the capability data (see the data appendix). For over 80\% of the BM units, the observed entries do not change over the sample period.

\textsuperscript{5}As a result, a BM unit does not have to withhold generating capacity from the main market in order to
The market for FR was proposed by one of the largest firms in the UK electricity market, RWE. This proposal was bitterly opposed by NG, who argued that since its demand for FR is regulated and thus inelastic, generators would be able to exploit their market power and the price of FR would rise. The government regulator dismissed these concerns, and on November 1, 2005 introduced CAP047. Figure 2 shows that NG had every reason to worry about CAP047, as the holding payment doubled within the year.

From the pre-CAP047 period, firms likely had an understanding of the response requirement NG is obligated to satisfy and the relative desirability of their BM units, as well as the cost of providing FR. However, firms faced uncertainty as to the demand for their FR services because they did not know how their rivals would bid in the auction. In addition to this strategic uncertainty, the firms faced demand uncertainty in that they did not know how price sensitive NG was. Our goal is to understand how firms learned to bid in the presence of this uncertainty, and how this contributed to the evolution of the holding payment in Figure 2.

Data. Our empirical analysis focuses on the first six years of the operation of the FR market from November 2005 to October 2011. We collected most of our data from two public sources. Our data on the FR market comes from NG. For the post-CAP047 period we have the bids submitted by each BM unit at a monthly level and the quantities provided of each type of FR (in MWh) by each BM unit at a daily level. The combination of bid and quantity data allows us to calculate the holding payment received by each BM unit.

Our data on the BM comes from Elexon Ltd. Elexon is contracted by the government regulator to manage measurement and financial settlement in the BM. For every BM unit we have data on the bids and acceptances in the BM every half-hour. In combination with data on the contracted position that the BM unit submits to NG one hour prior to the settlement period, this allows us to assess the operating position of the BM unit.

Finally, we collected data on ownership and characteristics of power stations and fuel prices from various sources. See the data appendix for further details on data sources as well as sample and variable construction.
### Table 1: Firms with the largest frequency response revenues

<table>
<thead>
<tr>
<th>Rank</th>
<th>Firm name</th>
<th>Num Units Owned</th>
<th>Total Revenue</th>
<th>Revenue Share (%)</th>
<th>Cumulative Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Drax Power Ltd.</td>
<td>6</td>
<td>99.4</td>
<td>23.8</td>
<td>23.8</td>
</tr>
<tr>
<td>2</td>
<td>E.ON UK plc</td>
<td>20</td>
<td>67</td>
<td>16</td>
<td>39.9</td>
</tr>
<tr>
<td>3</td>
<td>RWE plc</td>
<td>23</td>
<td>48.4</td>
<td>11.6</td>
<td>51.6</td>
</tr>
<tr>
<td>4</td>
<td>Eggborough Power Ltd</td>
<td>4</td>
<td>29.8</td>
<td>7.1</td>
<td>58.7</td>
</tr>
<tr>
<td>5</td>
<td>Keadby Generation Ltd</td>
<td>9</td>
<td>24.2</td>
<td>5.8</td>
<td>64.5</td>
</tr>
<tr>
<td>6</td>
<td>Barking Power Ltd</td>
<td>2</td>
<td>17.8</td>
<td>4.2</td>
<td>68.8</td>
</tr>
<tr>
<td>7</td>
<td>SSE Generation Ltd</td>
<td>4</td>
<td>15.2</td>
<td>3.6</td>
<td>72.5</td>
</tr>
<tr>
<td>8</td>
<td>Jade Power Generation Ltd</td>
<td>4</td>
<td>15</td>
<td>3.6</td>
<td>76.1</td>
</tr>
<tr>
<td>9</td>
<td>Centrica plc</td>
<td>8</td>
<td>14.7</td>
<td>3.5</td>
<td>79.6</td>
</tr>
<tr>
<td>10</td>
<td>Seabank Power Ltd</td>
<td>2</td>
<td>14</td>
<td>3.3</td>
<td>83</td>
</tr>
</tbody>
</table>

Inflation-adjusted revenue in millions of British pounds (base period is October 2011). There is information on 72 months in the data. The number of units owned is the maximum ever owned by that firm during the sample period.

**Market participants.** There are 130 BM units grouped into 61 power stations owned by 29 firms. The FR market is mildly concentrated with a ten-firm-concentration ratio of just over 80% and an HHI of 76.5. Table 1 summarizes revenue in the FR market for the ten largest firms over the first six years of the market’s existence.

The largest firm, Drax, had over 20% of the FR market and earned just £100,000,000 over the sample period, or about £1,400,000 per month. Drax is a single-station firm, while the next two largest firms, E.ON and RWE, are multi-station firms. Anecdotally, Drax’s disproportionate share is attributable to having a relatively new plant, with accurate governor controls, making it attractive for providing FR. The smallest firm, Seabank, still makes around £200,000 per month. This suggests that the FR market was big enough that firms may have been willing to devote an employee’s time to actively managing their bidding strategy, at least when the profitability of the market became apparent. In 2006 Drax indeed hired a trader to specifically deal with the FR market.\(^6\) Within a year, Drax’s revenue from the FR market increased more than threefold.

**Physical environment.** The FR market operates within a relatively stable environment. Starting with the demand for FR, the left panel of Figure 3 plots the monthly quantity of

\(^6\)Source: private discussion with Ian Foy, Head of Energy Management at Drax.
FR. Though this series is clearly volatile, it is no more volatile at the beginning than at the end of the period we study (and as we show below, the bids are). The right panel of Figure 3 shows some evidence of modest seasonality.

Figure 3: FR quantity by month (left panel) and by month-of-year (6 observations per month-of-year, right panel).

As further evidence of the stability of the demand for FR, the left panel of Figure 4 plots the daily total amount of electricity demand over the sample period. The response requirement NG is obligated to satisfy derives from electricity demanded and risk-response curves that did not change over the sample period. The right panel shows that electricity demanded is seasonal like the demand for FR, but that there are no obvious trends. Because electricity demanded is stable, the demand for FR is likely stable as well.

In addition to the mandatory frequency response (MFR) that is the focus of this paper, NG uses long-term contracts with BM units to procure FR services. This is known as firm frequency response (FFR). Figure 5 plots the monthly quantity of FFR and, for comparison purposes, that of MFR (see also the left panel of Figure 3). The quantity of FFR remains relatively stable over our sample period up until July 2010, when it almost doubles and thereafter remains stable at the new level. Nonetheless, the quantity of MFR remains relatively stable throughout the period.

Turning from the demand to the supply of FR, a BM unit can opt out of the FR market by submitting an unreasonably high bid. The left panel of Figure 6 plots the number of “active” power stations over time, where we define a station as active if one of its BM units submits a competitive bid of less than or equal to £23/MWh (see Appendix A for details).
Figure 4: Daily total amount of electricity demanded (left panel) and average monthly total amount by month-of-year (right panel).

The number of active stations fluctuates a bit, ranging from 53 to 61 over the sample period. However, the movements are relatively small and none of the stations who become active or inactive is particularly large: the right panel of Figure 6 shows that the share of stations that are always active is steady at around 95% until the last part of the sample period.

Finally, Figure 7 plots quarterly fuel prices paid by power stations in the UK over time. Fuel prices may matter for the FR market in that they change the “merit order” in the main market. For example, when gas is relatively expensive, gas-powered BM units may be part-loaded and therefore available for FR, whereas coal-powered BM units may be operating at full capacity and thus require repositioning in the BM in preparation for providing FR.

Up until the middle of 2009, the physical environment and demand and supply conditions are stable. After that date, FFR plays a larger role and the number of active BM units rises, as do oil and gas prices. Thus, any volatility in bids is unlikely to be caused by changes in demand or supply conditions, at least prior to 2009.

3 Evolution of the FR market

We divide the evolution of the FR market into three phases. To illustrate, we compute the monthly price of FR as the quantity-weighted average of bids. Figure 8 shows this series, with vertical lines separating the three phases. For comparison purposes, Figure 8 also shows the unweighted average of bids. During the early phase from November 2005 to February 2007,
the price exhibits a noticeable upward trend, moving from an initial price of £3.1/MWh to a final price of £7.2/MWh. The upward trend culminates in a “price bubble.” During the middle phase from March 2007 to May 2009, this trend reverses itself and the price falls back down to £4.8/MWh. From June 2009 to the end of our study period in October 2011 there is no obvious trend at all. While there are fluctuations during this late phase, they are smaller, and the price stays in the range of £4.3/MWh to £5.1/MWh. The sharper movements in one direction are relatively (to the prior periods) quickly corrected by movements in the opposite direction.

The movements in the price of FR in Figure 8 occurred despite the relative stability of the demand and supply conditions documented in section 2. The movements in the early and middle phases are too persistent to be driven by seasonality in the demand for FR. Although there are some upward trends in the number of active power stations as well as in the gas and oil prices, most of the action occurs towards the end of the sample period, when the price of FR has become quite stable. We therefore look for an alternative explanation for the changes in bidding behavior over time. In particular since none of the participants in this market had any experience with it, it seems unlikely that they had strong priors about either how their competitors would bid, or about how their allocation of FR would vary with their bid conditional on how their competitors would bid.

Figure 5: MFR and FFR quantities by month.
We begin with a summary of how the bidding behavior changed from one phase to the next. After providing the overview, we look more closely at the role of individual power stations.

**Early or rising-price phase (November 2005 – February 2007).** In the early or rising-price phase, firms change the bids of their BM units more often and by larger amounts (in absolute value) than in the middle and late phases. On average, the bids of 4 out of 10 BM units change each month by between £1/MWh and £3/MWh (conditional on changing). This is illustrated in Figures 9 and 10.

In addition to changing their bids more often and by larger amounts, firms tender very different bids in the early phase. Figure 11 shows that the range of bids as measured by the variance of bids across BM units is an order of magnitude larger than in the middle and late phases.

Comparing the left and right panels of Figure 12 shows that most of the variance stems from differences in bids between firms (within-firm variance, left panel) rather than from differences between BM units within firm (across-firm variance, right panel). The within-firm variance is highest in the early phase and then declines, suggesting that firms initially experimented by submitting different bids for their BM units, and that such experimentation became less prevalent over time.

Figure 13 shows the monthly bids of the eight largest power stations by revenue in the FR market. The top left panel provides a more detailed look at the early phase. In line with
the wide range of bids documented in Figure 11, the levels and trends of the bids are quite different across stations. Firms seem to experiment with different bids during the early phase of the FR market. Barking, Peterhead and Seabank bid very high early on — pricing themselves out of the market — and then drift back down into contention. The remaining stations start low and then gradually ramp up. The big increase in bids by Drax during late 2006 and early 2007 leads to the “price bubble” in Figure 8.

Middle or falling-price phase (March 2007 – May 2009). In the middle or falling-price phase, firms change the bids of their BM units less often and by smaller amounts (in absolute value) than in the early phase. As Figures 9 and 10 illustrate, on average, the bids of 3 out of 10 BM units change each month by around £1/MWh (conditional on changing). Figure 11 shows that the range of bids is much narrower than in the early phase.

The top right panel of Figure 13 provides more detail. The “price bubble” bursts when Seabank and Barking sharply decrease their bids and steal significant market share from Drax. Drax follows Seabank and Barking down, and this inaugurates intense competition and the noticeable downward trend in the price of FR in Figure 8. Experiments with increased bids are not successful. Drax, for example, increased its bid at the end of 2007 for exactly two months, giving its rivals an opportunity to see its increased bid and respond. When no
one followed suit, Drax decreased its bid. Toward the middle of 2008, Eggborough increased its bid. While this was followed by lesser increases by Drax and Connah’s Quay, Eggborough soon undercut Drax and Connah’s Quay and was subsequently followed down by them.

The dominant trend in the top right panel of Figure 13 is for the bids of the different power stations to move toward one another. The way this happens is that the stations that entered the middle phase with relatively high bids decreased their bids while the firms that entered the phase with relatively low bids maintained those bids. This intense competition generated the marked decrease in the range of bids in Figure 11.

**Late or stable-price phase (June 2009 – October 2011).** In the late or stable-price phase, firms change the bids for their BM units less often and by smaller amounts (in absolute value) than in the early and middle phase. As Figures 9 and 10 illustrate, on average, the bids of 3 out of 10 BM units change each month by around £0.5/MWh (conditional on changing). Figure 11 shows that the range of bids is again much narrower than in either of the earlier phases.

The bottom panel of Figure 13 provides more detail. While bids at some power stations continue to fall (Rats and Cottam), others are more erratic or rise (Drax and Eggborough),
Figure 9: Probability of a bid change between month $t$ and $t-1$. Weights are based in month $t-1$.

and others are almost completely flat (Peterhead). Overall, however, the bids of the different stations are noticeably closer to one another in this phase. By the time the FR market has entered its late phase, the impression prevails that it has reached a rest point that is, perhaps, periodically shocked by relatively small changes in the environment in which the FR market operates.

**Summary.** The early phase of the FR market was characterized by heterogeneous bidding behavior and frequent and sizeable adjustments of bids. During the middle and late phases, bids grew closer and the frequency and size of adjustments to bids fell.

At least to some extent, bidding behavior may reasonably be expected to be relatively volatile in the early or rising-price phase as firms had no prior experience of bidding in this market. We thus view the early phase as a period where those firms who thought they had a lot at stake experimented. This view is consistent with a comment by Ian Foy whose e-mail states: “The initial rush by market participants to test the waters having no history to rely upon; to some extent it was guess work, follow the price of others and try to figure out whether you have a competitive edge.” The experimentation appears to be quite complex. Different firms apparently pursued different strategies and some firms reacted to the past experience of rivals in addition to their own. As a result an appropriate model for this phase would require
Figure 10: Absolute value of bid change conditional on changing between month $t$ and $t - 1$. Weights are based on month $t - 1$ and are zero if the BM unit’s bid did not change.

We view the middle or falling-price phase as a period of firms learning about how best to maximize current profits. That is, we treat the middle phase as a period dominated by “exploitation” rather than experimentation. It is this phase which we consider in light of the available learning models in section 5.

Finally, we view the late or stable-price phase as the FR market having reached a rest point (possibly slightly perturbed by the increase in FFR, the number of active BM units, and the rise of oil and gas prices).

4 Demand and cost estimation

In this section we model and estimate the demand and cost primitives under relatively weak rationality assumptions. These primitives serve as an input to the learning models we use to better understand the data from the middle phase of the FR market in section 5.
Figure 11: Quantity-weighted and unweighted variance across BM units by month. Weights are based in month $t$.

4.1 Demand

We estimate a logit demand model at the BM unit-month level to approximate the market shares that are being generated by the proprietary linear program that NG solves in real time to satisfy its response requirement and equate supply and demand. We focus on the $J = 72$ BM units owned by the ten largest firms in Table 1. Together these “inside goods” account for just over 80% of revenue in the FR market. We treat the remaining BM units as parts of the “outside good.”

In addition to parsimoniously parameterizing own- and cross-price elasticities when there are many goods, an advantage of using a logit demand model for market shares is that it avoids having to model market size. As the right panel of Figure 3 shows, the monthly quantity of FR is seasonal. A disadvantage of using a logit demand model is that we have to explicitly deal with a BM unit that has a zero share in a month. To do so, we combine our logit demand model with a probit model that captures whether the BM unit is eligible for providing FR services.

\[\text{Due to non-competitive or missing bids, we subsume 10 of the 82 BM units into the outside good.}\]
Figure 12: Variance in bids within a firm (left panel) and across firms (right panel) over time. In the right panel, the volume-weighted variance across firms is the volume-weighted variance in the volume-weighted mean firm bids; and the unweighted variance is the unweighted variance across firms in the unweighted mean firm bids.

Model. Let $i$ index firms, $j$ BM units, and $t$ months. In month $t - 1$ firm $i$ submits a bid $b_{j,t}$ for BM unit $j$ in month $t$. Let $\mathcal{J}_i$ denote the indices of the BM units that are owned by firm $i$ and $b_{i,t} = (b_{j,t})_{j \in \mathcal{J}_i}$ the bids for these BM units. We adopt the usual convention to denote the bids for all BM units in month $t$ by $b_t = (b_{i,t}, b_{-i,t}) = (b_{j,t}, b_{-j,t})$.

Let $s_{j,t}$ denote the market share of BM unit $j$ in month $t$ and $s_{0,t} = 1 - \sum_j s_{j,t}$ the market share of the outside good. Let $e_{j,t} = 1(s_{j,t} > 0)$ be the indicator for BM unit $j$ being eligible for providing FR services — and thus having a positive market share — in month $t$. Accounting for eligibility, we specify a logit demand model for the market share of BM unit $j$ in month $t$ as

$$s_{j,t} = \frac{e_{j,t} \exp(\alpha \ln b_{j,t} + \beta x_{j,t} + \gamma_j + \mu_t + \xi_{j,t})}{1 + \sum_k e_{k,t} \exp(\alpha \ln b_{k,t} + \beta x_{k,t} + \gamma_k + \mu_t + \xi_{k,t})}$$

(1)

where $\gamma_j$ and $\mu_t$ are BM-unit and month fixed effects and $x_{j,t}$ and $\xi_{j,t}$ are observable and unobservable (to the econometrician) characteristics of BM unit $j$ in month $t$.

We choose to err on the side of flexibility in modeling demand by including a rich set of controls. The month fixed effect $\mu_t$ subsumes any time-varying characteristics of the outside good. The BM-unit fixed effect $\gamma_j$ captures the time-invariant preferences of NG for a BM unit due to, e.g., the precision of its governor controls and possibly also transmission constraints. In addition to its bid $b_{j,t}$, BM unit $j$ has time-varying characteristics $x_{j,t}$ and
Figure 13: Volume-weighted average bids of the largest power stations by month. November 2005 – February 2007 (top left panel), March 2007 – May 2009 (top right panel), and June 2009 – October 2011 (bottom panel). Stations ranked by revenue in the FR market during early and middle phases. Bids are censored above at £10/MWh to improve visual presentation.
\( \xi_{j,t} \) in month \( t \) that capture the main forces at work in the FR market. The observable characteristics \( x_{j,t} \) include two controls for the operating position of the BM unit, namely the fraction of the month the BM unit is fully loaded and the fraction of the month it is part-loaded. As discussed in section 2 NG uses long-term contracts to procure FFR services that may be a substitute for MFR services. To capture this \( x_{j,t} \) further includes a dummy for whether BM unit \( j \) is under contract with NG in month \( t \). Finally, we allow the unobservable characteristics \( \xi_{j,t} \) to follow an AR(1) process with

\[
\xi_{j,t} = \rho \xi_{j,t-1} + \nu_{j,t},
\]

where the innovation \( \nu_{j,t} \) is iid across BM units and months and mean independent of current and past bids \( b_{j,t} \) and observable characteristics \( x_{j,t} \). This setup allows a firm to condition its current bid on past unobservable (to the econometrician) characteristics but not on the current innovation, in line with the fact that the bid for the current month is submitted before the 20th of the previous month.

Our probit model for BM unit \( j \) being eligible for providing FR services in month \( t \) is

\[
e_{j,t} = 1(\tilde{\beta} x_{j,t} + \gamma_j + \tilde{\mu}_t + \eta_{j,t} > 0),
\]

where \( \eta_{j,t} \sim N(0,1) \) is a standard normally distributed disturbance that is iid across BM units and months. It follows that

\[
\Pr(e_{j,t} = 1|x_{j,t}) = 1 - \Phi \left( -\tilde{\beta} x_{j,t} - \gamma_j - \tilde{\mu}_t \right) = \Phi \left( \tilde{\beta} x_{j,t} + \gamma_j + \tilde{\mu}_t \right),
\]

where \( \gamma_j \) and \( \tilde{\mu}_t \) are BM-unit and month fixed effects, \( x_{j,t} \) are the same observable characteristics of BM unit \( j \) in month \( t \) as in equation (1), and \( \Phi(\cdot) \) is the standard normal cdf. Note that while the probability of having a positive market share differs across BM units and months, it is not affected by the bid itself. In the online appendix we include the log bid \( \ln b_{j,t} \) in a number of ways and show that although it is statistically significant, its impact in economic terms is small (in our preferred specification, a £1/MWh increase in bid decreases the probability of being eligible by -0.021 on a baseline of 0.75).

Equation (1) implies

\[
\ln s_{j,t} - \ln s_{0,t} \equiv \delta_{j,t} = \alpha \ln b_{j,t} + \beta x_{j,t} + \gamma_j + \mu_t + \xi_{j,t}
\]
as long as $e_{jt} = 1$. We can estimate equation (3) by OLS if $\rho = 0$ and $\nu_{j,t}$ is independent of $\eta_{j,t}$. We maintain the latter assumption of no selection on unobservables throughout. In the online appendix, we investigate selection in more detail. In a Mills ratio test (Heckman 1979) the coefficient on the inverse Mills ratio is statistically significant but the remaining coefficients are similar to our leading estimates in Table 3. Since they are so similar, we choose to proceed without accounting for the correlation between $\nu_{j,t}$ and $\eta_{j,t}$ to avoid making a joint normality assumption on these unobservables.

If $\rho > 0$, then OLS is biased to the extent that $\xi_{j,t}$ is correlated with $b_{j,t}$ or $x_{j,t}$. In particular, $b_{j,t}$ can reasonably be expected to be a function of $\xi_{j,t-1}$ which, in turn, is correlated with $\xi_{j,t}$ if $\rho > 0$. To deal with this, we quasi-first-difference equation (3) to obtain

$$
\delta_{j,t} - \rho \delta_{j,t-1} = \alpha (\ln b_{j,t} - \rho \ln b_{j,t-1}) + \beta (x_{j,t} - \rho x_{j,t-1}) + \gamma_j + \mu_t + \nu_{j,t},
$$

where $\gamma_j = (1 - \rho) \gamma_j$ and $\mu_t = \mu_t - \rho \mu_{t-1}$. As long as $e_{jt} = e_{jt-1} = 1$, we can estimate equation (4) by non-linear least squares (NLLS).

Data. Table 2 summarizes the data used in the estimation. Over the first six years of the operation of the FR market, we have 5175 observations at the BM unit-month level. Market shares are small with an average of 1%, although there is heterogeneity (in one month, a single BM unit got 13% share). In about 25% of observations, the market share is zero. Bids are £5.5/MWh on average. Some data on operating position is missing, and where it is, we dummy out for it by including a dummy for missing operating position in $x_{j,t}$ and interacting it with the controls for being fully and part loaded.

Results. The first column of Table 3 shows OLS estimates from equation (3) and the second column NLLS estimates from equation (4). The number of observations differs because we require $s_{j,t} > 0$ for OLS and $s_{j,t} > 0$ and $s_{j,t-1} > 0$ for NLLS.

The estimates are remarkably similar across specifications. The coefficient on log bid $\ln b_{j,t}$ is negative and significantly less than $-1$. Because market shares are small, it closely approxi-

---

8We ignore an “incidental parameter” problem. The “within” estimator generated by the BM-unit fixed effects $\tilde{\gamma}_j$ generates an error that depends on the mean of the quasi first difference in $\xi_{j,t}$ which, in turn, may be correlated with the quasi first difference in $\ln b_{j,t}$ which appears on the right hand side of equation (3). Following Nickell (1981), the bias in a linear (balanced) panel model is of the order $\rho/T$. We observe a BM unit for a median number of $T = 72$ months, so the bias is probably relatively small.
Table 2: Summary Statistics (top 10 firms only)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>0.0115</td>
<td>0.0160</td>
<td>0</td>
<td>0.131</td>
</tr>
<tr>
<td>Eligibility</td>
<td>0.752</td>
<td>0.432</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Bid</td>
<td>5.453</td>
<td>2.759</td>
<td>1.515</td>
<td>21.00</td>
</tr>
<tr>
<td>Fully loaded</td>
<td>0.133</td>
<td>0.236</td>
<td>0</td>
<td>0.997</td>
</tr>
<tr>
<td>Part loaded</td>
<td>0.551</td>
<td>0.373</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Missing operating position</td>
<td>0.115</td>
<td>0.319</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Positive FFR volume</td>
<td>0.00734</td>
<td>0.0854</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of observations</td>
<td>5175</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summary statistics on the frequency response market. An observation is a bmunit-month, and the sample is restricted to units owned by the top 10 biggest firms (ranked by revenue over the sample period). Fully loaded is the fraction of time the unit’s final physical notification is that it is fully loaded (i.e. operating at or close to capacity). Part loaded is the corresponding fraction when it is operating below capacity. FFR volume is the quantity of FR provided through firm frequency response contracts (i.e. outside of this market).

mates the price elasticity of demand. The coefficients on fully loaded and part-loaded in $x_{j,t}$ are positive and significant. This makes sense because a BM unit can provide FR only if it is currently operating. The coefficient on part-loaded is larger than that on fully loaded in line with our expectation that NG prefers to call upon a BM unit in the middle of its operating range. The coefficient on positive FFR volume in $x_{j,t}$ is negative and significant, indicating that a BM unit has a smaller share of the FR market if it is already under contract with NG to provides FFR services, also as expected. Finally, the NLLS estimates from equation (4) in the second column of Table 3 provide evidence of persistence in the unobservable characteristics $\xi_{j,t}$ as the $AR(1)$ coefficient $\rho$ is positive and significant.

To assess goodness of fit, we predict the market share of BM unit $j$ in month $t$ conditional on $s_{j,t} > 0$. To do so, we sample independently and uniformly from the empirical distribution of residuals $\hat{\xi}_{j,t}$ for the OLS specification in equation (3), and from the empirical distribution of innovations $\hat{\nu}_{j,t}$ for the NLLS specification in equation (4). The logit demand model fits the data reasonably well. Comparing realized and predicted market shares from equation (3) and equation (4), we get an $R^2$ of 0.49 and 0.66. This reinforces the importance of persistence.

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9In the latter case, we proceed as follows: Using the NLLS estimates, we obtain the residuals $\hat{\nu}_{j,t}$ from equation (4) and the residuals $\hat{\xi}_{j,t}$ from equation (3). We simulate $\xi_{j,t}$ by substituting $\hat{\xi}_{j,t-1}$ and a draw from the empirical distribution of residuals $\hat{\nu}_{j,t}$ into the law of motion $\xi_{j,t} = \rho \xi_{j,t-1} + \nu_{j,t}$. If BM unit $j$ has a zero share in month $t-1$, then we go back to the first month $\tau$ such that $s_{j,\tau} > 0$ and substitute into $\xi_{j,t} = \rho^{t-\tau} \xi_{j,\tau} + \nu_{j,t}$. 

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### Table 3: Demand System Estimates

<table>
<thead>
<tr>
<th></th>
<th>Market Share</th>
<th>Eligibility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>NLLS</td>
</tr>
<tr>
<td>Log bid</td>
<td>-1.648***</td>
<td>-1.614***</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>Fully loaded</td>
<td>1.666***</td>
<td>1.949***</td>
</tr>
<tr>
<td></td>
<td>(0.220)</td>
<td>(0.182)</td>
</tr>
<tr>
<td>Part loaded</td>
<td>2.111***</td>
<td>2.234***</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>Positive FFR volume</td>
<td>-0.794***</td>
<td>-0.587**</td>
</tr>
<tr>
<td></td>
<td>(0.200)</td>
<td>(0.245)</td>
</tr>
<tr>
<td>Unit and Month FE</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( \rho )</td>
<td>–</td>
<td>0.41</td>
</tr>
<tr>
<td>s.e. ( \rho )</td>
<td>–</td>
<td>0.03</td>
</tr>
<tr>
<td>( R^2 ) (in shares)</td>
<td>0.49</td>
<td>0.66</td>
</tr>
<tr>
<td>N</td>
<td>3831</td>
<td>3509</td>
</tr>
</tbody>
</table>

In the first two columns, the dependent variable is the log ratio of the share to the outside good share. In the last column it is an indicator for eligibility. The second market share specification allows for an AR(1) process in the error term, and we estimate the quasi-first-differenced equation by non-linear least squares (we provide an estimate of the autocorrelation coefficient \( \rho \) and the standard error of that estimate). The \( R^2 \) measure reported is for the fit of predicted versus actual shares (again omitting zero-share observations). Standard errors are clustered by bmunit. Significance levels are denoted by asterisks (* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)).

in the unobservable characteristics \( \xi_{j,t} \) and prompts us to take the NLLS estimates from equation (4) in the second column of Table 3 as our leading estimates.

Figure 14 shows that the fit is good even for the largest power stations, whose market shares change quite dramatically from one month to the next. This indicates that the good fit is not solely a consequence of having BM-unit fixed effects.\(^{10}\)

The third column of Table 3 show ML estimates from equation (2). They are in line with our logit demand model: In particular, the coefficients on fully loaded and part-loaded in \( x_{j,t} \) are positive and significant.

\(^{10}\)The fact that we see amplification in that realized shares move more than the predicted shares may be an indication that the firm knows \( \nu_{j,t} \), at least in part, so if it is favorable, it bids high and yet its share does not fall as much as our logit demand model predicts.
4.2 Cost

With demand estimated, we turn to estimating cost. As discussed in section 2, the main source of cost is the additional wear and tear that a BM unit incurs while in FR mode, which we expect to be relatively stable over time. A BM unit does not have to withhold generating capacity from the main market in order to participate in the FR market. Indeed, our data shows that BM units can — and do — contract out all of their capacity in the forward market while still actively participating in the FR market.\footnote{We thank Frank Wolak for pointing out to us that in many other countries a BM unit has to withhold generating capacity from the main market to participate in the FR market. Because of the resulting opportunity cost, the holding payment is an order of magnitude larger than in the UK.}

Let $c_j$ denote the constant marginal cost of BM unit $j$ for providing FR. The realized profit
of firm $i$ in month $t$ is

$$
\pi_{i,t} = \sum_{j \in J_i} (b_{j,t} - c_j) M_t s_j (b_t, x_t, \xi_t, e_t; \theta),
$$

(5)

where $M_t$ is market size in month $t$ and our notation emphasizes that the market share of BM unit $j$ in month $t$ depends on the bids $b_t$, characteristics $x_t$ and $\xi_t$ and eligibilities $e_t$ of all BM units as well as on the parameters $\theta$ of our logit demand model. In contrast, market size $M_t$ is independent of bids $b_t$ because the response requirement NG is obligated to satisfy is exogenously determined by government regulation.

To recover marginal cost $c_i = (c_j)_{j \in J_i}$ for the BM units that are owned by firm $i$ we invert the system of first-order conditions that describes the bidding behavior of the firm during the late or stable-price phase from June 2009 to October 2011. We assume that a firm “does its best” in the sense of choosing its bid to maximize its expected profits conditional on the information available to it. Let $\Omega_{i,t}$ denote the information of firm $i$ in month $t$. The following assumption makes the firm’s objective precise.

**Assumption 1 (static profit maximization)** Firm $i$ chooses its bids $b_{i,t}$ to maximize its perception of expected profit in month $t$ given its information $\Omega_{i,t}$:

$$
\max_{b_{i,t}} E_{b_{-i,t}, \xi_t, e_t, \theta} \left[ \sum_{j \in J_i} (b_{j,t} - c_j) M_t s_j (b_t, x_t, \xi_t, e_t; \theta) \bigg| \Omega_{i,t} \right].
$$

Note that an implication of assumption \[\text{I}\] is that firms do not expect current price to impact future profits. This rules out many standard models of experimentation and collusion. Moreover collusion is explicitly ruled out here, as firms maximize the joint (static) profit of only their BM units.

The notation in assumption \[\text{I}\] is designed to stress the two main sources of uncertainty that a firm faces, namely strategic uncertainty about its rivals’ bids $b_{-i,t}$ and demand uncertainty generated by the realizations of $\xi_t$ and $e_t$ and the fact that the demand parameters, $\theta$, may not be known. Using the information available to it, the firm forms perceptions (in the form of a subjective probability distribution) about $b_{-i,t}$, $\xi_t$, $e_t$, and $\theta$. These perceptions underlie the expectation operator $E_{b_{-i,t}, \xi_t, e_t, \theta} \left[ \cdot \bigg| \Omega_{i,t} \right]$ in Assumption \[\text{I}\]. How perceptions are formed is the central question for learning models that we turn to in section \[\text{S}\], but for now we remain agnostic. Finally, throughout the paper we make the simplifying assumption that the firm
has perfect foresight about market size $M_t$ and the characteristics $x_t$. Marginal costs are common knowledge as well.

Assumption 1 implies that the bids $b_{i,t}$ of firm $i$ in month $t$ solve the system of first-order conditions

$$
E_{b_{-i,t},\xi_t,e_t,\theta} \left[ s_k(b_t, x_t, \xi_t, e_t; \theta) + \sum_{j \in J_i} (b_{j,t} - c_j) \frac{\partial s_j(b_t, x_t, \xi_t, e_t; \theta)}{\partial b_{k,t}} \right] \bigg|_{\Omega_{i,t}} = 0, \quad k \in J_i. \quad (6)
$$

Assumption 1 and equation (6) do not suffice to recover marginal cost $c_i$, however, for the simple reason that we have not yet specified how agents form their perceptions, and any behavior can be rationalized by some perceptions.

We therefore restrict perceptions in a way that we view as appropriate for the late phase. By the time the FR market enters this phase, each firm has had ample opportunity to observe its rivals bid in the auction as well as the resulting allocation of market shares, and bidding behavior appears to have reached a rest point as discussed in section 3. Accordingly we assume that agents are “weakly rational” in the following sense:

**Assumption 2 (first-order conditions on average correct in late phase)** The bids $b_{i,t}$ of firm $i$ in month $t \in \{43, \ldots, 72\}$ during the late phase and its perceptions about $b_{-i,t}$, $\xi_t$, $e_t$, and $\theta$ satisfy the system of equations

$$\frac{1}{30} \sum_{t=43}^{72} \left[ s_k(b_t, x_t, \xi_t, e_t; \theta) + \sum_{j \in J_i} (b_{j,t} - c_j) \frac{\partial s_j(b_t, x_t, \xi_t, e_t; \theta)}{\partial b_{k,t}} \right] = 0, \quad k \in J_i.
$$

Assumption 2 is implied by rational expectations in the limit when the time horizon goes to infinity. In the limit it may also hold if agents’ perceptions converge to the objective probability distribution over time. Neither of these limiting assumptions is necessary though, and in that sense this is a weak rationality condition.

An advantage of assumption 2 is that it provides just as many equations as there are unknowns, namely the marginal cost $c_i = (c_j)_{j \in J_i}$ for the BM units that are owned by firm $i$. Because our logit demand model implies

$$\frac{\partial s_j(b_t, x_t, \xi_t, e_t; \theta)}{\partial b_{k,t}} = s_j(b_t, x_t, \xi_t, e_t; \theta) (1(k = j) - s_k(b_t, x_t, \xi_t, e_t; \theta)) \frac{\alpha}{b_{k,t}},$$

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Table 4: Cost estimates for the top 8 stations (by total revenue)

<table>
<thead>
<tr>
<th>Station</th>
<th># Units</th>
<th>Fuel</th>
<th>Vintage</th>
<th>Mean</th>
<th>Std. Dev. (within station)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barking</td>
<td>2</td>
<td>CCGT</td>
<td>1994</td>
<td>1.2</td>
<td>.01</td>
</tr>
<tr>
<td>Connah’s Quay</td>
<td>4</td>
<td>CCGT</td>
<td>1996</td>
<td>1.04</td>
<td>.03</td>
</tr>
<tr>
<td>Cottam</td>
<td>4</td>
<td>Coal</td>
<td>1969</td>
<td>1.35</td>
<td>.05</td>
</tr>
<tr>
<td>Drax</td>
<td>6</td>
<td>Coal</td>
<td>1974</td>
<td>1.05</td>
<td>.03</td>
</tr>
<tr>
<td>Eggborough</td>
<td>4</td>
<td>Coal</td>
<td>1968</td>
<td>1.53</td>
<td>.06</td>
</tr>
<tr>
<td>Peterhead</td>
<td>1</td>
<td>CCGT</td>
<td>2000</td>
<td>1.54</td>
<td>0</td>
</tr>
<tr>
<td>Ratcliffe</td>
<td>4</td>
<td>Coal</td>
<td>1968</td>
<td>1.33</td>
<td>.06</td>
</tr>
<tr>
<td>Seabank</td>
<td>2</td>
<td>CCGT</td>
<td>1998</td>
<td>1.6</td>
<td>.01</td>
</tr>
</tbody>
</table>

Summary statistics on the unit-specific cost estimates derived from solving the firm first order condition arising from the demand system, reported as the within-station average cost and standard deviation in costs.

where \( I(\cdot) \) is the indicator function, if we substitute in realized market size, market shares, and bids along with parameter estimates, we can solve the \(|J_i|\) equations in assumption 2 for the \(|J_i|\) unknowns \( c_i \). This is made especially easy by the fact that the equations are linear in the unknowns.

**Results: estimates.** The average of the 72 marginal costs that we estimate is £1.41/MWh, with a standard deviation across units of £0.66/MWh\(^{12}\). The cost estimates are reasonably precise, with an average standard error of 0.04. Table 4 shows the average marginal cost for the BM units belonging to the eight largest power stations. They are quite reasonable and vary between £1.04/MWh and £1.6/MWh for the eight largest stations. The standard deviation in estimated marginal costs within a station is very small (on the same order as the standard error), and so most of the variance in marginal costs is across power stations. By comparison, pre CAP047 the “cost reflective” administered price was around £1.7/MWh\(^{13}\). Since we expect some markup to be built into the administered price, the marginal cost we recover is in the right ballpark.

Table 5 shows the results from projecting our estimated costs \( c_j \) onto unit characteristics.

\(^{12}\)One BM unit has zero share during the late phase, and so we impute its marginal cost with that of the other BM unit in the same plant.

\(^{13}\)We have two sources: Figure 2 and a document prepared just prior to CAP047 by NG for Ofgem, the government regulator (www.ofgem.gov.uk/ofgem-publications/62273/8407-21104ngc.pdf). It states in paragraph 5.3 that the holding payment is “of the order of £5/MWh” for the bundle of primary, secondary, and high response, implying an average of £1.67/MWh per type of FR.
Table 5: Projecting costs onto unit characteristics

<table>
<thead>
<tr>
<th></th>
<th>Cost estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit vintage</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
</tr>
<tr>
<td>Dual Fuel</td>
<td>-0.822*</td>
</tr>
<tr>
<td></td>
<td>(0.467)</td>
</tr>
<tr>
<td>Large Coal</td>
<td>-0.464</td>
</tr>
<tr>
<td></td>
<td>(0.429)</td>
</tr>
<tr>
<td>Medium Coal</td>
<td>-0.683</td>
</tr>
<tr>
<td></td>
<td>(0.544)</td>
</tr>
<tr>
<td>Oil</td>
<td>-0.966**</td>
</tr>
<tr>
<td></td>
<td>(0.397)</td>
</tr>
</tbody>
</table>

$R^2$ 0.13
N 71

The dependent variable is the cost estimate $c_j$. The omitted fuel type is combined cycle gas turbines (CCGT). One observation is dropped because of missing vintage data. Significance levels are denoted by asterisks (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$).

As one might have expected the (typically smaller) Dual Fuel and Oil plants have lower costs than plants with other fuel types, while units of later vintage have lower costs (although the latter result is not statistically significant).

**Results: residuals.** Once we have the parameter estimates, we can evaluate the realized value of the profit derivative in assumption 2 at the parameter estimates for each BM unit and month. For simplicity, we call these values “residuals”. In the last phase, by construction, the average residual is zero, with a standard deviation of 0.001. By contrast, during the early phase, the average residual is positive (0.002) and more variable (standard deviation of 0.006), and in the middle phase it is generally negative (-0.0004) with intermediate variability (standard deviation 0.002). The declining volatility of the residuals over the time series is consistent with our earlier observations about convergence to a rest point.

Under rational expectations, the expected value of the residuals is zero at each point in time, though of course any particular realization may be non-zero. In addition, the residuals should not be autocorrelated since at any point in time the agent can condition on all past information in choosing their bid. For the same reason, the residuals should not be correlated with anything else in their information set.
Table 6: Autocorrelation in residuals

<table>
<thead>
<tr>
<th>Lagged residual</th>
<th>Early All</th>
<th>Middle</th>
<th>Late</th>
<th>Early Bid changes only</th>
<th>Middle</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.550***</td>
<td>0.335***</td>
<td>0.454***</td>
<td>0.397***</td>
<td>0.115***</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.051)</td>
<td>(0.065)</td>
<td>(0.064)</td>
<td>(0.038)</td>
<td>(0.080)</td>
</tr>
</tbody>
</table>

\[ R^2 \] | 0.63 | 0.48 | 0.21 | 0.72 | 0.39 | 0.08 |
\[ N \] | 1080 | 1931 | 2088 | 355 | 449 | 401 |

The dependent variable is the residual in the FOC at the estimated costs. Controls are the lagged residual and unit fixed effects. The regressions with bid changes only include only observations in which the unit’s bid was different from its bid in the previous period. Standard errors are clustered by unit. Significance levels are denoted by asterisks (* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \)).

Table 6 displays the coefficients from regressions of the residuals on their lagged values for different set of observations (in all cases unit fixed effects are included). We find significant evidence of autocorrelation in all regressions but the last, in which we restrict attention to the late phase of the data and to observations in which the unit’s bid changed between months. We believe the autocorrelation reflects persistent differences between the firms’ perceptions of their expected profits and those under the objective probability distribution. This makes sense in particular in the early and middle phases of the FR market where firms had little experience and behaved quite differently. In the late phase, one would expect little autocorrelation, if firms actively re-evaluated their bids each period, incorporating all available information. However the bottom panel of figure 13 suggests that there are long stretches in which bids do not change. This can generate autocorrelation (see column 2). This autocorrelation vanishes when we restrict attention to active bid choices (see column 4).

In table 6 we report results from regressing the residuals on lagged values of the variables in table 2 as well as controls for total market size and total FFR volume. It is hard to know what to make of the estimated coefficients, but it is striking how the \( R^2 \) falls over time, indicating that the available information explains less and less of the residuals.

Finally we look at the time series of residuals, contrasting the early and middle periods of the data with the late phase. The left panel of figure 15 shows the early and middle periods. The average residuals start well above zero before falling below zero in the middle period, while the standard deviation falls throughout (consistent with our earlier discussion of convergence). In the right panel, we document the residuals during the late phase of the
Table 7: Correlation of residuals and available information

<table>
<thead>
<tr>
<th></th>
<th>Early</th>
<th>Middle</th>
<th>Late</th>
<th>Early</th>
<th>Middle</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged share</td>
<td>119.958***</td>
<td>2.812</td>
<td>14.123***</td>
<td>130.471***</td>
<td>-6.656</td>
<td>-11.403</td>
</tr>
<tr>
<td></td>
<td>(39.846)</td>
<td>(7.026)</td>
<td>(3.820)</td>
<td>(47.468)</td>
<td>(12.539)</td>
<td>(13.008)</td>
</tr>
<tr>
<td>Lagged eligibility</td>
<td>-0.841</td>
<td>-0.141</td>
<td>-0.103**</td>
<td>-1.840**</td>
<td>-0.619**</td>
<td>-0.362</td>
</tr>
<tr>
<td></td>
<td>(0.560)</td>
<td>(0.113)</td>
<td>(0.050)</td>
<td>(0.861)</td>
<td>(0.306)</td>
<td>(0.229)</td>
</tr>
<tr>
<td>Lagged bid</td>
<td>-0.156**</td>
<td>-0.288**</td>
<td>-0.799***</td>
<td>-0.069</td>
<td>-0.129**</td>
<td>-0.156</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.114)</td>
<td>(0.287)</td>
<td>(0.073)</td>
<td>(0.053)</td>
<td>(0.183)</td>
</tr>
<tr>
<td>Lagged fullyloaded</td>
<td>3.480*</td>
<td>-0.558**</td>
<td>0.135</td>
<td>2.179</td>
<td>0.713</td>
<td>4.212***</td>
</tr>
<tr>
<td></td>
<td>(2.000)</td>
<td>(0.248)</td>
<td>(0.233)</td>
<td>(2.661)</td>
<td>(0.690)</td>
<td>(1.380)</td>
</tr>
<tr>
<td>Lagged partloaded</td>
<td>0.650</td>
<td>0.026</td>
<td>-0.125</td>
<td>2.004</td>
<td>-0.034</td>
<td>1.005*</td>
</tr>
<tr>
<td></td>
<td>(0.794)</td>
<td>(0.188)</td>
<td>(0.267)</td>
<td>(1.353)</td>
<td>(0.608)</td>
<td>(0.514)</td>
</tr>
<tr>
<td>Lagged missing</td>
<td>-1.476</td>
<td>-0.508***</td>
<td>0.057</td>
<td>-1.028</td>
<td>-0.753</td>
<td>0.569</td>
</tr>
<tr>
<td></td>
<td>(1.087)</td>
<td>(0.186)</td>
<td>(0.301)</td>
<td>(2.155)</td>
<td>(0.476)</td>
<td>(0.405)</td>
</tr>
<tr>
<td>Lagged total market size</td>
<td>-0.000***</td>
<td>-0.000</td>
<td>0.000*</td>
<td>0.000</td>
<td>-0.000***</td>
<td>0.000***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Lagged total FFR volume</td>
<td>-0.000***</td>
<td>-0.000</td>
<td>-0.000</td>
<td>-0.000**</td>
<td>-0.000**</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Lagged FFR contract</td>
<td>-1.538***</td>
<td>-0.388</td>
<td>-1.488***</td>
<td>-3.952***</td>
<td>0.41</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.471)</td>
<td>(0.306)</td>
<td>(0.471)</td>
<td>(0.915)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[R^2\] 0.51 0.43 0.11 0.61 0.41 0.26
\[N\] 1080 1931 2088 355 449 401

The dependent variable is the residual in the FOC at the estimated costs, multiplied by 1000. Unit fixed effects are included. Standard errors are clustered by unit. Significance levels are denoted by asterisks (* \(p < 0.1\), ** \(p < 0.05\), *** \(p < 0.01\)).

Data. As earlier shown in figure 5, the FFR volume substantially increases from July 2010 onwards, presumably perturbing MFR market. Yet we see that after this shock, the residuals continue to be close to zero on average, although there is a small and temporary increase in the cross-sectional standard deviation of the residuals. This is evidence that firms are able to quickly adjust to minor changes in the environment, in the sense that the realized profit derivative remains close to zero. This is a nice counterpoint to the rest of the paper, which argues that it takes quite a while for an entirely new market to reach a rest point.

Collusion. We briefly examine the possibility of collusion between the firms in our data. To begin with, we re-estimate the costs during the stable period using first-order condi-
Figure 15: Average and standard deviation of residuals during the early and middle phases (left panel) and late phase (right panel). In the right panel, the dotted line indicates July 2010, when FFR volume nearly doubles.

tions that reflect different collusive arrangements. If all top 10 firms were colluding and therefore maximized the combined profits of all their units, the implied average costs are £-9.8/MWh. If only the top 3 firms colluded, their implied average costs are £-0.26/MWh. We obtain negative cost estimates because demand is estimated to be relatively inelastic, and so rationalizing their bids in the face of increased market power requires them to have low costs.

We also look for coordination in the timing and direction of bid changes across firms, as this could be a sign of collusion being established or breaking down. To do this, we compute all pairwise correlations between BM units in a dummy variable for changing their bid; and (conditional on both units in the pair changing) a dummy variable for increasing their bid. We plot the distribution of correlation coefficients separately for units within a firm and for units that are owned by different firms. This gives 4 histograms: within/across firm cross change/direction. This is Figure 16.

The two left hand side pictures suggest that there substantial positive correlation in changes within the firm. This reinforces our contention that decisions are made at the firm rather than BM unit level. The two right hand side pictures show much less correlation, consistent with independent decision making across firms. It also suggests that the environment is relatively stable, as even without collusion we would expect positively correlated changes in bids in response to exogenous demand changes or cost shocks.

14We have repeated the exercise separately for each of our three phases, but find little difference across
Repositioning in the BM. One might worry that our first-order conditions do not reflect the full set of incentives firms face, as they do not account for the profit that accrues to the BM unit as it is repositioned in the BM in preparation for providing FR. In the online appendix we show how one can incorporate these incentives into our first-order condition. We provide a demand system (analogous to the one for the FR market) and estimate demand using additional data on repositioning operations. Extending assumption 2 to the new first-order condition, we simultaneously estimate marginal costs for providing FR and a markup on repositioning operations. The estimated markup parameter is very small and not statistically different from zero, and the marginal costs of providing FR do not change materially. The markup parameter in part reflects the amount of attention paid to profits from repositioning phases in the correlation histograms.
when deciding on the FR bid. Our small estimated markup may thus be a result of the fact that FR bids and bids in the BM are made by different people within the firm, and those making FR decisions may not pay attention to the BM market. This is consistent with our conversations with Ian Foy, who has told us that people in the industry do not think of repositioning incentives when making FR bids. See the online appendix for additional details.

5 Learning

[TO BE UPDATED]

6 Conclusion

We have documented what happened to the FR market following its deregulation in the UK. All our analysis suggests that the market converged to a rest point that appears to be a static Nash equilibrium, although this process took over 3 years. The opening period of the market seemed to be characterized by substantial uncertainty with firms taking different approaches. Some firms explored different bids with an eye to the value of the information they obtained from those experiments, while others did little at all. An actual model of this period is beyond the scope of this paper.

By contrast, the middle period is more amenable to analysis. We find that simple fictitious play models fit the data better than assuming firms play according to a full information Nash equilibrium. Assuming that firms are uncertain about the price elasticity and update over time as data comes improves the fit still further.

References


A Data appendix

Data sources. Since a redesign on November 1, 2013, the data website of NG is available at [http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmission-operational-data/Data-explorer/Outcome-Energy-Services/](http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmission-operational-data/Data-explorer/Outcome-Energy-Services/). The data on the FR market is available under the tab “Frequency Response “FFR and Mandatory.” We downloaded our data from a previous version of the NG data website. In those cases detailed below where the data is no longer available on the NG data website, it is available from the authors on request. NG used to publish Seven Year Statements detailing their projections of energy supply and demand and upcoming challenges. These used to be available at [http://www.nationalgrid.com/uk/Electricity/SYS/archive/](http://www.nationalgrid.com/uk/Electricity/SYS/archive/).

- **Bids:** We obtained FFR bid data directly from the NG data website. The relevant file is labeled “Prices.” Currently, a version is available that starts in January 2007 and is updated every month. From the old version of the data website, we downloaded one file for the period from November 2005 to January 2010, and another file for January 2007 to July 2013. These files contain monthly bids (in £/MWh) by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. The combined data period from the two files is November 2005 to July 2013.

- **Capabilities:** We obtained FR capabilities data directly from the NG data website. The relevant file is labeled “Capabilities.” Currently, a version is available that starts in January 2006 and is updated every month. From the old version of the data website, we downloaded one file for the period from November 2005 to January 2010, and another file for January 2006 to August 2013. The former file reports that November and December 2005 are not available, so only the latter file is relevant, since it contains all the data that is available. The file contains monthly response capabilities by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. For the market segment primary, response capabilities in MWh are given at 0.2 Hz, 0.5 Hz, and 0.8 Hz, while for the market segments secondary and high, only response capabilities at 0.2 Hz and 0.5 Hz are listed. In each case, the column on the right represents the maximum over the operating range. These values are constant over the sample period for more than 80% of the generators. The data period is January 2006 to August 2013.
• **Quantities**: We obtained FR quantity data directly from the NG data website. Unfortunately, the new data website no longer provides historic quantities, and only a file that holds quantities from August 2013 is available. We downloaded monthly quantity files for November 2005 thru June 2013. Each of these files contains one month of daily holding quantities in MWh by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. The combined data period of these monthly files is November 2005 to June 2013.

• **Main market position**: Elexon publishes all messages submitted to the Balancing Mechanism Reporting System on a given day at [http://www.bmreports.com/](http://www.bmreports.com/). An example for a daily file is [http://www.bmreports.com/tibcodata/tib_messages.2003-01-01.gz](http://www.bmreports.com/tibcodata/tib_messages.2003-01-01.gz). Each file collects the messages submitted as part of the balancing mechanism on a given day. These messages contain information on Final Physical Notification (FPN), Maximum Export Limit (MEL), Bid-Offer Data (BOD), or Bid-Offer Acceptance Level (BOAL) for typically a half-hour interval.

• **Electricity demanded**: We take information on electricity demanded from National Grid, at [http://www.nationalgrid.com/UK/Electricity/Data/Demand+Data/](http://www.nationalgrid.com/UK/Electricity/Data/Demand+Data/). The data is stored in a sequence of excel spreadsheets, each of which has the quantity demanded on a given day.

• **Firm frequency response (FFR)**: We obtain information on firm frequency response from the reports published at [http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmission-operational-data/Report-explorer/Services-Reports/](http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmission-operational-data/Report-explorer/Services-Reports/). The data is stored in a sequence of excel spreadsheets published monthly, each of which has FFR volumes by day.

• **Fuel type**: We take fuel type information from appendix F1 of the Seven-Year Statement prepared by NG in 2011: [http://nationalgrid.com/NR/rdonlyres/3B1B4AE4-236B-4B6E-8DB5-47211/NETSSYS2011AppendixF1.xls](http://nationalgrid.com/NR/rdonlyres/3B1B4AE4-236B-4B6E-8DB5-47211/NETSSYS2011AppendixF1.xls). The sheet “F-2,” corresponding to table F.2, provides fuel type for every BM unit listed under the column “Plant type.” For an additional eleven stations, we take information on fuel type from the Variable Pitch project: [http://www.variablepitch.co.uk/grid/](http://www.variablepitch.co.uk/grid/).

• **Fuel prices**: The UK Department of Energy and Climate Change publishes quarterly and annual prices of fuels purchased by power generators and of gas at UK delivery
points. A file titled “Average prices of fuels purchased by the major UK power producers and of gas at UK delivery points (QEP 3.2.1)” is available at https://www.gov.uk/government/statistical-data-sets/prices-of-fuels-purchased-by-major-power-producers. The sheet “Quarterly” contains the quarterly price of coal, oil, and gas, measured in pence per kWh, in columns D, F, and G.

- **Vintage**: We take fuel type information from appendix F1 of the Seven-Year Statement prepared by NG in 2011: http://nationalgrid.com/NR/rdonlyres/3B1B4AE4-2368-4B6E-8DA4-539A67EAD41F/47211/NETSSYS2011AppendixF1.xls. The sheet “F-2,” corresponding to table F.2, provides vintages for most BM units under the column “Commissioning Year.” The cell is empty for almost all hydro plants, so we take this information from the website of the British Hydropower Association: http://www.british-hydro.org/. For an additional eleven stations, we take this information from Wikipedia (5), from press releases prepared by the respective operator (5), and the website www.scottish-places.info (1). We are missing vintage for FAWN-1, which is connected with the Esso refinery in Fawley.

- **Ownership**: After registration on https://www.elexonportal.co.uk/ information on the registered party is contained in the file reg_bm_units.csv available under “Operational Data” → “Registration Information” → “Registered BM units” or under https://www.elexonportal.co.uk/REGISTEREDBMWUNITS. It is based on registration data at the Central Registration Agency and under “Party Name”, it lists the registered party. We downloaded a version of this file on December 29, 2009, and July 15, 2013, but there were no conflicts.

**Sample and variable construction.** The unit of observation is BM unit by month. We consider the time period November 2005 to October 2011. We include BM units in the analysis if they provided positive FR quantity in at least one of these months.

We aggregate quantities for the three market segments primary, secondary, and high (see footnote[1]) by summing daily quantities across segments and days. For BM unit $j$ in month $t$ we thus obtain FR quantity as

$$q_{j,t} = \sum_{k=P,S,H} \sum_{d \in M_t} q_{k,d,j,t},$$
where \( k \) indexes market segments, \( M_t \) is the set of days in month \( t \), and \( d \) indexes days. The FR bids are constructed as quantity-weighted averages of segment-specific bids, where the weights are constant and given by the overall quantities of the three segments over the sample period:

\[
b_{j,t} = \left( \sum_{k=P,S,H} Q_k b_{k,j,t} \right) / Q,
\]

where \( Q_k = \sum_j \sum_t \sum_{d \in M_t} q_{k,d,j,t} \) and \( Q = Q_P + Q_S + Q_H \).

Because a bid above £23/MWh is only accepted 12 times in our dataset of over 9000 observations, we label such a bid non-competitive; we otherwise label the bid competitive. One reason to opt out of the FR market by submitting a non-competitive bid is that the BM unit undergoes maintenance that month. Modelling maintenance and other reasons a BM unit opts out of the FR market is beyond the scope of this paper, and throughout the subsequent analysis we simply drop the corresponding observations. We also drop observations if the bid is missing.

Table 8 summarizes the main variables we use in the analysis.

## B Online appendix

### B.1 Selection

**Selection on observables.** To investigate selection on observables, we extend the probit model in equation (2) to include the log bid \( \ln b_{j,t} \):

\[
\Pr(e_{j,t} = 1|x_{j,t}) = 1 - \Phi \left( -\alpha \ln b_{j,t} - \beta x_{j,t} - \gamma_j - \mu_t \right) = \Phi \left( \alpha \ln b_{j,t} + \beta x_{j,t} + \gamma_j + \mu_t \right). \tag{7}
\]

Table 9 shows ML estimates. In the first column, we omit \( x_{j,t} \). In the second column, the independent variables are the same as in equation (1). In the last column, the bid enters more flexibly through a series of dummies for the bid being in each decile of the distribution of bids. The coefficient on bid is statistically significant in all specifications, including many of the decile coefficients in the flexible specification. However, as noted in the main text, the magnitude is small in all cases.
Table 8: Sources and Definitions of Variables used in the Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Sample</th>
<th>Source</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td>£/MWh</td>
<td>All</td>
<td>Bids</td>
<td>Monthly bid</td>
</tr>
<tr>
<td>Quantity</td>
<td>MWh</td>
<td>All</td>
<td>Quantities</td>
<td>Sum of segment quantities</td>
</tr>
<tr>
<td>Fully loaded</td>
<td>Fraction in [0, 1] &gt; 75% in merged_data</td>
<td>Main market position</td>
<td>Fraction of the month in which the unit is fully loaded (i.e. operating at or very close to their maximum export limit)</td>
<td></td>
</tr>
<tr>
<td>Part loaded</td>
<td>Fraction in [0, 1] &gt; 75% in merged_data</td>
<td>Main market position</td>
<td>Fraction of the month in which the unit is part loaded (i.e. neither off nor close to their maximum export limit)</td>
<td></td>
</tr>
<tr>
<td>Missing</td>
<td>Binary</td>
<td>All</td>
<td>Main market position</td>
<td>Indicator for missing main market position data</td>
</tr>
<tr>
<td>Positive FFR volume</td>
<td>Binary</td>
<td>All</td>
<td>FFR volumes</td>
<td>Indicates if a unit received positive FFR volume in that month</td>
</tr>
<tr>
<td>Fuel type</td>
<td>Categorical</td>
<td>All but one BM unit</td>
<td>Fuel type</td>
<td>Type of fuel (e.g., oil, large Coal, OCGT)</td>
</tr>
<tr>
<td>Fuel price</td>
<td>£/MWh</td>
<td>All</td>
<td>Fuel prices</td>
<td>Quarterly fuel price</td>
</tr>
<tr>
<td>Vintage</td>
<td>Year</td>
<td>All</td>
<td>Vintage</td>
<td>Commissioning year</td>
</tr>
<tr>
<td>Owner</td>
<td>Categorical</td>
<td>All</td>
<td>Ownership</td>
<td>Registered party</td>
</tr>
</tbody>
</table>
Selection on unobservables. To examine selection on unobservables, we revert to the probit model in equation (2). We allow for correlation between $\nu_{j,t}$ and $\eta_{j,t}$ (and hence $\xi_{j,t}$ and $\eta_{j,t}$) and assume that they are iid across BM units and months and jointly normal distributed as

$$\begin{pmatrix} \nu_{j,t} \\ \eta_{j,t} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \lambda \sigma \\ \lambda \sigma & 1 \end{pmatrix} \right).$$

It follows that

$$\mathbb{E}(\nu_{j,t}|e_{j,t} = e_{j,t-1} = 1, x_{j,t}) = \mathbb{E}(\nu_{j,t}|\eta_{j,t} > -\tilde{\beta}x_{j,t} - \tilde{\gamma}_j - \tilde{\mu}_t, \eta_{j,t-1} > -\tilde{\beta}x_{j,t-1} - \tilde{\gamma}_j - \tilde{\mu}_{t-1}, x_{j,t})$$

$$= \mathbb{E}(\nu_{j,t}|\eta_{j,t} > -\tilde{\beta}x_{j,t} - \tilde{\gamma}_j - \tilde{\mu}_t, x_{j,t})$$

$$= \lambda \sigma \frac{\phi \left( -\tilde{\beta}x_{j,t} - \tilde{\gamma}_j - \tilde{\mu}_t \right)}{1 - \Phi \left( -\tilde{\beta}x_{j,t} - \tilde{\gamma}_j - \tilde{\mu}_t \right)} = \lambda \sigma \frac{\phi \left( \tilde{\beta}x_{j,t} + \tilde{\gamma}_j + \tilde{\mu}_t \right)}{\Phi \left( \tilde{\beta}x_{j,t} + \tilde{\gamma}_j + \tilde{\mu}_t \right)},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf. Hence, $\mathbb{E}(\nu_{j,t}|e_{j,t} = e_{j,t-1} = 1, x_{j,t}) \neq 0$ as long as $\lambda \neq 0$ and there is correlation between the unobservables in the selection and market share equations.

Estimating equation (4) requires adding an inverse Mills ratio selection correction. Table 10 shows the resulting NLLS estimates. The coefficient on the inverse Mills ratio is significant but the remaining coefficients are very similar to our leading estimates in Table 3.
Table 9: Determinants of positive volume

<table>
<thead>
<tr>
<th></th>
<th>Indicator for positive share</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log bid</td>
<td>-0.302***</td>
<td>-0.526***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.165)</td>
<td>(0.203)</td>
<td></td>
</tr>
<tr>
<td>Fully loaded</td>
<td>2.604***</td>
<td>2.591***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.365)</td>
<td>(0.349)</td>
<td></td>
</tr>
<tr>
<td>Part loaded</td>
<td>2.277***</td>
<td>2.436***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.344)</td>
<td>(0.300)</td>
<td></td>
</tr>
<tr>
<td>Positive FFR volume</td>
<td>-0.581</td>
<td>-0.527</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.481)</td>
<td>(0.451)</td>
<td></td>
</tr>
<tr>
<td>Bid decile 2</td>
<td>-0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.360)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 3</td>
<td>0.442</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.314)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 4</td>
<td>-0.430</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.360)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 5</td>
<td>-0.699**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 6</td>
<td>-0.959***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.335)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 7</td>
<td>-0.729**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.356)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 8</td>
<td>-0.693**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.341)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 9</td>
<td>-0.443</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.317)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bid decile 10</td>
<td>-0.866***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.320)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                         | yes                          | yes | yes |
| Unit and Month FE       |                              |     |     |
| Flexible bid controls   | no                           | no  | yes |
| N                       | 5175                         | 5175 | 5175|

An observation is a unit-month, and the dependent variable is an indicator for a unit having positive volume. Inactive units are omitted. The regressors are the bid (either in logs, or with indicators for the bid being in decile bins), the fraction of time the unit’s final physical notification is that it is fully loaded (i.e. operating at capacity) and part loaded (i.e. operating below capacity); and whether the unit is under a firm frequency response contract. Standard errors are clustered by bmunit. Significance levels are denoted by asterisks (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$).
Table 10: Demand System Estimates

<table>
<thead>
<tr>
<th></th>
<th>Log share ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QFD</td>
</tr>
<tr>
<td>Log bid</td>
<td>-1.649***</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
</tr>
<tr>
<td>Fully loaded</td>
<td>1.580***</td>
</tr>
<tr>
<td></td>
<td>(0.226)</td>
</tr>
<tr>
<td>Part loaded</td>
<td>1.927***</td>
</tr>
<tr>
<td></td>
<td>(0.185)</td>
</tr>
<tr>
<td>Positive FFR volume</td>
<td>-0.573**</td>
</tr>
<tr>
<td></td>
<td>(0.246)</td>
</tr>
<tr>
<td>Mills ratio</td>
<td>-0.517***</td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
</tr>
<tr>
<td>Unit and Month FE</td>
<td>yes</td>
</tr>
<tr>
<td>ρ</td>
<td>0.40</td>
</tr>
<tr>
<td>s.e. ρ</td>
<td>0.03</td>
</tr>
<tr>
<td>N</td>
<td>3509</td>
</tr>
</tbody>
</table>

The dependent variable is the log ratio of the unit share to the outside good share (an observation is a unit-month), coded as missing where the share is zero and omitted in estimation. The specification allows for an AR(1) process in the error term, and we estimate the quasi-first-differenced equation by non-linear least squares (we provide an estimate of the autocorrelation coefficient ρ and the standard error of that estimate). Standard errors are clustered by bmunit. Significance levels are denoted by asterisks (* p < 0.1, ** p < 0.05, *** p < 0.01).
B.2 Repositioning in the BM

We account for the profit that accrues to a BM unit as it is repositioned in the BM in preparation for providing FR. The BM is a multi-unit discriminatory auction that is held every half-hour. Prior to this auction, a BM unit submits its contracted position to NG along with its bid. A bid in the BM is essentially a supply curve that is centered at the BM unit’s contracted position. This supply curve is described by price-quantity pairs through which the BM unit can offer to increase its energy production in up to five increments above its contracted position. If NG accepts an offer, the BM unit is paid by NG accordingly. The supply curve is further described by up to five price-quantity pairs through which the BM unit can bid to decrease its energy production below its contracted position. If NG accepts a bid, the BM unit pays NG accordingly.

The BM in other countries has been studied in great detail by Borenstein, Bushnell, and Wolak (2002), Wolak (2003), Wolak (2007), and Sweeting (2007). In line with our focus on the FR market, we work with a much simpler model of the BM that is designed to merely give us a sense of the profit that accrues to a BM unit as it is repositioned in the BM and how that profit changes with its bid for providing FR. We proceed in two steps. First, we estimate a demand model for repositioning. To account for the interdependency between the BM and the FR market, we include the bid for providing FR in the demand model. Second, to obtain profit, we estimate the markup in the BM jointly with the cost for providing FR.

Data. For every BM unit we have data on bids and offers (up to ten price-quantity pairs), contracted position, and actual position every half-hour. The quantity of upward repositioning $q^+_{j,\tau}$ of BM unit $j$ in half-hour $\tau$ effected through the BM is therefore the larger of zero and the difference between actual and contracted position; the quantity of downward repositioning $q^-_{j,\tau}$ is the larger of zero and the difference between contracted and actual position. Market size $M^+_{\tau} = \sum_j q^+_{j,\tau}$ and $M^-_{\tau} = \sum_j q^-_{j,\tau}$ is the total amount of upward, respectively, downward repositioning in half-hour $\tau$.

We face two problems with the data. First, if BM unit $j$ is not repositioned up or down in the BM in half-hour $\tau$, then $q^+_{j,\tau} = 0$, respectively, $q^-_{j,\tau} = 0$. This happens quite frequently, and we account for it in our demand model. Second, the bids and offers can take on extreme values. This sometimes happens even though the BM unit is repositioned so that $q^+_{j,\tau} > 0$ or $q^-_{j,\tau} > 0$. Hence, taken at face value, the bids and offers imply an implausibly huge profit.
We deal with this by directly estimating the markup rather than marginal cost in the BM.
The only place in which the offers are used in what follows is to construct a grid of 24 prices for upward repositioning as follows: Pooling across all BM units and half-hours, we consider the distribution of offers and take the 4th through 96th percentiles. We proceed analogously to fix a grid of 24 prices for downward repositioning.

**Demand.** As with the FR market, the “inside goods” are the $J = 72$ BM units owned by the ten largest firms in Table 1 and the “outside good” encompasses the remaining BM units. To simplify the exposition, we focus on the demand model for upward repositioning. The demand model for downward repositioning is analogous.

Let $s^+_j,\tau$ denote the market share of upward repositioning of BM unit $j$ in half-hour $\tau$ and $s^+_0,\tau = 1 - \sum_j s^+_j,\tau$ the market share of the outside good. Let $e^+_j,\tau = 1(s^+_j,\tau > 0)$ be the indicator for BM unit $j$ being eligible for repositioning in the BM — and thus having a positive market share — in half-hour $\tau$. Accounting for eligibility, we use a logit demand model for the market share of BM unit $j$ in half-hour $\tau$ with

$$s^+_j,\tau = \frac{e^+_j,\tau \exp\left(\alpha^+ \ln b_{j,t} + \beta^+ x^+_j,\tau + \gamma^+_j + \xi^+_j,\tau\right)}{1 + \sum_k e^+_k,\tau \exp\left(\alpha^+ \ln b_{k,t} + \beta^+ x^+_k,\tau + \gamma^+_k + \xi^+_k,\tau\right)}. \quad (8)$$

$\gamma^+_j$ is a BM-unit fixed effect. $b_{j,t}$ is the bid for providing FR of BM unit $j$ in the month $t$ to which half-hour $\tau$ belongs. $x^+_j,\tau$ are controls that parsimoniously represent the supply curves that the BM units bid in the BM. We include in $x^+_j,\tau$ the hypothetical market share of BM unit $j$ in half-hour $\tau$ at each of the 24 prices in the grid for upward repositing.\[15\] Finally, $\xi^+_j,\tau$ is a disturbance that, we assume, is mean independent of $b_{j,t}$ and $x^+_j,\tau$. This rules out that a firm conditions its bid in the BM on $\xi^+_j,\tau$.

We use a probit model for BM unit $j$ being eligible for repositioning in the BM in half-hour $\tau$ with

$$e^+_j,\tau = 1(\tilde{\alpha}^+ \ln b_{j,t} + \tilde{\beta}^+ \tilde{x}^+_j,\tau + \tilde{\gamma}^+_j + \eta^+_j,\tau > 0).$$

$\tilde{\gamma}^+_j$ is a BM-unit fixed effect. $b_{j,t}$ is the bid for providing FR of BM unit $j$ in the month $t$ to which half-hour $\tau$ belongs. $\tilde{x}^+_j,\tau$ contains additional half-hour-of-day (same for each day), day-

\[15\] From its supply curve we can infer a hypothetical quantity of upward repositioning for BM unit $j$ in half-hour $\tau$ at any given price. We compute the hypothetical market share of BM unit $j$ in half-hour $\tau$ from the hypothetical quantities of all BM units, irrespective of whether they are part of the inside or outside goods.
of-week (same for each week), week-of-year (same for each year), and year fixed effects and controls that parsimoniously represent the supply curves that the BM units bid in the BM. We include in $\tilde{x}_{j,\tau}^+$ the lowest offer of BM unit $j$ in half-hour $\tau$ along with the corresponding quantity. Next we compute the distribution of lowest offers of all BM units (irrespective of whether they are part of the inside or outside goods) in half-hour $\tau$. We include in $\tilde{x}_{j,\tau}^+$ ten dummies for the decile in which the lowest offer of BM unit $j$ in half-hour $\tau$ falls. We proceed similarly for the quantity corresponding to the lowest offer and include in $\tilde{x}_{j,\tau}^+$ another ten dummies for the decile in which the quantity corresponding to the lowest offer of BM unit $j$ in half-hour $\tau$ falls. Finally, $\eta_{j,\tau}^+ \sim N(0, 1)$ is a standard normally distributed disturbance that, we assume, is mean independent of $b_{j,t}$ and $\tilde{x}_{j,\tau}^+$ and independent across BM units and half-hours.

It follows that

$$\Pr(e_{j,\tau}^+ = 1|b_{j,t}, \tilde{x}_{j,\tau}^+) = 1 - \Phi \left( -\tilde{\alpha}^+ \ln b_{j,t} - \tilde{\beta}^+ \tilde{x}_{j,\tau}^+ - \tilde{\gamma}_j^+ \right) = \Phi \left( \tilde{\alpha}^+ \ln b_{j,t} + \tilde{\beta}^+ \tilde{x}_{j,\tau}^+ + \tilde{\gamma}_j^+ \right),$$

where $\Phi(\cdot)$ is the standard normal cdf. We estimate equation (9) by ML.

Equation (8) implies

$$\ln s_{j,\tau}^+ - \ln s_{0,\tau}^+ \equiv \delta_{j,\tau}^+ = \alpha^+ \ln b_{j,t} + \beta^+ \tilde{x}_{j,\tau}^+ + \gamma_j^+ + \xi_{j,\tau}^+$$

as long as $e_{j,\tau}^+ = 1$. We assume $\xi_{j,\tau}^+$ and $\eta_{j,\tau}^+$ are independent of each other and estimate by OLS on the subsample of observations with a positive share of upward repositioning.

**Results.** See Tables [12 and 13]. The coefficient on log FR bid is significantly different from zero and negative in both the logit and probit specifications, and also for both upward and downward repositionings. This indicates that units who submit attractive FR bids are more likely to be repositioned, presumably so that they can supply FR services. However the coefficients involved are reasonably small: for example, in the logit regressions, the elasticity of share with respect to FR bid is on the order of -0.1, compared to around -1.6 in the FR market.

**Markup and profit.** To simplify the exposition, we again focus on upward repositioning. Conditional on eligibility (or in realization), the market share of BM $j$ in half-hour $\tau$ is
Table 12: Repositioning Probit Analysis

<table>
<thead>
<tr>
<th>Probability of Repositioning</th>
<th>Upward repositions</th>
<th>Downward repositions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>logFRbid</td>
<td></td>
</tr>
<tr>
<td>Probability</td>
<td>-0.04833***</td>
<td>-0.03793***</td>
</tr>
<tr>
<td>logFRbid</td>
<td>-0.20608***</td>
<td>-0.09922***</td>
</tr>
</tbody>
</table>

N 1511737 1511737 1511734 1508599

Estimates from a 20% random sample of observations. In the first pair of regressions, the dependent variable is the indicator variable of whether a unit get repositioned upward; in the next two columns, it is the corresponding indicator for downward repositions. Controls for the bid/offer closest to the current contracted position, and dummies for the percentile of the bid and offer (relative to contemporaneous offers) are included in the second and fourth columns but suppressed. Month-of-year, day-of-week and hour-of-day dummies fixed effects are included in all specifications. Significance levels are denoted by asterisks (* p < 0.1, ** p < 0.05, *** p < 0.01).

Table 13: Repositioning Share Analysis

<table>
<thead>
<tr>
<th>Log relative share of repositioning (if positive)</th>
<th>Upward repositions</th>
<th>Downward repositions</th>
</tr>
</thead>
<tbody>
<tr>
<td>logFRbid</td>
<td>-0.086***</td>
<td>-0.108***</td>
</tr>
<tr>
<td></td>
<td>-0.076***</td>
<td>-0.100***</td>
</tr>
</tbody>
</table>

N 260482 260482 885659 885659

R² 0.57 0.58 0.53 0.54

In the first pair of regressions, the dependent variable is the log share ratio of upward repositioning volume; in the next two columns, it is the corresponding log share ratio of downward repositioning volume. Controls for the share of volume that a uniform auction would assign this unit based on its offers (upward) and bids (downward) at a set of 24 increasing prices are included, although their coefficients are omitted. Unit fixed effects are also included. Significance levels are denoted by asterisks (* p < 0.1, ** p < 0.05, *** p < 0.01), and standard errors are clustered by half-hour periods (an observation is a unit-half-hour).

\[ s^j_+ \left( b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+ \right), \] as defined on the right-hand side of equation (8). We use the shorthands \( x^+_\tau = \left( x^+_j \right)_{j=1,...,J_\tau}, \xi^+_\tau = \left( \xi^+_j \right)_{j=1,...,J}, \) and \( e^+_\tau = \left( e^+_j \right)_{j=1,...,J}, \) \( \theta^+ \) denotes the parameters of the logit demand model in equation (8). Unconditionally (or in expectation), the market share of BM \( j \) in half-hour \( \tau \) is

\[
s^j_+ \left( b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+ \right) = \sum_{e^+_\tau \in \{0,1\}^J} s^j_+ \left( b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+ \right) w^+ \left( b_t, x^+_\tau, e^+_\tau, \theta^+ \right) w^j_+ \left( b_t, x^+_\tau, e^+_\tau, \theta^+ \right),
\]

where

\[
w^+ \left( b_t, x^+_\tau, e^+_\tau, \theta^+ \right) \equiv \prod_{l=1,...,J} \Phi \left( \alpha^+ \ln b^t_l + \beta^+ \tilde{x}^+_l + \gamma^+_l \right)^{e^+_l} \left( 1 - \Phi \left( \alpha^+ \ln b^t_l + \beta^+ \tilde{x}^+_l + \gamma^+_l \right) \right)^{1-e^+_l}.
\]

(10)
and the summation is over all $2^J$ possible values of $\epsilon^+_i$. $\tilde{\theta}^+$ denotes the parameters of the probit model in equation (9).

We assume that the profit that accrues to BM unit $j$ as it is repositioned in the BM over the course of month $t$ (again unconditionally or in expectation) can be written as

$$\mu_j \sum_{\tau \in t} \left( M^+_\tau s^+_j(b_t, x^+_\tau, \xi^+_\tau, \bar{x}^+_\tau, \theta^+, \tilde{\theta}^+) + M^-_\tau s^-_j(b_t, x^-_\tau, \xi^-_\tau, \bar{x}^-_\tau; \theta^-, \tilde{\theta}^-) \right),$$

where we abuse notation to denote as $\tau \in t$ the half-hours in month $t$. $\mu_j$ is a common markup for upward and downward repositioning. If NG accepts an offer to increase energy production, then the BM unit is paid by NG according to its offer but bears the cost of the additional fuel. If NG accepts a bid to decrease energy production, then the BM unit pays NG according to its bid but saves on fuel cost. Because bids and offers are under the control of the firm owning the BM unit, we expect the markup to be nonnegative.

Recalling that $\mathcal{J}_i$ denotes the indices of the BM units that are owned by firm $i$, the profit of firm $i$ in the BM over the course of month $t$ (again unconditionally or in expectation) is

$$\sum_{j \in \mathcal{J}_i} \mu_j \sum_{\tau \in t} \left( M^+_\tau s^+_j(b_t, x^+_\tau, \xi^+_\tau, \bar{x}^+_\tau, \theta^+, \tilde{\theta}^+) + M^-_\tau s^-_j(b_t, x^-_\tau, \xi^-_\tau, \bar{x}^-_\tau; \theta^-, \tilde{\theta}^-) \right).$$

We are interested in how this profit changes with the bid for providing FR. Recall that the bid for the current month is submitted before the 20th of the previous month while bidding in the BM takes place during the current month. We simplify and assume that in preparing its bid for providing FR a firm ignores $\frac{\partial x^+_\tau}{\partial b_{j,t}}$, $\frac{\partial x^+_\tau}{\partial b_{j,t}}$, $\frac{\partial x^-_\tau}{\partial b_{j,t}}$, and $\frac{\partial x^-_\tau}{\partial b_{j,t}}$ for all $\tau \in t$. In essence, this says that the firm ignores that through its bid for providing FR it can influence the competitive landscape for the subsequent bidding in the BM. Under some conditions the envelope theorem ensures that this assumption is satisfied with respect to the bids and offers for the BM units that are owned by the firm. We emphasize, however, that this assumption has bite with respect to the bids and offers for the BM units that are owned by the firm’s rivals.
It remains to compute \( \frac{\partial s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \bar{x}^+_{\tau}; \theta^+, \bar{\theta}^+)}{\partial b_{j,t}} \) and \( \frac{\partial s_j^-(b_t, x^-_{\tau}, \xi^-_{\tau}, \bar{x}^-_{\tau}; \theta^-, \bar{\theta}^-)}{\partial b_{k,t}} \). We have

\[
\frac{\partial s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \bar{x}^+_{\tau}; \theta^+, \bar{\theta}^+)}{\partial b_{j,t}} = \sum_{\epsilon^+_{\tau} \in \{0, 1\}} \left( s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+)(1 - s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+)) \frac{\alpha^+}{b_{j,t}} + s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+) \frac{\tilde{\alpha}^+ \phi \left( \tilde{\alpha}^+ \ln b_{j,t} + \tilde{\beta}^+ \bar{x}^+_{j,\tau} + \tilde{\gamma}^+_{j} \right)}{b_{j,t} \left( \Phi \left( \tilde{\alpha}^+ \ln b_{j,t} + \tilde{\beta}^+ \bar{x}^+_{j,\tau} + \tilde{\gamma}^+_{j} \right) + e^+_{j,\tau} - 1 \right)} \right) w^+(b_t, \bar{x}^+_{\tau}, e^+_{\tau}; \bar{\theta}^+)
\]

for \( k = j \) and

\[
\frac{\partial s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \bar{x}^+_{\tau}; \theta^+, \bar{\theta}^+)}{\partial b_{k,t}} = \sum_{\epsilon^+_{\tau} \in \{0, 1\}} \left( -s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+) s_k^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+) \frac{\alpha^+}{b_{k,t}} + s_j^+(b_t, x^+_{\tau}, \xi^+_{\tau}, \epsilon^+_{\tau}; \theta^+) \frac{\tilde{\alpha}^+ \phi \left( \tilde{\alpha}^+ \ln b_{k,t} + \tilde{\beta}^+ \bar{x}^+_{k,\tau} + \tilde{\gamma}^+_{k} \right)}{b_{k,t} \left( \Phi \left( \tilde{\alpha}^+ \ln b_{k,t} + \tilde{\beta}^+ \bar{x}^+_{k,\tau} + \tilde{\gamma}^+_{k} \right) + e^+_{k,\tau} - 1 \right)} \right) w^+(b_t, \bar{x}^+_{\tau}, e^+_{\tau}; \bar{\theta}^+)
\]

for \( k \neq j \). Note that these derivatives are themselves expectations over eligibility \( e^+_{\tau} \) using probability weights \( w^+(b_t, \bar{x}^+_{\tau}, e^+_{\tau}; \bar{\theta}^+) \).

To jointly estimate the marginal cost for providing FR and the markup on repositioning operations, we extend assumption 2 as follows: The bids \( b_{i,t} \) of firm \( i \) in month \( t \in \{43, \ldots, 72\} \) during the late phase and its perceptions about \( b_{-i,t}, \xi_t, \xi^+_t = (\xi^+_{\tau})_{\tau \in t}, \xi^-_t = (\xi^-_{\tau})_{\tau \in t}, e_t, \)
\[ e_t^+ = (e^+_{\tau})_{\tau \in \ell}, e_t^- = (e^-_{\tau})_{\tau \in \ell}, \theta, \theta^+, \bar{\theta}^+, \bar{\theta}^-, \text{ and } \bar{\theta}^- \text{ satisfy the system of equations}^{16} \]

\[
\frac{1}{30} \sum_{t=43}^{72} \left[ M_t s_k(b_t, x_t, \xi_t, e_t; \theta) + \sum_{j \in J_t} (b_{j,t} - c_j) M_t s_j(b_t, x_t, \xi_t, e_t; \theta) (1(k = j) - s_k(b_t, x_t, \xi_t, e_t; \theta)) \right] \frac{\alpha}{b_{k,t}}
\]

\[
+ \sum_{j \in J_t} \sum_{\tau \in \ell} \left( M^+_t \left( s_j^+(b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+) (1(k = j) - s_k^+(b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+)) \right) \frac{\alpha^+}{b_{k,t}}
\]

\[
+ s_j^+(b_t, x^+_\tau, \xi^+_\tau, e^+_\tau, \theta^+) \frac{\bar{\alpha}^+ \phi \left( \bar{\alpha}^+ \ln b_{k,t} + \bar{\beta}^+ x^+_\tau + \bar{\gamma}^+_k \right) \Phi \left( \bar{\alpha}^+ \ln b_{k,t} + \bar{\beta}^+ x^+_\tau + \bar{\gamma}^+_k \right) + e^+_k - 1)}{b_{k,t}}
\]

\[
+ M^-_t \left( s_j^-(b_t, x^-_\tau, \xi^-_\tau, e^-_\tau, \theta^-) (1(k = j) - s_k^-(b_t, x^-_\tau, \xi^-_\tau, e^-_\tau, \theta^-)) \right) \frac{\alpha^-}{b_{k,t}}
\]

\[
+ s_j^-(b_t, x^-_\tau, \xi^-_\tau, e^-_\tau, \theta^-) \frac{\bar{\alpha}^- \phi \left( \bar{\alpha}^- \ln b_{k,t} + \bar{\beta}^- x^-_\tau + \bar{\gamma}^-_k \right) \Phi \left( \bar{\alpha}^- \ln b_{k,t} + \bar{\beta}^- x^-_\tau + \bar{\gamma}^-_k \right) + e^-_k - 1)}{b_{k,t}} \right) \otimes (1, f_{k,t-1}) = 0, \quad k \in J_t,
\]

where \( \otimes \) denotes the Kronecker product, 1 the constant, and \( f_{k,t-1} \) the fuel price relevant for BM unit \( k \) in month \( t - 1 \). As in the main text, we evaluate derivatives by substituting in realizations and parameter estimates.

These \( 2|J_t| \) equations not only require that the first-order conditions are on average correct in the late phase but also that they are uncorrelated with the lagged fuel price that is known to the firm at the time it prepares its current FR bid. To facilitate the estimation, we assume the markup is common across BM units and firms and solve the resulting overdetermined system of linear equations by OLS.

**Results.** Including the repositioning incentives has a relatively small impact on the estimated marginal costs: they fall from an average of 1.41 to 1.36. The estimated markup is not significantly different from zero.

---

\( ^{16} \)We make the simplifying assumption that the firm has perfect foresight about \( M_t^+ = (M^+_{\tau})_{\tau \in \ell}, M_t^- = (M^-_{\tau})_{\tau \in \ell}, x^+_t = (x^+_{\tau})_{\tau \in \ell}, \bar{x}^+_t = (\bar{x}^+_{\tau})_{\tau \in \ell}, x^-_t = (x^-_{\tau})_{\tau \in \ell}, \) and \( \bar{x}^-_t = (\bar{x}^-_{\tau})_{\tau \in \ell} \).
Table 14: Cost Estimates with Repositioning Incentives

<table>
<thead>
<tr>
<th></th>
<th>Without main market</th>
<th>With main market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average marginal cost</td>
<td>1.41</td>
<td>1.36</td>
</tr>
<tr>
<td>Main market markup</td>
<td>–</td>
<td>-0.0023</td>
</tr>
<tr>
<td>s.e. markup</td>
<td>–</td>
<td>0.0039</td>
</tr>
</tbody>
</table>

Cost and markup estimates, with and without accounting for main market incentives. Estimation is by generalized method of moments. The first column estimates are the same as those discussed in the main text.