

Price Caps, Oligopoly, and Entry *

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Abstract

We examine the impact of price caps in oligopoly markets with endogenous entry. In the case of deterministic demand and constant marginal cost, reducing a price cap yields increased total output, consumer welfare, and total welfare. This result falls in line with classic results on price caps in monopoly markets, and with recent results for oligopoly markets with a fixed number of firms. However, when demand is deterministic and marginal cost is increasing, a welfare improving price cap may not exist. We also show that a welfare-improving cap need not exist in the case where demand is stochastic. The fact that a welfare-improving cap cannot be guaranteed in these two cases points to a sharp difference in results between an endogenous entry model and oligopoly models with a fixed number of firms. Finally, we provide sufficient conditions that ensure the existence of a welfare-improving cap in the case of stochastic demand.

Keywords: Price Caps; Market Entry; Oligopoly; Demand Uncertainty

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1 Introduction

Price ceilings or caps are relevant in many areas including: wholesale electricity markets, interest on loans and credit, telecommunications services, taxi services, and housing in densely populated areas. For instance, regulators have imposed price caps in a number of U.S. regional wholesale electricity markets, including ERCOT (Texas), New England, and PJM. A key goal for price caps in wholesale electricity markets is to limit the exercise of market power. The principle that a price cap can limit market power is well understood in the case of a monopolist with constant marginal cost in a perfect-information environment. A price cap increases marginal revenue in those situations where it is binding and incentivizes the monopolist to increase output. Total output, consumer surplus, and total welfare are decreasing in the price cap.

Recent papers by Earle, Schmedders and Tatur [2007] and Grimm and Zottl [2010] examine the effectiveness of price caps in Cournot oligopolies with constant marginal cost. Earle et al. [2007] show that while the classic monopoly results for price caps carry over to oligopoly when demand is certain, these results do not hold under demand uncertainty. In particular, they demonstrate that when firms must make output decisions prior to the realization of demand, total output, welfare, and consumer surplus may be locally increasing in the price cap. This result raises into question the effectiveness of price caps as a welfare-enhancing policy tool. On the other hand, Grimm and Zottl [2010] demonstrate that, within the framework of Cournot oligopoly with uncertain demand analyzed by Earle, et al., there exists an interval of prices such that any price cap in this interval increases both total market output and welfare compared to the no-cap case. Thus, while the standard comparative statics results of price caps do not hold with uncertain demand, there always exists a welfare-improving price cap.

Importantly, prior analyses of oligopoly markets with price caps assume that the number of firms is held fixed. In this paper we explore an additional margin upon which a price cap may operate; firm entry decisions. We modify the analyses of Earle et al. [2007] and Grimm and Zottl [2010] by introducing an initial market entry period prior to a second period of product market competition. Market entry requires a firm to incur a sunk cost prior to period two product market competition. The inclusion of a sunk entry cost introduces economies of scale into the analysis. This would seem to be a natural formulation, since an oligopolistic market structure in a homogeneous product market may well be present because of economies of scale.¹

¹Cottle and Wallace [1983] consider a possible reduction in the number of firms in their analysis

Our model of endogenous entry builds on results and insights from Mankiw and Whinston [1986] and Amir and Lambson [2000]. Mankiw and Whinston show that when total output is increasing in the number of firms but per-firm output is decreasing in the number of firms (the term for the latter is the business-stealing effect), the socially optimal number of firms will be less than the free-entry number of firms when the number of firms, n , is continuous. For discrete n the free entry number of firms may be less than the socially optimal number of firms, but never by more than one. Intuitively, when a firm chooses to enter, it does not take into account decreases in per-firm output and profit of the other active firms. Thus, the social gain from entry may be less than the private gain to the firm. Amir and Lambson [2000] provide a taxonomy of the effects of entry on output in Cournot markets. In particular, they provide a very general condition under which equilibrium total output is increasing in the number of firms. Our analysis relies heavily on their approach and results. We demonstrate that Amir and Lambson's condition under which total output is increasing in the number of firms is also a sufficient condition for total output to be increasing in the number of firms when there is a price cap.

Our objective is to analyze the impact of price caps in oligopoly markets in which entry decisions are endogenous. Given the prominent use of price ceilings or caps as a regulatory tool in settings with multiple suppliers, we believe that an analysis that fails to consider their impact on market entry decisions may be missing a potentially vital component. We show that when entry is endogenous, demand is deterministic, and marginal cost is constant, the standard comparative statics results continue to hold. In this case, a price cap may result in fewer firms, but the incentive provided by the cap to increase output overwhelms the incentive to withhold output due to a decrease in competition. It follows that, regardless of the number of firms that enter the market, output will always increase as the cap is lowered. Welfare gains are realized on two fronts. First, the cap increases total output. Second, the cap may deter entry, and in doing so, reduce the total cost associated with entry.

We also consider the case of increasing marginal costs of production. Given our assumption of a sunk cost of entry, this case implies U-shaped average cost curves for firms. We show that when demand is deterministic and marginal cost is increasing, the standard comparative statics results do not hold when entry is endogenous. In contrast to results for a fixed number of firms, it may be the case that *any* price cap

of a price ceiling in a perfectly competitive market subject to demand uncertainty. Our interest is in the impact of price caps in oligopoly markets in which entry is endogenous.

reduces total output and welfare (i.e., there does not exist a welfare improving cap). Moreover, we demonstrate that if marginal cost is increasing sufficiently fast, it may be the case that the standard comparative statics results are reversed; that is, welfare and output are increasing in the price cap.

Finally, we demonstrate that a welfare-improving price cap may not exist when demand is uncertain and entry is endogenous (for constant marginal cost). Thus, the results of Grimm and Zottl do not generalize to the case of endogenous entry. We provide sufficient conditions for existence of a welfare-improving price cap. These conditions restrict the curvature of inverse demand, which in turn influences the extent of the business-stealing effect when an additional firm enters the market. We also consider a version of the model with disposal; firms do not have to sell the entire quantity they produced, but instead may choose the amount to sell after demand uncertainty has been resolved. We show that the sufficient condition for existence of a welfare improving price cap for the no-disposal model carries over to the model with disposal.

2 The Model

We assume there is an arbitrarily large number, $N \in \mathbb{N}$, of symmetric potential market entrants. We formulate a two period game. The N potential entrants are ordered in a queue and make sequential entry decisions in period one. Each firm's entry decision is assumed to be observed by the other firms. We assume that there is a cost of entry $K > 0$ which is sunk if a firm enters. If a firm does not enter it receives a payoff of zero.

Each of the n market entrants produces a homogeneous good in period two. Each firm has a convex production cost function $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. Output decisions are made simultaneously. The inverse demand function is given by $P(Q, \theta)$ which depends on total output, Q , and a random variable, θ . θ is continuously distributed according to CDF F with corresponding density f . The support of θ is bounded and given by $\Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. Each firm knows the distribution of θ but must make its output decision prior to its realization. We assume that a regulator may impose a price cap, denoted \bar{p} . The following assumption is in effect throughout the paper.

Assumption 1.

(a) P is continuous in Q and θ , strictly decreasing in Q for fixed θ , and strictly increasing in θ for fixed Q .

(b) $\lim_{Q \rightarrow \infty} [P(Q, \bar{\theta}) - C'(Q)] < 0$

(c) $\max_{Q \in \mathbb{R}_+} E[QP(Q, \theta) - C(Q)] > K$

Assumption (1a) matches the assumptions imposed by Earle et al. [2007]; Grimm and Zottl [2010] additionally assume differentiability of inverse demand in Q and θ . Assumption (1b) insures that a profit-maximizing quantity exists for period two decisions.

Earle et al. [2007] assumed that $E[P(0, \theta)] > c$ (they assume that marginal cost is a constant, c). Their assumption ensured that “production is gainful”; that is, given a fixed number, $n > 0$, of market participants, there exist price caps such that equilibrium market output will be strictly positive. Our assumption (1c) is a “profitable entry” condition which guarantees that there exist price caps such that at least one firm enters the market and that equilibrium output will be strictly positive. We let \mathbb{P} denote the set of price caps which induce at least one market entrant. That is

$$\mathbb{P} \equiv \left\{ \bar{p} > 0 \mid \max_{Q \in \mathbb{R}_+} E [Q \min\{P(Q, \theta), \bar{p}\} - C(Q)] \geq K \right\}$$

In this paper we are only concerned with price caps $\bar{p} \in \mathbb{P}$. In the analysis that follows, we restrict attention to subgame-perfect pure strategy equilibria and focus on period two subgame equilibria that are symmetric with respect to the set of market entrants.

For a given price cap and a fixed number of firms, there may exist multiple period two subgame equilibria. As is common in the oligopoly literature we focus on extremal equilibria - the equilibria with the smallest and largest total output levels - and comparisons between extremal equilibria. So when there is a change in the price cap we compare equilibrium outcomes before and after the change, taking into account the change (if any) in the equilibrium number of firms, while supposing that subgame equilibria involve either maximal output or minimal output.

One other point to note. Imposing a price cap may require demand rationning. When rationning occurs, we assume rationning is efficient; i.e., buyers with the lowest WTPs do not receive output. This is consistent with prior analyses of oligopoly with price caps.

We denote by $Q_n^*(\bar{p})$, period two subgame extremal equilibrium total output² when n firms enter and the price cap is \bar{p} . We let $q_n^*(\bar{p})$ be per-firm output in this equilibrium and let $\pi_n^*(\bar{p})$ denote each firm's expected period two profit in this equilibrium. We let $Q_n^\infty = Q_n^*(\infty)$ be the period two equilibrium total output when n firms enter with no price cap and let q_n^∞ denote the corresponding per-firm output. Let π_n^∞ denote each firm's expected period two profit in this equilibrium. Firms are risk neutral and make output decisions to maximize expected profit. That is, each firm i takes the total output of its rivals, y , as given and chooses q to maximize

$$\pi(q, y, \bar{p}) = E[q \min\{P(q + y, \theta), \bar{p}\} - C(q)]$$

After being placed in the queue, firms have an incentive to enter as long as their expected period two equilibrium profit is at least as large as the cost of entry. We assume that firms whose expected second stage profits are exactly equal to the cost of entry will choose to enter. As mentioned above, we restrict attention to subgame-perfect equilibria. For a fixed price cap, \bar{p} , subgame perfection in the entry stage (along with the indifference assumption) implies that the equilibrium number of firms, n^* , is the largest positive integer less than (or equal to) N such that $\pi_{n^*}^*(\bar{p}) \geq K$. Clearly, n^* exists and is unique. Moreover, for any $\bar{p} \in \mathbb{P}$ we also have $1 \leq n^* \leq N$, due to Assumption (1c).

For convenience and clarity, we will define $Q^*(\bar{p}) \equiv Q_{n^*}^*(\bar{p})$, $q^*(\bar{p}) \equiv q_{n^*}^*(\bar{p})$, and $\pi^*(\bar{p}) \equiv \pi_{n^*}^*(\bar{p})$. Similarly, we define $Q^\infty = Q_{n^*}^\infty$, $q^\infty = q_{n^*}^\infty$, and $\pi^\infty = \pi_{n^*}^\infty$.

3 Deterministic Demand

We begin our analysis by considering a deterministic inverse demand function. That is, the distribution of θ places unit mass at some particular $\tilde{\theta} \in \Theta$. In this section, we suppress the second argument in the inverse demand function and simply write $P(Q)$.

3.1 Constant Marginal Cost

Consider the case in which marginal cost is constant; $C(x) = cx$, where $c \geq 0$. For a given number, $n \in \mathbb{N}$, of market participants Earle et al. [2007] prove the existence of

²We do not introduce notation to distinguish between maximal and minimal equilibrium output. In most cases our arguments and results are identical for equilibria with maximal and minimal total outputs. We will indicate where arguments and/or results differ for the two types of equilibrium.

a period two equilibrium that is symmetric for the n firms. Our main result in this section demonstrates that the classic results on price caps continue to hold when entry is endogenous. We first state three lemmas that are used in the proof of this result; all proofs are in the Appendix.

Lemma 3.1. *For fixed \bar{p} , extremal subgame equilibrium total output, $Q_n^*(\bar{p})$ is non-decreasing in the number of firms, n . Moreover, extremal subgame equilibrium profit $\pi_n^*(\bar{p})$ is non-increasing in the number of firms, n .*

Lemma 3.2. *For fixed n , extremal subgame equilibrium profit $\pi_n^*(\bar{p})$ is non-decreasing in the price cap \bar{p} .*

Lemma 3.3. *In equilibrium the number of firms is non-decreasing in the price cap, \bar{p} .*

Proposition 1. *Restrict attention to $\bar{p} \in \mathbb{P}$. Then in equilibrium, total output, total welfare, and consumer surplus are non-increasing in the price cap.*

Proposition 1 is similar to Theorem 1 in Earle, et al.³ However, our model takes into account the effects of price caps on firm entry decisions. This is an important consideration, given that Lemma 3.1 ensures that for a fixed price cap, total equilibrium output is non-decreasing in the number of firms. This fact, along with the fact that a lower price cap may deter entry, suggest that a reduction in the cap could have the effect of hindering competition and reducing total output. Our result shows that with constant marginal cost and non-stochastic demand, even if entry is reduced, the incentive for increased production with a cap dominates the possible reduction in output due to less entry. There are two sources of welfare gains. First, total output is decreasing in the price cap, so a lower price cap yields either constant or reduced deadweight loss. Second, a lower price cap may reduce the number of firms, and thereby decrease the total sunk costs of entry.

Assumption 1 allows for a very general demand function, and because of this, there may be multiple equilibria. Proposition 1 provides results for extremal equilibria of period two subgames for cases with multiple equilibria. With an additional restriction on the class of demand functions the equilibrium is unique and we achieve a stronger result on the impact of changes in the price cap.

³As suggested by the discussion in Section 2, an equilibrium referred to in Lemma 3.3 and Proposition 1 involves period two subgames in which firms play either maximal or minimal subgame equilibrium output.

Proposition 2. *Suppose P is log-concave in output. Then for any $\bar{p} \in \mathbb{P}$ there exists a unique symmetric subgame equilibrium in the period 2 Cournot game. Moreover, equilibrium output, welfare, and consumer surplus are strictly decreasing in the cap for all $\bar{p} < P(Q^\infty)$ and $\bar{p} \in \mathbb{P}$.*

The intuition behind Proposition 2 is straightforward. When inverse demand is log-concave, there is a unique symmetric period two subgame equilibrium for each n and \bar{p} . If \bar{p} is less than the equilibrium price when there is no cap then \bar{p} must be binding in the subgame equilibrium. In addition, the subgame equilibrium price when there is no cap is non-decreasing in n , from Amir and Lambson [2000]. So any price cap below the no-cap free-entry equilibrium price is strictly binding in equilibrium, since the number of firms that enter will be no greater than the number of firms that enter in the absence of a cap. A reduction in the cap will therefore yield strictly greater total output.

A consequence of our results for constant marginal cost is that the welfare-maximizing price cap is the lowest cap which induces exactly one firm to enter. Imposing such a cap both increases output and reduces entry costs. Since marginal cost is constant, the total cost of producing a fixed total output does not depend on the number of firms that enter. By contrast, if marginal cost is increasing, the total production cost associated with a given level of output is decreasing in the number of firms. Thus, a price cap which increases output and reduces entry may result in a significant increase in production costs. Finally, note that with constant marginal cost, a shortage will never arise in equilibrium. This is driven solely by the fact that $\bar{p} > c$ for any level of production. However, if marginal cost is strictly increasing in output, then firms will produce no more than the quantity at which their marginal cost is equal to the price cap, and this could result in a shortage in the market. In the next sub-section we examine the impact of price caps in an environment in which firms have an increasing marginal cost of production.

3.2 Increasing Marginal Cost

In this section, we assume that the cost function, $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, is twice continuously differentiable with $C'(x) > 0$ and $C''(x) > 0$ for all $x \in \mathbb{R}_+$. While our focus in this paper is on models with endogenous entry, we begin this section with results for games with a fixed number of firms, since there do not appear to be results of this type in the

literature for increasing marginal costs.⁴ This will provide a benchmark against which our results for endogenous entry may be compared.

Our first result below demonstrates that when the number of firms is fixed, there exists a range of caps under which extremal equilibrium output and associated welfare are monotonically non-increasing in the cap. This range of caps consists of all price caps above the n -firm competitive equilibrium price. Intuitively, price caps above this threshold are high enough that marginal cost in equilibrium is strictly below the price cap for each firm. A slight decrease in the price cap means the incentive to increase output created by a lower cap outweighs the fact that marginal cost has increased (since the cap still lies above marginal cost).

Proposition 3. *Suppose that, in addition to Assumption 1, P is differentiable in Q . Fix $n \in \mathbb{N}$ and define the n -firm competitive price as the unique price satisfying $\hat{p} = P(nC'^{-1}(\hat{p}))$. Then for extremal equilibria:*

- (i) *Total output is non-increasing in the price cap for all $\bar{p} > \hat{p}$.*
- (ii) *Welfare is non-increasing in the price cap for all $\bar{p} > \hat{p}$.*

A price cap equal to \hat{p} maximizes welfare.

The proof that total output and welfare are non-increasing in the price cap for the case of constant marginal cost is based on showing that payoff function $\pi(q, y, \bar{p})$ satisfies the single crossing property in $(q, -\bar{p})$ for all y (Lemma 2 in Earle, et al.). The SCP property does not carry over to the general case of convex production cost. Our proof of Proposition 3 establishes that SCP in $(q, -\bar{p})$ holds for a modified game with a restricted choice set, and then shows that an output level is a symmetric equilibrium of the modified game if and only if it is a symmetric equilibrium of the original game.

In contrast to the result above with a fixed number of firms, we now provide an example which demonstrates that when entry is endogenous, a welfare-improving price cap may fail to exist.

⁴Neither Earle et al. [2007] nor Grimm and Zottl [2010] devote significant attention to the issue of increasing marginal cost. Both papers state that their main results for stochastic demand hold for increasing marginal cost as well as for constant marginal cost. Neither paper addresses whether the classical monotonicity results hold for a fixed number of firms, deterministic demand, and increasing marginal cost.

Example 1. Consider the inverse demand, cost function and entry cost given below:

$$P(Q) = 17 - Q, \quad C(q) = \frac{4}{3}q^2 \quad \text{and} \quad K = 21$$

In the absence of a price cap, it may easily be verified that two firms enter with each producing 3 units of output. This results in an equilibrium price of 11 and equilibrium period two profit equal to the entry cost, 21. Equilibrium welfare with no cap is therefore given by:

$$W^{nc} = \int_0^6 (17 - t) dt - 2 \left(\frac{4}{3}3^2 + 21 \right) = 18$$

Any price cap less than the equilibrium price of 11 results in at most one entrant. In order to ensure entry of at least one firm, the price cap must be greater than or equal to minimum of average total cost (ATC^m) ≈ 10.58 . So, any price cap $\bar{p} \in [ATC^m, 11)$ results in the entry of exactly one firm. It is straightforward to show that all such price caps require rationing of buyers and yield strictly lower total output, total welfare and consumer surplus than the no-cap equilibrium. A welfare-improving price cap does not exist for this example. In fact, total output and welfare are *increasing* in the price cap for $\bar{p} \in [ATC^m, 11)$. A welfare improvement could be achieved by a policy that combines an entry subsidy - to encourage entry - with a price cap - to incentivize increased output.

A key feature of Example 1 is that marginal cost rises sharply enough with output so that the output at which MC equals the price cap is less than the quantity demanded at the cap. So by reducing the number of firms from two to one, imposition of a price cap also results in a discrete reduction in total output. Note that this effect is absent when marginal cost is constant.

Proposition 4 below provides general conditions under which price caps yield outcomes like those illustrated by Example 1; that is, equilibrium output and welfare are monotonically increasing in a price cap. Prior to stating the Proposition, we state and prove two lemmas used in its proof. In what follows, we will let $p^\infty = P(Q^\infty)$.

Lemma 3.4. *Fix $\bar{p} \in \mathbb{P}$. Then extremal equilibrium total output is non-decreasing in the number of firms. Moreover, extremal equilibrium profit is non-increasing in the number of firms.*

Lemma 3.5. *Suppose, $n^\infty \geq 2$ and $\pi^\infty = K$. Consider a price cap $\bar{p} \in [ATC^m, p^\infty)$. Then $1 \leq n^* < n^\infty$.*

Proposition 4. *Suppose $\pi^\infty = K$. If $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$, then for all $\bar{p} \in [ATC^m, p^\infty)$:*

- (i) *The equilibrium number of firms is $n^\infty - 1$*
- (ii) *Total output is strictly increasing in \bar{p}*
- (iii) *Total output under the price cap is strictly lower than in the absence of a cap*
- (iv) *Welfare is strictly increasing in \bar{p}*

Proposition 4 is based on two conditions. The first is that firms earn zero economic profit in the free entry equilibrium with no cap. This implies that when a binding price cap is imposed, at least one less firm will enter (using Lemmas 3.4 and 3.5). The second is that if total output in the free entry equilibrium in the no-cap case is equally divided among one less firm then marginal cost exceeds the equilibrium price in the no-cap case. Clearly this condition is satisfied only if marginal cost rises relatively rapidly as output increases. The introduction of a price cap in this case must reduce total output since no firm will produce output in excess of the quantity q where $C'(q) = \bar{p}$. The result is demand rationing and a market shortage. This shortage creates deadweight loss which grows as the price cap decreases.

Still, a comparison of welfare before-and-after the introduction of the price cap is ambiguous in general, as the introduction of a cap may decrease the number of firms. Example One satisfies the hypotheses of Proposition 4 and illustrates a case in which any binding price cap reduces welfare. On the other hand, one can construct examples which satisfy the hypotheses of Proposition 4 and for which a welfare-improving price cap exists. Under the hypotheses of Proposition 4 there are three competing forces which affect welfare. First, a decrease in the number of firms results in welfare gains associated with entry-cost savings. Second, as described above, the introduction of a price cap creates deadweight loss and decreases welfare. Third, the combination of a decrease in output and a decrease in the number of firms has an ambiguous impact on firms' production costs. The effect on welfare depends on the interplay of these three forces.

4 Stochastic Demand

We now investigate the impact of price caps when demand is stochastic. We assume that marginal cost is constant and given by $c \geq 0$. For the fixed n model Grimm and Zottl [2010] demonstrate that under a generic distribution of demand uncertainty, there exists a range of price caps which strictly increases output and welfare as compared to the case with no cap. Their result is driven by the following observation. Fix an extremal symmetric equilibrium of the game with n firms and no price cap. Let $\bar{p}^\infty = P(Q^\infty, \bar{\theta})$ denote the lowest price cap that does not affect prices; i.e., \bar{p}^∞ is the maximum price in the no-cap equilibrium. And let \overline{MR}^∞ be a firm's maximum marginal revenue in this equilibrium. If firms choose their equilibrium outputs and a cap is set between \overline{MR}^∞ and \bar{p}^∞ then the cap will bind for an interval of high demand shocks; for these shocks marginal revenue will exceed what marginal revenue would have been in the absence of a cap, and for other shocks marginal revenue is unchanged. Firms therefore have an incentive to increase output relative to the no cap case for caps between \overline{MR}^∞ and \bar{p}^∞ .⁵ Earle et al. [2007] provide a quite different result for price caps when demand is stochastic. They show that raising a price cap can increase both total output and welfare. This is a comparative static result, in contrast to Grimm and Zottl's result on the existence of welfare improving price caps.

We begin this section by providing an example which demonstrates that a welfare improving price cap may not exist when entry is endogenous.

Example 2. Consider the following inverse demand, costs and distribution for θ :

$$P(Q, \theta) = \theta + \exp(-Q), \quad K = \exp(-2), \quad c = 1, \quad \theta \in \{0, 10\}, \quad Prob[\theta = 10] = 0.1$$

With no cap, each firm has a dominant strategy in the period 2 subgame to choose an output of 1. This leads to 2 market entrants; each earning second period profit exactly equal to the cost of entry. This results in total output of 2 units, total welfare of approximately 0.59, and $\bar{p}^\infty = 10 + \exp(-2)$. Consider a price cap slightly less than \bar{p}^∞ . Clearly such a cap will deter at least one firm from entering as the profit constraint is binding in the absence of a cap. Indeed it may be verified that one firm enters in equilibrium, and per-firm output increases (slightly) to approximately 1.04.

⁵When there are multiple equilibria of the game with no cap, the argument of Grimm and Zottl [2010] is tied to a particular equilibrium. It is possible that there is no price cap that would increase output and welfare across multiple equilibria.

The impact of the price cap is to reduce total output from 2 to 1.04, and to reduce total welfare from 0.59 to 0.51. It may be verified for this example that any price cap $\bar{p} \in (c, \bar{p}^\infty)$ will deter entry, reduce total output, and reduce total welfare.

Example 2 demonstrates that when demand is stochastic and entry is endogenous, there need not exist a welfare improving price cap. There are two important features of this example. First, the particular inverse demand and marginal cost selected imply that firms have a dominant strategy to choose an output of exactly one unit. Hence, the business-stealing effect is absent and total output increases linearly in the number of firms.⁶ Thus, a price cap can lead to lower total output by reducing the number of market entrants, as long as the cap doesn't stimulate too much output per firm. This is where the second feature comes in. As explained in Earle et al. (p.95), when demand is uncertain firms maximize a weighted average of profit when the cap is non-binding (low demand realizations) and profit when the cap is binding (high demand realizations). These two scenarios provide conflicting incentives for firms. The first effect is that a higher price cap creates an incentive to expand output as the benefits of increasing quantity increase when the cap is binding (and are not affected when the cap is not binding). The second effect is that a higher price cap may reduce the probability that the cap will bind, and this reduces the incentive to increase quantity. For Example 2, when the price cap is set slightly below \bar{p}^∞ output per firm increases slightly compared to the duopoly no-cap equilibrium; this increase in output per firm is much smaller than the reduction in total output due to fewer market entrants. Further reductions in the price cap in the interval (c, \bar{p}^∞) result in monotonic reductions in output as the first effect outlined above dominates the second effect for the distribution of demand shocks in this example.⁷

Example 2 suggests that a zero or weak business stealing effect is one source of failure of existence of welfare improving price caps. The Proposition below provides sufficient conditions on demand that ensure the existence of a welfare-maximizing cap. These conditions ensure that the business stealing effect is relatively strong, so that reduced entry does not have a large effect on total output. Assumption 2 is in effect for the remainder of the paper.⁸

⁶The same result holds for similar examples with a small business-stealing effect.

⁷The probability that the price cap is binding is constant at 0.1 for $\bar{p} \in (c, \bar{p}^\infty)$. So the second effect is zero for this example. This is an artifact of our use of a two-point distribution coupled with a low value for the low demand shock. However, analogous results are possible with a continuous distribution, provided that the second effect is smaller than the first.

⁸Part (d) may be viewed as a normalization of the support of θ ; it is used only for the free disposal

Assumption 2.

- (a) $f(\theta)$ is positive and continuous for all $\theta \in \Theta$
- (b) P is additively separable in Q and θ with: $P(Q, \theta) = \theta + p(Q)$
- (c) $p(\cdot)$ is twice continuously differentiable with $p' > 0$ and $p'' \leq 0$
- (d) $\underline{\theta} + p(0) = 0$

Lemma 4.1. *For fixed $\bar{p} \in \mathbb{P}$, extremal subgame equilibrium total output, $Q_n^*(\bar{p})$ is non-decreasing in the number of firms, n , and extremal equilibrium profit $\pi_n^*(\bar{p})$ is non-increasing in the number of firms, n .*

Proposition 5. *There exists a price cap which strictly increases equilibrium welfare.*

Assumption 2 provides sufficient conditions for existence of a welfare improving price cap. This assumption places strong restrictions on the form of inverse demand, but no restrictions other than a positive and continuous density on the form of demand uncertainty. The business-stealing effect is relatively strong when demand is concave in output. As demonstrated in Mankiw and Whinston [1986], when the business-stealing effect is present and n is continuous, the free-entry number of firms entering a market exceeds the socially optimal number of firms.⁹ The proof of Proposition 5 first establishes that, when the entry constraint is not binding in the absence of a cap, then there is an interval of prices such that a price cap chosen from this interval will yield the same number of firms, but higher total output and welfare. This follows directly from Theorem 1 in Grimm and Zottl [2010]. The proof proceeds to show that when the entry constraint is binding (i.e., zero equilibrium profit in the endogenous entry game) in the absence of a cap, then a reduction in the number of firms by one will increase total welfare. Indeed, it may be shown that when the entry constraint is binding (for $n \geq 2$) and there is no price cap, the socially optimal number of firms is strictly less than the free-entry number of firms. The imposition of a price cap in this case has two welfare-enhancing effects. First, the cap deters entry, and entry deterrence (by exactly one firm) is welfare enhancing. Second, the cap increases total output and welfare

case. This assumption was made in Grimm and Zottl [2006]. Lemma 4.1 is used in the proof of Proposition 5.

⁹Their result is for a model in which n is a continuous variable, whereas our analysis restricts n to whole numbers. Nonetheless, we are able to apply the intuition on excess entry to prove our result.

relative to what output and welfare *would* be in the new entry scenario (i.e., with one less firm) in the absence of a cap.¹⁰

4.1 Free Disposal

We now examine a variation of the game examined in the previous sections. This model is a three period game. In the first period, firms sequentially decide whether to enter or not (again, with each firm's entry decision observed by the next). Entry entails a sunk cost $K > 0$. In the second period, before θ is realized, firms that entered simultaneously choose capacities, with x_i designating the capacity choice of firm, i ; capacity is built at constant marginal cost $c > 0$. In the third period, firms observe θ and simultaneously choose outputs, with firm i choosing output $q_i \in [0, x_i]$; output is produced at zero cost.¹¹ Versions of this model with a fixed number of firms have been analyzed by Earle et al. [2007] and Grimm and Zottl [2010].

This model with free disposal may be interpreted as one in which the firms that have entered make long run capacity investment decisions prior to observing the level of demand, and then make output decisions after observing demand. Under this interpretation, c is the marginal cost of capacity investment, and the marginal cost of output is constant and normalized to zero. As with the proof of Proposition 5, we will demonstrate that in the absence of a price cap, if the free-entry constraint binds (ie $\pi_n^\infty = K$), then welfare is strictly higher if entry is reduced by one.

Before stating the main result for free disposal we introduce some of the key expressions. In the absence of a price cap, in the third period each firm solves:

$$\max_{q_i} (\theta + P(q_i + y))q_i - cx_i \text{ such that } q_i \leq x_i$$

Let n denote the number of entrants in the first period in the absence of a price cap. Under Assumption 2, there exists a unique symmetric subgame equilibrium level of capacity for any $n \in \mathbb{N}$ (see Grimm and Zottl [2006]). Denote by $X_n > 0$ the total

¹⁰The assumption of additively separable demand shocks is important for the second effect. It implies that the maximum marginal revenue in symmetric subgame equilibria is invariant to the number of firms. So if n is the equilibrium number of firms with no cap, maximum marginal revenue in a subgame with $n - 1$ firms and no cap is less than the maximum equilibrium price in a subgame with n firms and no cap (\bar{p}^∞). This means that a price cap between maximum marginal revenue and \bar{p}^∞ will both reduce the number of entrants *and* induce the firms that enter to produce more output than they would in the absence of a cap.

¹¹In the version of the model examined by Earle et al. [2007], output is produced at marginal cost δ which may be positive or negative. Our results continue to hold in this case.

equilibrium capacity and let $x_n = \frac{X_n}{n}$ denote the equilibrium capacity per firm. Let $Q_n(\theta)$ denote the total equilibrium output in the third period. Let $\tilde{\theta}(X_n)$ satisfy:

$$\tilde{\theta}(X_n) + p(X_n) + p'(X_n)x_n = 0$$

$\tilde{\theta}(X_n)$ is the lowest value of the demand shock at which the price cap binds in equilibrium, given total capacity X_n . Assumption (2d) above ensures that $\tilde{\theta}(X_n) > \underline{\theta}$ and Assumptions (2a) and (2d) ensure that $\tilde{\theta}(X_n)$ is single-valued. Then for $\theta \in [\underline{\theta}, \tilde{\theta}(X_n))$, it holds that no firm is constrained in equilibrium. In the unconstrained case, our assumptions above are sufficient to guarantee that a unique equilibrium exists in the third period. let $\tilde{Q}_n(\theta)$ denote the total equilibrium output in this case and let $\tilde{q}_n(\theta) = \frac{\tilde{Q}_n(\theta)}{n}$ denote the equilibrium per-firm output. Note that for each $\theta \in [\underline{\theta}, \tilde{\theta}(X_n))$, $\tilde{Q}_n(\theta)$ is given by the first-order condition:

$$\theta + p(\tilde{Q}_n(\theta)) + p'(\tilde{Q}_n(\theta))\tilde{q}_n(\theta) = 0$$

So, total equilibrium output in the third stage is given by:

$$Q_n^*(\theta) = \begin{cases} \tilde{Q}_n(\theta), & \underline{\theta} \leq \theta < \tilde{\theta}(X_n) \\ X_n, & \tilde{\theta}(X_n) \leq \theta \leq \bar{\theta} \end{cases}$$

We may write equilibrium profit per firm as:

$$\pi_n^* = \int_{\underline{\theta}}^{\tilde{\theta}(X_n)} \tilde{q}_n(\theta)(\theta + p(\tilde{Q}_n(\theta))) dF(\theta) + \int_{\tilde{\theta}(X_n)}^{\bar{\theta}} x_n(\theta + p(X_n)) dF(\theta) - cx_n$$

Grimm and Zottl [2006] prove that equilibrium capacities satisfy the following condition:

$$\int_{\tilde{\theta}(X_n)}^{\bar{\theta}} [\theta + p(X_n) + x_n p'(X_n)] dF(\theta) = c$$

This can be interpreted as saying that the expected marginal revenue for capacity is equal to the marginal cost of capacity in a symmetric equilibrium. Since $c > 0$ and F contains no mass points, it must be that $\tilde{\theta}(X_n) < \bar{\theta}$.

Lemma 4.2. *Let $m, n \in \mathbb{N}$ and let X_m and X_n denote total equilibrium output when*

m and n firms enter, respectively. Then $\tilde{\theta}(X_m) = \tilde{\theta}(X_n)$.

Lemma 4.3. Fix $n \geq 2$. Then for all $\theta \in \Theta$

(i) $Q_n^*(\theta) > Q_{n-1}^*(\theta)$

(ii) $q_n^*(\theta) < q_{n-1}^*(\theta)$

(iii) $\pi_n^* < \pi_{n-1}^*$

Proposition 6. The statement of Proposition 5 remains valid in the version of the model with disposal.

5 Conclusion

This paper has analyzed the welfare impact of price caps, taking into account the possibility that a price cap may reduce the number of firms that enter a market. The vehicle for the analysis is a two period oligopoly model in which product market competition in quantity choices follows endogenous entry with a sunk cost of entry. First, we analyzed the impact of price caps when there is no uncertainty about demand when firms make their output decisions. For this case, we showed that when marginal cost is constant, the standard comparative statics results remain true. That is, output, welfare, and consumer surplus all increase as the price cap is lowered. We then showed that when demand is known but marginal cost is strictly increasing, it may be the case that welfare is lower under any price cap than in the absence of a cap. We also demonstrated that if marginal cost increases sufficiently fast, welfare and output may be monotonically increasing in the price cap.

Next we analyzed the impact of price caps when demand is stochastic and firms must make output decisions prior to the realization of demand. We showed that, once again the existence of a welfare-improving price cap cannot be guaranteed. We then provided sufficient conditions on demand under which a range of welfare-improving price caps will always exist. The sufficient conditions restrict the curvature of the inverse demand function, which in turn influences the welfare impact of entry. We also extended this result to an environment with free disposal. This type of environment can be viewed as one in which there is endogenous entry, capacity investment decisions are made prior to observing demand, and output decisions are made after observing demand.

Appendix

Proof of Lemma 3.1

By Assumption (1c) there exists some $M > 0$ such that a firm's best response is bounded by M . As in Amir and Lambson [2000], we can express a firm's problem as choosing total output, Q , given total rivals' output, y . Define a payoff function,

$$\tilde{\pi}(Q, y, \bar{p}) = (Q - y)[\min\{P(Q), \bar{p}\} - c], \quad (1)$$

and a lattice,

$$\Phi \equiv \{(Q, y) : 0 \leq y \leq (n - 1)M, y \leq Q \leq y + M\}. \quad (2)$$

First we show that $\tilde{\pi}$ has increasing differences (ID) in (Q, y) on the lattice Φ . Consider $Q_1 \geq Q_2$ and $y_1 \geq y_2$ such that the points $(Q_1, y_1), (Q_1, y_2), (Q_2, y_1), (Q_2, y_2)$ are all in Φ . Since $y_1 \geq y_2$ and $P(Q_2) \geq P(Q_1)$, we have,

$$(y_2 - y_1) \min\{P(Q_1), \bar{p}\} \geq (y_2 - y_1) \min\{P(Q_2), \bar{p}\}. \quad (3)$$

Add $(Q_1 - Q_1) \min\{P(Q_1), \bar{p}\} = 0$ and $(Q_2 - Q_2) \min\{P(Q_2), \bar{p}\} = 0$ to the left and right hand sides of (3), respectively, to yield,

$$\begin{aligned} & ((Q_1 - y_1) \min\{P(Q_1), \bar{p}\} - (Q_1 - y_2) \min\{P(Q_1), \bar{p}\}) \geq \\ & (Q_2 - y_1) \min\{P(Q_2), \bar{p}\} - (Q_2 - y_2) \min\{P(Q_2), \bar{p}\}. \end{aligned} \quad (4)$$

Subtracting $c(y_1 - y_2)$ from both sides of (4) yields,

$$\tilde{\pi}(Q_1, y_1, \bar{p}) - \tilde{\pi}(Q_1, y_2, \bar{p}) \geq \tilde{\pi}(Q_2, y_1, \bar{p}) - \tilde{\pi}(Q_2, y_2, \bar{p}),$$

which is equivalent to,

$$\tilde{\pi}(Q_1, y_1, \bar{p}) - \tilde{\pi}(Q_2, y_1, \bar{p}) \geq \tilde{\pi}(Q_1, y_2, \bar{p}) - \tilde{\pi}(Q_2, y_2, \bar{p}). \quad (5)$$

Inequality (5) establishes that $\tilde{\pi}$ has increasing differences in (Q, y) on Φ .

Note that the choice set Φ is ascending in y and $\tilde{\pi}$ is continuous in Q and satisfies ID in (Q, y) . Then as shown in Topkis [1978], it follows that the maximal and minimal selections of $\operatorname{argmax}\{(Q - y)[\min\{P(Q), \bar{p}\} - c] : y \leq Q \leq y + M\}$ are nondecreasing

in y . The remainder of the proof follows almost directly from the proofs of Theorems 2.1 and 2.2 in Amir and Lambson [2000]. A symmetric equilibrium exists for the subgame, via the same argument employed in Amir and Lambson [2000]. However, asymmetric subgame equilibria may exist for our formulation, since the price function, $\min\{P(Q), \bar{p}\}$, is not strictly decreasing in Q . For symmetric equilibria, extremal total output is non-decreasing in n and extremal profit per firm is non-increasing in n .

Proof of Lemma 3.2

Fix $n \in \mathbb{N}$. Let $\bar{p}_1 > \bar{p}_2$ and let $q_n^*(\bar{p}_i)$ and $Q_n^*(\bar{p}_i)$ be minimal extremal equilibrium output per firm and total output, respectively, in the subgame with n firms and cap \bar{p}_i . Then,

$$\begin{aligned} \pi_n^*(\bar{p}_1) &= q_n^*(\bar{p}_1)(\min\{P(Q_n^*(\bar{p}_1)), \bar{p}_1\} - c) \\ &\geq q_n^*(\bar{p}_2)(\min\{P(q_n^*(\bar{p}_2) + (n-1)q_n^*(\bar{p}_1)), \bar{p}_1\} - c) \\ &\geq q_n^*(\bar{p}_2)(\min\{P(q_n^*(\bar{p}_2) + (n-1)q_n^*(\bar{p}_1)), \bar{p}_2\} - c) \\ &\geq q_n^*(\bar{p}_2)(\min\{P(q_n^*(\bar{p}_2) + (n-1)q_n^*(\bar{p}_2)), \bar{p}_2\} - c) = \pi_n^*(\bar{p}_2) \end{aligned}$$

The first inequality follows from the fact that $q_n^*(\bar{p}_1)$ is a best response to $(n-1)q_n^*(\bar{p}_1)$. The second inequality follows since $\bar{p}_1 > \bar{p}_2$. For fixed n Earle et al. [2007] prove (in Theorem 1) that minimal equilibrium output per firm is non-increasing in the price cap, so that $q_n^*(\bar{p}_2) \geq q_n^*(\bar{p}_1)$. Since $P(\cdot)$ is decreasing, the third inequality holds. The argument for maximal extremal equilibrium subgame output is analogous.

Proof of Lemma 3.3

Let $\bar{p}_1 > \bar{p}_2$. Assume that firms play minimum extremal equilibrium output in each possible subgame. Let n_i be the equilibrium number of firms under \bar{p}_i , $i \in \{1, 2\}$. We will establish the claim by contradiction. So, suppose $n_2 > n_1$. By definition, n_i must satisfy $\pi_{n_i}^*(\bar{p}_i) \geq K$ and $\pi_{n_i}^*(\bar{p}_i) < K$ for $m > n_i$. Since $n_2 > n_1$, we must have $\pi_{n_2}^*(\bar{p}_1) < K$. But by Lemma 3.2 we must have $\pi_{n_2}^*(\bar{p}_1) \geq \pi_{n_2}^*(\bar{p}_2) \geq K$ which is a contradiction. The argument for maximal extremal equilibrium subgame output is identical.

Proof of Proposition 1

Let $\bar{p}_1 > \bar{p}_2$. Let n_i be the equilibrium number of firms under \bar{p}_i , $i \in \{1, 2\}$. From Lemma 3.3 we know that $n_1 \geq n_2$. Let $\hat{Q}_i = P^{-1}(\bar{p}_i)$. We must have $Q_{n_i}^*(\bar{p}_i) \geq \hat{Q}_i$,

otherwise any one firm could increase output slightly and increase profit. Moreover, since $\bar{p}_1 > \bar{p}_2$ Assumption (1a) implies that $\hat{Q}_2 > \hat{Q}_1$.

Part (i) We must show that $Q_{n_2}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$. Earle et al. [2007] prove in their Theorem 1 that the desired result holds if $n_1 = n_2$. So the remainder of part (i) deals with the case $n_1 > n_2$. The arguments for the equilibrium with the smallest subgame outputs are different from those for the equilibrium with the largest subgame outputs. We provide the argument for the smallest subgame outputs first, followed by the argument for the largest subgame outputs.

We will proceed by contradiction. That is, suppose that $Q_{n_2}^*(\bar{p}_2) < Q_{n_1}^*(\bar{p}_1)$. Immediately we have $Q_{n_1}^*(\bar{p}_1) > Q_{n_2}^*(\bar{p}_2) \geq \hat{Q}_2 > \hat{Q}_1$. Now, consider the subgame with price cap \bar{p}_2 and n_1 active firms; let q be any non-negative output.

$$\begin{aligned}
\pi_{n_1}(q_{n_1}^*(\bar{p}_1), (n_1 - 1)q_{n_1}^*(\bar{p}_1), \bar{p}_2) &= q_{n_1}^*(\bar{p}_1)(\min\{P(Q_{n_1}^*(\bar{p}_1)), \bar{p}_2\} - c) \\
&= q_{n_1}^*(\bar{p}_1)(\min\{P(Q_{n_1}^*(\bar{p}_1)), \bar{p}_1\} - c) \\
&\geq q(\min\{P(q + (n_1 - 1)q_{n_1}^*(\bar{p}_1)), \bar{p}_1\} - c) \\
&\geq q(\min\{P(q + (n_1 - 1)q_{n_1}^*(\bar{p}_1)), \bar{p}_2\} - c) \\
&= \pi_{n_1}(q, (n_1 - 1)q_{n_1}^*(\bar{p}_1), \bar{p}_2)
\end{aligned}$$

The first equality follows from the definition of subgame payoffs. The second equality follows from the fact that $Q_{n_1}^*(\bar{p}_1) > \hat{Q}_2 > \hat{Q}_1$ (ie, the price cap is not binding under either cap). The first inequality follows by definition of $q_{n_1}^*(\bar{p}_1)$. The second inequality clearly holds since $\bar{p}_1 > \bar{p}_2$. This establishes that $Q_{n_1}^*(\bar{p}_1)$ is an equilibrium total quantity in the subgame with cap \bar{p}_2 and n_1 firms. In addition we know that (1) $Q_{n_1}^*(\bar{p}_2)$ is the extremal (minimum) equilibrium total output in this subgame, and (2) $Q_{n_1}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$ by the result on price caps in Earle et al. [2007] for a fixed number of firms. Taking these results together yields $Q_{n_1}^*(\bar{p}_2) = Q_{n_1}^*(\bar{p}_1)$.

Now since $Q_{n_1}^*(\bar{p}_2) = Q_{n_1}^*(\bar{p}_1)$ and $Q_{n_1}^*(\bar{p}_1) > \hat{Q}_2 > \hat{Q}_1$ this means that the equilibrium payoff for the subgame with n_1 firms and price cap \bar{p}_2 satisfies the following:

$$\begin{aligned}
\pi_{n_1}^*(\bar{p}_2) &= q_{n_1}^*(\bar{p}_2) [\min\{P(Q_{n_1}^*(\bar{p}_2)), \bar{p}_2\} - c] \\
&= q_{n_1}^*(\bar{p}_1) [\min\{P(Q_{n_1}^*(\bar{p}_1)), \bar{p}_2\} - c] \\
&= q_{n_1}^*(\bar{p}_1) [\min\{P(Q_{n_1}^*(\bar{p}_1)), \bar{p}_1\} - c] \\
&= \pi_{n_1}^*(\bar{p}_1) \\
&\geq K
\end{aligned}$$

But this contradicts the fact that $n_2 < n_1$ is the equilibrium number of entering firms when the price cap is \bar{p}_2 ; the subgame equilibrium payoff for n_1 firms and price cap \bar{p}_2 must be less than K . So we have the result, $Q_{n_2}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$.

The argument above explicitly relies on the fact that the equilibrium under consideration is the smallest equilibrium output level. We now provide an alternative proof of this result for the largest equilibrium output level. As before, let $\bar{p}_1 > \bar{p}_2$. Let $Q_n^*(\bar{p})$ be the maximal equilibrium output when the cap is \bar{p} and n firms are active. We aim to show that $Q_{n_2}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$. We will proceed by contradiction. So, assume that $Q_{n_2}^*(\bar{p}_2) < Q_{n_1}^*(\bar{p}_1)$. Immediately we have that $\hat{Q}_1 < \hat{Q}_2 \leq Q_{n_2}^*(\bar{p}_2) < Q_{n_1}^*(\bar{p}_1)$

Claim. $Q_{n_1}^*(\bar{p}_2)$ is an equilibrium output level in the subgame with n_1 firms and price cap \bar{p}_1

Proof of Claim: All subgames dealt with in this Claim have n_1 firms; so $n = n_1$ and we drop the n subscript in the proof of the Claim. We proceed by contradiction. So, suppose $Q^*(\bar{p}_2)$ is not an equilibrium in the subgame with cap \bar{p}_1 and n_1 firms. By EST it must be that $Q^*(\bar{p}_2) > Q^*(\bar{p}_1)$. Let $y^*(\bar{p}_k) \equiv (n_k - 1)q^*(\bar{p}_k)$. Now, as in the proof of Lemma 3.1, let $\tilde{\pi}(Q, y, \bar{p}) \equiv (Q - y)[\min\{P(Q), \bar{p}\} - c]$. We may again think of a single firm choosing total output, Q , when the other $n_1 - 1$ firms choose y . Let $Q(y, \bar{p})$ be the maximal selection from the best response for a firm when the total output of the other $n_1 - 1$ firms is y and the price cap is \bar{p} .

Since $Q^*(\bar{p}_2)$ is not an equilibrium in the subgame with cap \bar{p}_1 and n_1 firms, we have:

$$\tilde{\pi}(Q^*(\bar{p}_2), y^*(\bar{p}_2), \bar{p}_1) < \tilde{\pi}(Q(y^*(\bar{p}_2), \bar{p}_1), y^*(\bar{p}_2), \bar{p}_1) \quad (6)$$

The inequality $Q^*(\bar{p}_1) < Q^*(\bar{p}_2)$ implies that $y^*(\bar{p}_1) < y^*(\bar{p}_2)$. Recall from the proof of Lemma 3.1 that $Q(y, \bar{p})$ is nondecreasing in y for fixed \bar{p} . Hence, we must have $Q(y^*(\bar{p}_1), \bar{p}_1) \leq Q(y^*(\bar{p}_2), \bar{p}_1)$. But since $Q^*(\bar{p}_1)$ is maximal subgame equilibrium total output, we must have $Q(y^*(\bar{p}_1), \bar{p}_1) = Q^*(\bar{p}_1)$. Hence:

$$Q^*(\bar{p}_1) \leq Q(y^*(\bar{p}_2), \bar{p}_1) \quad (7)$$

Recall that $\hat{Q}_1 < \hat{Q}_2 < Q^*(\bar{p}_1) < Q^*(\bar{p}_2)$. Equation (7) therefore implies $\hat{Q}_1 < \hat{Q}_2 < Q(y^*(\bar{p}_2), \bar{p}_1)$. But then (6) implies,

$$\tilde{\pi}(Q^*(\bar{p}_2), y^*(\bar{p}_2), \bar{p}_2) < \tilde{\pi}(Q(y^*(\bar{p}_2), \bar{p}_1), y^*(\bar{p}_2), \bar{p}_2)$$

since neither cap binds under either output level. The above equation contradicts the definition of $Q^*(\bar{p}_2)$. Hence, the claim is established.

The Claim establishes that $Q_{n_1}^*(\bar{p}_2)$ is an equilibrium total output for the subgame with n_1 firms and cap \bar{p}_1 . This output cannot exceed maximal equilibrium output for this subgame, so $Q_{n_1}^*(\bar{p}_2) \leq Q_{n_1}^*(\bar{p}_1)$. By Theorem 1 in EST, we must have $Q_{n_1}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$. Combining these two inequalities yields, $Q_{n_1}^*(\bar{p}_2) = Q_{n_1}^*(\bar{p}_1)$. As in the proof for the minimal equilibrium output level, we can use this equality to show that n_1 firms would have an incentive to enter when the cap is \bar{p}_2 , contradicting the condition $n_1 > n_2$.

Part (ii) Let $W(\bar{p})$ be total welfare in the equilibrium with the lowest output when the price cap is \bar{p} . Let $Q_i^* = Q_{n_i}^*(\bar{p}_i)$, $i \in \{1, 2\}$. Now note:

$$\begin{aligned} W(\bar{p}_2) &= \int_0^{Q_2^*} [P(z) - c] dz - n_2 K \\ &\geq \int_0^{Q_2^*} [P(z) - c] dz - n_1 K \\ &\geq \int_0^{Q_1^*} [P(z) - c] dz - n_1 K \\ &= W(\bar{p}_1) \end{aligned}$$

The first inequality follows since $n_1 \geq n_2$. The second inequality follows from the fact that $Q_2^* \geq Q_1^*$ and that $P(Q_2^*) \geq c$ (otherwise any firm could increase its period two profit by reducing output).

Part (iii) Let $CS(Q, \bar{p})$ denote consumer surplus when total production is Q and the price cap is \bar{p} .

$$CS(Q, \bar{p}) = \int_0^Q [P(z) - \min\{P(Q), \bar{p}\}] dz$$

Note that $CS(Q, \bar{p})$ is increasing in Q and is decreasing in \bar{p} . Since $Q_{n_2}^*(\bar{p}_2) \geq Q_{n_1}^*(\bar{p}_1)$ and $\bar{p}_2 < \bar{p}_1$, immediately we have that $CS(Q_{n_2}^*(\bar{p}_2), \bar{p}_2) \geq CS(Q_{n_1}^*(\bar{p}_1), \bar{p}_1)$.

Proof of Proposition 2

Let $\bar{p} < P(Q^\infty)$ such that $\bar{p} \in \mathbb{P}$. We first claim that equilibrium output under the cap satisfies $P(Q^*(\bar{p})) = \bar{p}$.

To establish the claim we proceed by contradiction. So, suppose not. Then we must have $P(Q^*(\bar{p})) < \bar{p}$ or equivalently, $Q^*(\bar{p}) > \hat{Q}(\bar{p})$. Now, consider the subgame

with no cap and n^* firms. Since $n^* \leq n^\infty$ it must be that $Q_{n^*}^\infty \leq Q^\infty < \hat{Q}$. It follows that $Q^*(\bar{p}) > \hat{Q} > Q_{n^*}^\infty$.

Let $\tilde{\pi}_L(Q, y) \equiv \ln(\tilde{\pi}(Q, y))$. Where $\tilde{\pi}(Q, y) = (Q - y)(P(Q) - c)$. Note that since P is log-concave this implies $\tilde{\pi}_L$ is strictly concave in Q for all Q such that $P(Q) > c$.

Let $Q(y) \equiv \arg \max_{Q \geq y} \tilde{\pi}_L(Q, y)$. Since $\tilde{\pi}_L$ is strictly concave in Q , clearly $Q(y)$ is a single-valued function. In the subgame with n^* firms and no cap, using standard results, we must have a unique equilibrium since P is log-concave. This means $Q^*(\bar{p})$ cannot be an equilibrium output level with no cap and n^* firms. Let $y^*(\bar{p}) = \left(\frac{n^*-1}{n^*}\right)Q^*(\bar{p})$. Then since $Q^*(\bar{p})$ is not an equilibrium output in the absence of a price cap it follows that $Q(y^*(\bar{p})) \neq Q^*(\bar{p})$. In particular, it must be that $Q(y^*(\bar{p})) < \hat{Q} < Q^*(\bar{p})$. If $Q(y^*(\bar{p})) \geq \hat{Q}$ then $Q(y^*(\bar{p}))$ would also be the best response to $y^*(\bar{p})$ under the price cap as well. This would contradict the definition of $Q^*(\bar{p})$. Then since in any equilibrium price must be strictly above marginal cost, the strict concavity of $\tilde{\pi}_L$ implies:

$$\tilde{\pi}_L(Q(y^*(\bar{p})), y^*(\bar{p})) > \tilde{\pi}_L(\hat{Q}, y^*(\bar{p})) > \tilde{\pi}_L(Q^*(\bar{p}), y^*(\bar{p}))$$

This implies that $\tilde{\pi}(\hat{Q}, y^*(\bar{p})) > \tilde{\pi}(Q^*(\bar{p}), y^*(\bar{p}))$. But since $P(\hat{Q}) = \bar{p}$ and $P(Q^*(\bar{p})) < \bar{p}$, we have:

$$(\hat{Q} - y^*(\bar{p}))(\min\{P(\hat{Q}), \bar{p}\} - c) > (Q^*(\bar{p}) - y^*(\bar{p}))(\min\{P(Q^*(\bar{p})), \bar{p}\} - c)$$

This contradicts the definition of $Q^*(\bar{p})$ and establishes the claim. Now under any relevant cap, equilibrium output must satisfy $P(Q^*(\bar{p})) = \bar{p}$. Clearly, this means there may exist only one symmetric equilibrium.

Consider two caps $\bar{p}_2 < \bar{p}_1 < P(Q^\infty)$ such that $\bar{p}_i \in \mathbb{P}$ for each $i \in \{1, 2\}$. By the claim established in the first part of the proof, we must have $\bar{p}_1 = P(Q^*(\bar{p}_1)) > P(Q^*(\bar{p}_2)) = \bar{p}_2$. Obviously, since P is strictly decreasing $Q^*(\bar{p}_1) < Q^*(\bar{p}_2)$. The result is immediate.

Proof of Proposition 3

As in Proposition 1, Assumption (1b) implies that each firm's output choice can be restricted to a convex set, $[0, M]$, for some large positive M . We refer to the original game as the n player game in which each firm chooses output q from $[0, M]$ to maximize $\pi(q, y, \bar{p})$, where y is total output of rivals. However, a formulation of the output choice game with this choice set implies a payoff function that does not satisfy SCP in $(q, -\bar{p})$.

Define $\hat{q} \equiv C'^{-1}(\hat{p})$, and consider a modified game in which each firm's output choice is restricted to $[0, \hat{q}]$.

Claim 1: An output level q' is a symmetric Nash equilibrium of the original game if and only if it is a symmetric Nash equilibrium of the modified game.

(\Leftarrow) Let q^{**} be a symmetric Nash equilibrium of the original game. Suppose that $q^{**} > \hat{q}$. Then $C'(q^{**}) > \hat{p}$ since $C'(\cdot)$ is strictly increasing. Also, $P(nq^{**}) < P(n\hat{q}) = \hat{p}$ since $P(\cdot)$ is strictly decreasing. We have the following payoff relationships:

$$\begin{aligned}
\pi(q^{**}, (n-1)q^{**}, \bar{p}) &= q^{**} \min\{P(nq^{**}), \bar{p}\} - C(q^{**}) \\
&= q^{**}P(nq^{**}) - C(q^{**}) \\
&\leq \hat{q}P(nq^{**}) - C(\hat{q}) \\
&< \hat{q}P(\hat{q} + (n-1)q^{**}) - C(\hat{q}) \\
&= \hat{q} \min\{P(\hat{q} + (n-1)q^{**}), \bar{p}\} - C(\hat{q}) \\
&= \pi(\hat{q}, (n-1)q^{**}, \bar{p})
\end{aligned}$$

The first equality is definitional. The second equality holds since $P(nq^{**}) < \hat{p}$. The first inequality holds since $P(nq) < C'(q)$ for all $q \in [\hat{q}, q^{**}]$. The second inequality holds since $P(\cdot)$ is strictly decreasing and $\hat{q} < q^{**}$. The next equality holds since $P(\hat{q} + (n-1)q^{**}) < \hat{p} \leq \bar{p}$, and the final equality holds by the definition of the payoff function. These inequalities establish that q^{**} is not a best response to $y = (n-1)q^{**}$ and so q^{**} cannot be a symmetric Nash equilibrium output. The supposition that $q^{**} > \hat{q}$ has led to a contradiction; if q^{**} is a symmetric Nash equilibrium of the original game then $q^{**} \leq \hat{q}$. From this it follows that q^{**} is a symmetric Nash equilibrium of the modified game.

(\Rightarrow) Let q^* be a symmetric Nash equilibrium of the modified game and let q^{**} be a symmetric Nash equilibrium of the original game. Suppose that q^* is not a symmetric Nash equilibrium of the original game. Then there exists $\tilde{q} > \hat{q}$ such that,

$$\pi(\tilde{q}, (n-1)q^*, \bar{p}) > \pi(q^*, (n-1)q^*, \bar{p}) \quad (8)$$

Since \tilde{q} is a feasible choice for the original game, and q^{**} is an equilibrium output for the original game, we have:

$$\pi(q^{**}, (n-1)q^{**}, \bar{p}) \geq \pi(\tilde{q}, (n-1)q^{**}, \bar{p}) \quad (9)$$

And, since $q^{**} \leq \tilde{q}$ (established in proving the only-if part of the Claim) and therefore a feasible choice for the modified game, we have:

$$\pi(q^*, (n-1)q^*, \bar{p}) \geq \pi(q^{**}, (n-1)q^*, \bar{p}) \quad (10)$$

Combining inequalities (8) - (10) yields,

$$\pi(\tilde{q}, (n-1)q^*, \bar{p}) - \pi(\tilde{q}, (n-1)q^{**}, \bar{p}) > 0 \geq \pi(q^{**}, (n-1)q^*, \bar{p}) - \pi(q^{**}, (n-1)q^{**}, \bar{p}) \quad (11)$$

Recall that for any q and any \bar{p} , $\pi(q, y, \bar{p})$ is non-increasing in y . Then the left-hand side of (11) implies that $(n-1)q^* < (n-1)q^{**}$, while the right-hand side of (11) implies that $(n-1)q^* \geq (n-1)q^{**}$ which is a contradiction. Hence, it must be that q^* is an equilibrium output choice of the original game. This demonstrates Claim 1.

Claim 2: The payoff function $\pi(q, y, \bar{p})$ for the modified game satisfies the single crossing property (SCP) in $(q, -\bar{p})$ for all $y \geq 0$.

Proof of Claim 2: Let $\bar{p}' > \bar{p} > \hat{p}$ and let $q < q' \leq C'^{-1}(\hat{p})$. Assume that $\pi(q', y, \bar{p}') > \pi(q, y, \bar{p}')$. We will show that this implies $\pi(q', y, \bar{p}) > \pi(q, y, \bar{p})$. To do this, we will separately examine three cases.

- (a) First, suppose that the price cap \bar{p} binds for both quantities q and q' . Then since $\bar{p} > \hat{p}$ immediately it must be that $C'(q) < C'(q') \leq \hat{p} < \bar{p}$. Hence, $\pi(z, y, \bar{p}) = z\bar{p} - C(z)$ is strictly increasing in z for all $z \in [q, q']$. This immediately implies that $\pi(q', y, \bar{p}) > \pi(q, y, \bar{p})$
- (b) Next, suppose that the price cap, \bar{p} binds for q but not for q' . Since, the lower cap does not bind for output q' , the higher cap must not have been binding either. So, $\pi(q', y, \bar{p}) = \pi(q', y, \bar{p}') > \pi(q, y, \bar{p}') \geq \pi(q, y, \bar{p})$.
- (c) If the cap does not bind for either quantity, then the profits are the same under \bar{p} as they were under \bar{p}' . This establishes Claim 2.

The arguments that establish that total output in symmetric equilibria of the modified game with the smallest and largest outputs are nonincreasing in the price cap are analogous to the arguments used in Earle et al. [2007] in their proof of Theorem 1; because the arguments are so similar to those in Earle et al., we do not include them here. These arguments rely on the payoff function satisfying SCP in $(q, -\bar{p})$. Our

Claim 1 then extends the result to the original game. The welfare result follows from the result for total output. Finally, for fixed n , welfare is maximized when each firm chooses output so that its marginal cost equals the price determined via the inverse demand function. A price cap equal to \hat{p} implements these output choices by firms.

Proof of Lemma 3.4

In Lemma 3.1 we show that the payoff function has increasing differences (ID) in total output Q and rivals' output y for the constant marginal cost case. For the case of strictly increasing marginal cost, the payoff function can be viewed as the sum of the payoff in the constant marginal cost case (with $c = 0$) and $-C(Q - y)$. Each of the two payoff functions in this sum satisfy ID; for the latter function, this holds since $-C(\cdot)$ is strictly convex in output. Then since the sum of two functions that satisfy ID also satisfies ID, the payoff function for the increasing marginal cost case satisfies ID in (Q, y) on Φ . The remainder of the proof is identical to the proof in Lemma 3.1 for constant marginal cost.

Proof of Lemma 3.5

First, note that since $\bar{p} \geq ATC^m$ it is profitable for at least one firm to enter. Lemma 3.4 implies that it suffices to show that in the subgame with n^∞ firms and a price cap $\bar{p} \in [ATC^m, p^\infty)$, equilibrium profit is strictly less than the cost of entry.

Let Q^* be the equilibrium total output in the subgame with n^∞ firms with price cap \bar{p} and let $q^* = \frac{Q^*}{n^\infty}$ and π^* be the associated output per firm and profit per firm, respectively. First, suppose $q^* > q^\infty$. Then we have:

$$\begin{aligned}
K &= q^\infty P(Q^\infty) - C(q^\infty) \\
&\geq q^* P(q^* + (n^\infty - 1)q^\infty) - C(q^*) \\
&> q^* P(Q^*) - C(q^*) \\
&\geq q^* \min\{P(Q^*), \bar{p}\} - C(q^*) \\
&= \pi^*
\end{aligned}$$

The first inequality follows by definition of q^∞ , while the strict inequality follows since $q^* > q^\infty$. Now suppose that $q^* \leq q^\infty$. See that:

$$\begin{aligned}
K &= q^\infty P(Q^\infty) - C(q^\infty) \\
&\geq q^* P(Q^\infty) - C(q^*) \\
&> q^* \bar{p} - C(q^*) \\
&\geq q^* \min\{P(Q^*), \bar{p}\} - C(q^*) \\
&= \pi^*
\end{aligned}$$

To see why the first inequality holds, note that Q^∞ must satisfy $P(Q^\infty) \geq C'(q^\infty)$. Since $C'(\cdot)$ is strictly increasing, we must have $P(Q^\infty) > C'(x)$ for all $x \in [0, q^\infty)$. Hence, the function $xP(Q^\infty) - C(x)$ is strictly increasing in x for $x \in [0, q^\infty)$. Since $q^* \leq q^\infty$, the inequality holds. Finally, note that the strict inequality follows since $\bar{p} < P(Q^\infty)$.

Hence, we must have $\pi^* < K$.

Proof of Proposition 4

First note that since the entry constraint is binding, by Lemma 3.5 any price cap, $\bar{p} \in [ATC^m, p^\infty)$, results in the entrance of at least one firm and at most $n^\infty - 1$ firms. First, consider the subgame with $n^\infty - 1$ firms under a price cap $\bar{p} \in [ATC^m, p^\infty)$. We will then show that maximal equilibrium profit in this subgame is at least as large as the cost of entry.

Let $\pi(Q, y, \bar{p}) = (Q - y) \min\{P(Q), \bar{p}\} - C(Q - y)$ be profit for a firm if total output is Q , rivals' output is y , and the price cap is $\bar{p} \in [ATC^m, p^\infty)$. Initially we suppose that $y = (n^\infty - 2)C'^{-1}(\bar{p})$; that is, each rival firm sets its output such that marginal cost is equal to the price cap. Note that $\pi(Q, y, \bar{p})$ is continuous in Q . Also

$$\frac{\partial \pi(Q, y, \bar{p})}{\partial Q} = \begin{cases} \bar{p} - C'(Q - y), & Q < P^{-1}(\bar{p}) \\ P(Q) - (Q - y)P'(Q) - C'(Q - y), & Q > P^{-1}(\bar{p}) \end{cases}$$

For $Q > P^{-1}(\bar{p})$ we have the following inequalities:

$$\frac{\partial \pi(Q, y, \bar{p})}{\partial Q} < \bar{p} - C'(Q - y) < 0, \tag{12}$$

where the first inequality follows because the inverse demand function is strictly decreasing in Q . The second inequality is due to the following argument. First, the assumption that $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$ implies that, $Q^\infty > (n^\infty - 1)C'^{-1}(p^\infty) \geq (n^\infty - 1)C'^{-1}(\bar{p}) = y + C'^{-1}(\bar{p})$. Second, $Q > P^{-1}(\bar{p})$ implies $Q > Q^\infty$ so, $Q - y > C'^{-1}(\bar{p})$. Third, this implies that $\bar{p} - C'(Q - y) < 0$, which is the second inequality in equation

(12) above.

Since the payoff function is strictly decreasing in Q for $Q > P^{-1}(\bar{p})$, it must be that the best response Q to y is in the interval, $[y, P^{-1}(\bar{p})]$. Strict convexity of the cost function implies that the payoff function is strictly concave in Q in this interval. The unique best response to y satisfies, $\bar{p} = C'(Q - y)$; that is, it is optimal for a firm to set its output such that marginal cost is equal to the price cap when each of its rivals follows the same policy. The optimal choice of Q is in the interior of $[y, P^{-1}(\bar{p})]$ since (by assumption) $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$. $q^* = C'^{-1}(\bar{p})$ is equilibrium output per firm in the subgame with $n^\infty - 1$ firms. A proof-by-contradiction can be used to show that this is the unique subgame equilibrium. Equilibrium profit is given by $\pi^*(\bar{p}) = q^*\bar{p} - C(q^*)$

To demonstrate (i), we show that in the subgame with $n^\infty - 1$ firms for all $\bar{p} \in [ATC^m, p^\infty)$ we have $\pi^*(\bar{p}) \geq K$. Let q^m solve $\min_x ATC(x)$. Then see that:

$$\begin{aligned}\pi^*(\bar{p}) &= q^*\bar{p} - C(q^*) \\ &\geq q^m\bar{p} - C(q^m) \\ &\geq q^m ATC^m - C(q^m) = K\end{aligned}$$

The first inequality follows by the optimality of q^* and the second follows since $\bar{p} \geq ATC^m$. Hence, the equilibrium number of firms is $n^\infty - 1$. Then since $C'' > 0$, we have that q^* is increasing in \bar{p} . Moreover, since $\bar{p} < p^\infty$, we must have $P((n^\infty - 1)C'^{-1}(\bar{p})) > P((n^\infty - 1)C'^{-1}(p^\infty)) > p^\infty$. So, $(n^\infty - 1)C'^{-1}(\bar{p}) < Q^\infty$. This establishes (i) – (iii).

Finally, note that for any relevant cap, equilibrium welfare is given by:

$$W^*(\bar{p}) = \int_0^{(n^\infty - 1)C'^{-1}(\bar{p})} P(z) dz - (n^\infty - 1)C(C'^{-1}(\bar{p})) - (n^\infty - 1)K$$

Now see that

$$W^{*'}(\bar{p}) = \frac{n^\infty - 1}{C''(C'^{-1}(\bar{p}))} \left[P\left((n^\infty - 1)C'^{-1}(\bar{p})\right) - \bar{p} \right] > 0$$

,

which establishes (iv).

Proof of Lemma 4.1

As in Lemma 3.1 we define a firm's payoff function in terms of total output Q and rivals' output y :

$$\tilde{\pi}(Q, y, \bar{p}) = E[(Q - y) \min\{P(Q, \theta), \bar{p}\} - (Q - y)c]$$

Let $\theta^b(Q, \bar{p})$ satisfy $P(Q, \theta^b(Q, \bar{p})) = \bar{p}$. So when total output is Q and the cap is \bar{p} the price cap will bind for shocks $\theta \geq \theta^b(Q, \bar{p})$. Now define

$$\tilde{P}(Q, \bar{p}) = E[\min\{P(Q, \theta), \bar{p}\}] = \int_{\underline{\theta}}^{\theta^b(Q, \bar{p})} P(Q, \theta) dF(\theta) + \int_{\theta^b(Q, \bar{p})}^{\bar{\theta}} \bar{p} dF(\theta)$$

Since $f(\theta) > 0$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$, and $P_1(Q, \theta) < 0$ for all $Q \geq y$, $\theta \in \Theta$ we have:

$$\tilde{P}_1(Q, \bar{p}) = \int_{\underline{\theta}}^{\theta^b(Q, \bar{p})} P_1(Q, \theta) dF(\theta) \leq 0$$

Then $\tilde{\pi}(Q, y, \bar{p}) = (Q - y) \left(\tilde{P}(Q, \bar{p}) - c \right)$. The cross partial derivative of $\tilde{\pi}$ with respect to Q and y on Φ is given by $-\tilde{P}_1(Q, \bar{p}) \geq 0$. This establishes that $\tilde{\pi}$ has ID in (Q, y) on Φ . The remainder of the proof is analogous to Lemma 3.1.

Proof of Proposition 5

For notational convenience, in this proof we let $n^\infty \equiv n$. Concavity of $p(\cdot)$ implies the existence of a unique symmetric period two subgame equilibrium. Also, concavity of $p(\cdot)$ implies that, for a fixed number of firms, equilibrium profit is continuous in the price cap. If $\pi^\infty > K$ then by the continuity of period two profit in \bar{p} , there is an interval of price caps below \bar{p}_n such that the equilibrium number of entrants remains at n . For a fixed number of firms, Grimm and Zottl [2010] establish that any price cap $\bar{p} \in [\overline{MR}_n, \bar{p}_n)$ both increases output and total welfare. Thus, a price cap in the intersection of $[\overline{MR}_n, \bar{p}_n)$ and the set of price caps for which n firms enter will leave the equilibrium number of firms unchanged and will increase both output and welfare.

If $\pi^\infty = K$ then there exists a range of price caps below \bar{p}^∞ such that the equilibrium number of firms decreases by exactly one. Also, if $\pi^\infty = K$ then by Assumption (1d) we must have $n \geq 2$. We let Q_m denote equilibrium total output in the absence of a price cap when m firms enter in the first stage. We also define per-firm output and profit analogously. We begin by demonstrating that $\pi_{n-1} > K$.

In a subgame with no cap and m firms, the symmetric equilibrium condition is given by:

$$\mu - c + p(Q_m) + \frac{Q_m}{m} p'(Q_m) = 0 \quad (13)$$

where $\mu = E[\theta]$. Given that $p(Q)$ is strictly decreasing and concave, equation (13) implies that $Q_{n-1} < Q_n$ and $q_{n-1} > q_n$. The inequality for total output implies that $p(Q_{n-1}) > p(Q_n)$, and this inequality combined with the inequality for output per firm, implies that $\pi_{n-1} > \pi_n = \pi^\infty = K$.

We now demonstrate that in the absence of a price cap, total equilibrium welfare is higher in the subgame with $n - 1$ firms than with n firms. Let $\Delta Q \equiv Q_n - Q_{n-1}$. We claim that $\Delta Q \leq \frac{1}{n}q_n$. To establish the claim, we will proceed by contradiction. Suppose that $\Delta Q > \frac{1}{n}q_n$. Equivalently, $Q_{n-1} < g$ where $g \equiv Q_n - \frac{1}{n}q_n$.

Now, since $g > Q_{n-1}$ and since p is decreasing and concave, equation (13) implies that:

$$0 > \mu - c + p(g) + \frac{g}{n-1} p'(g) \quad (14)$$

Moreover, the concavity of p implies that:

$$p(g) \geq p(Q_n) - \left(\frac{1}{n}q_n\right) p'(g) \quad (15)$$

Then equations (14) and (15) together imply that

$$0 > \mu - c + p(Q_n) + q_n p'(g)$$

Note that $g < Q_n \implies p'(g) \geq p'(Q_n)$. So,

$$0 > \mu - c + p(Q_n) + q_n p'(Q_n)$$

But this contradicts (13) for $m = n$. So we must have $0 \leq \Delta Q_n \leq \frac{1}{n}q_n$.

Now, let

$$B(z) \equiv E \left[\int_0^z P(s, \theta) ds \right] = \mu z + b(z)$$

where

$$b(z) \equiv \int_0^z p(s) ds$$

Let $W(z, m) = B(z) - cz - mK$ denote total welfare when total production is z and

m firms enter. Let $\Delta W \equiv W(Q_{n-1}, n-1) - W(Q_n, n)$ denote the change in equilibrium welfare when the number of firms decreases by one in the absence of a cap. Note that $\pi_n = K$ implies that

$$\Delta W = -[B(Q_n) - B(Q_{n-1}) - (\Delta Q_n)c] + \pi_n$$

Note that the term in square brackets above is given by

$$(\mu - c)\Delta Q + \int_{Q_{n-1}}^{Q_n} p(s) ds$$

Now, let $T(s; x) = p'(x)s + p(x) - p'(x)x$ denote the equation of the line tangent to p at output x . Note that since p is concave and decreasing, for all $s \in [Q_{n-1}, Q_n]$, $p(s) \leq T(s, Q_n)$. This means

$$\int_{Q_{n-1}}^{Q_n} p(s) ds \leq \int_{Q_{n-1}}^{Q_n} T(s; Q_n) ds = \Delta Q p(Q_n) - \frac{1}{2}(\Delta Q)^2 p'(Q_n)$$

Plugging this back into the expression for ΔW we see that:

$$\Delta W \geq \frac{1}{2}p'(Q_n)(\Delta Q_n)^2 - (p(Q_n) + \mu - c)\Delta Q + \pi_n$$

Then, from (13), it follows that $p(Q_n) + \mu - c = -p'(Q_n)q_n$ and $\pi_n = -p'(Q_n)(q_n)^2$. Combining this with the fact that $\Delta Q \leq \frac{1}{n}q_n$, $p' < 0$ and $n \geq 2$ yields:

$$\begin{aligned} \Delta W &\geq \frac{1}{2}p'(Q_n)(\Delta Q)^2 + p'(Q_n)q_n\Delta Q - p'(Q_n)(q_n)^2 \\ &\geq \frac{1}{2}p'(Q_n)\left(\frac{1}{n}q_n\right)^2 + p'(Q_n)\frac{1}{n}(q_n)^2 - p'(Q_n)(q_n)^2 \\ &= p'(Q_n)q_n^2\left(\frac{1}{2n^2} + \frac{1}{n} - 1\right) > 0 \end{aligned}$$

Thus we see that, in the absence of a price cap total welfare increases when the number of firms that enter decreases by one. Let \overline{MR}_m denote the maximal equilibrium marginal revenue when m firms enter and there is no price cap. Clearly, by assumptions placed on P ,

$$\overline{MR}_m = \bar{\theta} + p(Q_m^\infty) + p'(Q_m^\infty)q_m^\infty$$

By the first order equilibrium conditions, it follows that $p(Q_m^\infty) + p'(Q_m^\infty)q_m^\infty = c - \mu$. Hence, independent of m it is clear that

$$\overline{MR}_m = \bar{\theta} + c - \mu$$

and since $p' < 0$:

$$\overline{MR}_m < \bar{\theta} + p(Q_m^\infty) = \bar{\rho}_m$$

Since \overline{MR}_m is independent of m , we drop this subscript and write \overline{MR} . Let $\bar{\rho}_m = P(Q_m^\infty, \bar{\theta})$. Now, since $\pi^\infty = K$ and $\pi_{n-1} > K$, there exists a range of price caps strictly less than $\bar{\rho}_n$ such that the equilibrium number of firms will decrease by exactly one¹². Let \hat{p}_1 denote the smallest of these price caps. Let $\hat{p} \equiv \max\{\hat{p}_1, \overline{MR}\}$. Then since $\overline{MR} < \bar{\rho}_n$ and $\hat{p}_1 < \bar{\rho}_n$ we have $\hat{p} < \bar{\rho}_n$. Also see that under any price cap $\bar{p} \in [\hat{p}, \bar{\rho}_n)$ the equilibrium number of firms is $n - 1$. Finally note that by lemma (4.1) it is clear that $\bar{\rho}_n \leq \bar{\rho}_{n-1}$.

Choose $\bar{p} \in [\hat{p}, \bar{\rho}_n)$. Then since $\bar{p} \in [\overline{MR}, \bar{\rho}_{n-1})$ by Grimm and Zottl [2010] Theorem 1, it follows that equilibrium welfare under the price cap is strictly greater than equilibrium welfare in the subgame with $n - 1$ firms and no cap. Since welfare in the absence of a cap when $n - 1$ firms enter is strictly higher than welfare when n firms enter, the result follows.

Proof of Lemma 4.2

Recall that the necessary condition for equilibrium capacities is:

$$\int_{\tilde{\theta}(X_n)}^{\bar{\theta}} [\theta + p(X_n) + x_n p'(X_n)] dF(\theta) = c \quad (16)$$

By the definition of $\tilde{\theta}(X_m)$ and $\tilde{\theta}(X_n)$ this necessary condition may be re-written as:

$$\int_{\tilde{\theta}(X_m)}^{\bar{\theta}} [\theta - \tilde{\theta}(X_m)] dF(\theta) = \int_{\tilde{\theta}(X_n)}^{\bar{\theta}} [\theta - \tilde{\theta}(X_n)] dF(\theta) = c$$

Let $G(s) = \int_s^{\bar{\theta}} [\theta - s] dF(\theta)$ and note that $G'(s) = \int_s^{\bar{\theta}} -1 dF(\theta)$. Note that $G'(s) < 0$ for all $s < \bar{\theta}$. Then, the first-order conditions imply that $G(\tilde{\theta}(X_m)) = G(\tilde{\theta}(X_n))$. Since

¹²Once again, this follows since concavity of P implies the continuity of equilibrium profit in the price cap

$\tilde{\theta}(X_m) < \bar{\theta}$ and $\tilde{\theta}(X_n) < \bar{\theta}$ it must be that $\tilde{\theta}(X_m) = \tilde{\theta}(X_n)$.

Proof of Lemma 4.3

By Lemma 4.2 it follows that $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1}) = \tilde{\theta}$. First, fix $\theta < \tilde{\theta}$. In this case we have $Q_n(\theta) = \tilde{Q}_n(\theta)$ and $Q_{n-1}(\theta) = \tilde{Q}_{n-1}(\theta)$. Both output choices must satisfy their respective first-order conditions:

$$\theta + p(\tilde{Q}_n(\theta)) + \tilde{q}_n(\theta)p'(\tilde{Q}_n(\theta)) = 0$$

and

$$\theta + p(\tilde{Q}_{n-1}(\theta)) + \tilde{q}_{n-1}(\theta)p'(\tilde{Q}_{n-1}(\theta)) = 0$$

From the first-order conditions, the concavity of p together with the fact that $p' < 0$ implies that $\tilde{Q}_n(\theta) > \tilde{Q}_{n-1}(\theta)$ and $\tilde{q}_n(\theta) < \tilde{q}_{n-1}(\theta)$

Now, since $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1}) = \tilde{\theta}$ it follows from the definitions of $\tilde{\theta}(X_n)$ and $\tilde{\theta}(X_{n-1})$ that

$$\tilde{\theta} + p(X_n) + x_n p'(X_n) = 0$$

and

$$\tilde{\theta} + p(X_{n-1}) + x_{n-1} p'(X_{n-1}) = 0$$

Once again, these equations together with our assumptions on p ensure that $X_n > X_{n-1}$ and $x_n < x_{n-1}$. This establishes (i) and (ii).

Note that the first-order condition given in (16) implies that

$$\left(\int_{\tilde{\theta}(X_{n-1})}^{\bar{\theta}} [\theta + p(X_{n-1})] dF(\theta) - c \right) > 0 \quad (17)$$

Then by parts (i) and (ii) above, we have:

$$\begin{aligned}
\pi_{n-1} &= \int_{\underline{\theta}}^{\tilde{\theta}(X_{n-1})} \tilde{q}_{n-1}(\theta) [\theta + p(\tilde{Q}_{n-1}(\theta))] dF(\theta) + x_{n-1} \left(\int_{\tilde{\theta}(X_{n-1})}^{\bar{\theta}} [\theta + p(X_{n-1})] dF(\theta) - c \right) \\
&> \int_{\underline{\theta}}^{\tilde{\theta}(X_n)} \tilde{q}_n(\theta) [\theta + p(\tilde{Q}_{n-1}(\theta))] dF(\theta) + x_n \left(\int_{\tilde{\theta}(X_n)}^{\bar{\theta}} [\theta + p(X_{n-1})] dF(\theta) - c \right) \\
&> \int_{\underline{\theta}}^{\tilde{\theta}(X_n)} \tilde{q}_n(\theta) [\theta + p(\tilde{Q}_n(\theta))] dF(\theta) + x_n \left(\int_{\tilde{\theta}(X_n)}^{\bar{\theta}} [\theta + p(X_n)] dF(\theta) - c \right) \\
&= \pi_n
\end{aligned}$$

The first inequality holds since $\tilde{\theta}(X_{n-1}) = \tilde{\theta}(X_n)$, $\tilde{q}_{n-1} > \tilde{q}_n$, $x_{n-1} > x_n$ and (17). The second inequality holds since $\tilde{Q}_n > \tilde{Q}_{n-1}$ and $X_n > X_{n-1}$.

Proof of Proposition 6

As in the proof of Proposition 4, we demonstrate our result by showing that welfare in the subgame with n firms and no price cap is strictly lower than welfare in the subgame with $n - 1$ firms and no cap. For each $\theta \in \Theta$ let $\Delta Q(\theta) \equiv Q_n(\theta) - Q_{n-1}(\theta)$. We will first demonstrate that for each θ , $\Delta Q(\theta) \leq \frac{1}{n} q_n(\theta)$.

Using the fact that $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1})$ we first examine the case when $\theta < \tilde{\theta}$. In this case, we use the first order conditions and the proof is identical to the proof in the previous section. For the case when $\theta > \tilde{\theta}$ we again use the definition of $\tilde{\theta}$ and follow a similar argument as in the previous section.

Thus, for each θ $\Delta Q(\theta) \leq \frac{1}{n} q_n(\theta)$

Now, let W_m denote equilibrium expected welfare in the subgame with m firms. Let $\Delta W \equiv W_{n-1} - W_n$. Note that

$$W_n = E \left[\int_0^{Q_n(\theta)} [\theta + p(s)] ds \right] - cX_n - n\pi_n$$

Which may be written:

$$W_n = \int_{\underline{\theta}}^{\tilde{\theta}} \left[\int_0^{\tilde{Q}_n(\theta)} [\theta + p(s)] ds \right] dF(\theta) + \int_{\tilde{\theta}}^{\bar{\theta}} \left[\int_0^{X_n} [\theta + p(s)] ds \right] dF(\theta) - cX_n - n\pi_n$$

Let $\Delta W = W_{n-1} - W_n$ and note that

$$\Delta W = - \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\tilde{Q}_{n-1}(\theta)}^{\tilde{Q}_n(\theta)} [\theta + p(s)] ds \right] dF(\theta) - \int_{\tilde{\theta}}^{\bar{\theta}} \left[\int_{X_{n-1}}^{X_n} [\theta + p(s)] ds \right] dF(\theta) + (\Delta X)c + \pi_n$$

Now, following a similar argument as in the previous section, using the concavity of p , we may show that for each $\theta \in [\underline{\theta}, \tilde{\theta}]$

$$\int_{\tilde{Q}_{n-1}(\theta)}^{\tilde{Q}_n(\theta)} [\theta + p(s)] ds \leq \Delta \tilde{Q}(\theta)(\theta + p(\tilde{Q}_n(\theta))) - \frac{1}{2}(\Delta \tilde{Q}(\theta))^2 p'(\tilde{Q}_n(\theta)) \equiv A(\theta)$$

Moreover, for each $\theta \in [\tilde{\theta}, \bar{\theta}]$

$$\int_{X_{n-1}}^{X_n} [\theta + p(s)] ds \leq \Delta X(\theta + p(X_n)) - \frac{1}{2}(\Delta X)^2 p'(X_n) \equiv B(\theta)$$

Now, using the first-order conditions we may write:

$$\pi_n = \left(- \int_{\underline{\theta}}^{\tilde{\theta}} (\tilde{q}_n(\theta))^2 p'(\tilde{Q}_n(\theta)) dF(\theta) \right) + \left(- \int_{\tilde{\theta}}^{\bar{\theta}} (x_n)^2 p'(X_n) dF(\theta) \right) \equiv \pi_n^A + \pi_n^B$$

Hence, it follows that

$$\Delta W \geq - \int_{\underline{\theta}}^{\tilde{\theta}} A(\theta) dF(\theta) - \int_{\tilde{\theta}}^{\bar{\theta}} B(\theta) dF(\theta) + (\Delta X)c + \pi_n^A + \pi_n^B$$

Now, note that

$$- \int_{\underline{\theta}}^{\tilde{\theta}} A(\theta) dF(\theta) + \pi_n^A = \int_{\underline{\theta}}^{\tilde{\theta}} \left[-\Delta \tilde{Q}(\theta)(\theta + p(\tilde{Q}_n(\theta))) + \frac{1}{2}(\Delta \tilde{Q}(\theta))^2 p'(\tilde{Q}_n(\theta)) - (\tilde{q}_n(\theta))^2 p'(\tilde{Q}_n(\theta)) \right] dF(\theta)$$

Using the first-order conditions once again, we know that for each $\theta \in [\underline{\theta}, \tilde{\theta}]$ it holds that $\theta + \tilde{Q}_n = -\tilde{q}_n(\theta)p'(\tilde{Q}_n)$. This, combined with the fact that $\Delta \tilde{Q}(\theta) \leq \frac{1}{n}\tilde{q}_n(\theta)$ allow us to write

$$-\int_{\underline{\theta}}^{\bar{\theta}} A(\theta) dF(\theta) + \pi_n^A \geq \int_{\underline{\theta}}^{\bar{\theta}} (\tilde{q}_n(\theta)p'(\tilde{Q}_n(\theta))) \left(\frac{1}{n} + \frac{1}{2n^2} - 1 \right) dF(\theta) > 0$$

Now also see that

$$\begin{aligned} & -\int_{\bar{\theta}}^{\bar{\theta}} B(\theta) dF(\theta) + (\Delta X)c + \pi_n^B \\ &= -\Delta X \left[\int_{\bar{\theta}}^{\bar{\theta}} (\theta + p(X_n)) dF(\theta) - c \right] + \int_{\bar{\theta}}^{\bar{\theta}} \left[\frac{1}{2}(\Delta X)^2 p'(X_n) - (x_n)^2 p'(X_n) \right] dF(\theta) \end{aligned}$$

From the first-order condition, it follows that

$$\int_{\bar{\theta}}^{\bar{\theta}} (\theta + p(X_n)) dF(\theta) - c = \int_{\bar{\theta}}^{\bar{\theta}} -x_n p'(X_n) dF(\theta)$$

This, combined with the fact that $\Delta X \leq \frac{1}{n}x_n$ allows us to write:

$$-\int_{\bar{\theta}}^{\bar{\theta}} B(\theta) dF(\theta) + (\Delta X)c + \pi_n^B \geq (x_n)^2 p'(X_n) \int_{\bar{\theta}}^{\bar{\theta}} \left(\frac{1}{n} + \frac{1}{2n^2} - 1 \right) dF(\theta) > 0$$

It follows immediately that $\Delta W > 0$. The remainder of the proof works identically to the proof of Proposition 5. The result follows.

References

- Amir, R. and Lambson, V. (2000). On the effects of entry in Cournot markets. *Review of Economic Studies*, 67.
- Cottle, R. and Wallace, M. (1983). Economic effects of non-binding price constraints. *The Journal of Industrial Economics*, 31(4):469–474.
- Earle, R., Schmedders, K., and Tatur, T. (2007). On price caps under uncertainty. *Review of Economic Studies*, 74:93–111.
- Grimm, V. and Zottl, G. (2006). Capacity choice under uncertainty: The impact of market structure. *University of Cologne Working Paper Series in Economics*, (23).

- Grimm, V. and Zottl, G. (2010). Price regulation under demand uncertainty. *The B.E. Journal of Theoretical Economics*, 10(1).
- Mankiw, N. G. and Whinston, M. (1986). Free entry and social inefficiency. *The Rand Journal of Economics*, 17(1):48–58.
- Topkis, D. (1978). Minimizing a submodular function on a lattice. *Operations Research*, 26(2).