Price Caps, Oligopoly, and Entry

Stanley S. Reynolds* David Rietzke†

September 7, 2012

Abstract

This paper investigates the impact of price caps in oligopoly markets with endogenous entry. In the case of deterministic demand and constant marginal cost, reducing a price cap yields increased total output, consumer welfare, and total welfare. This result falls in line with classic results on price caps in monopoly markets, and with results in Earle, Schmedders and Tatur (2007) and Grimm and Zottl (2010) for oligopoly markets with a fixed number of firms. With deterministic demand and increasing marginal cost, we show that these comparative statics results may be reversed if marginal cost increases sufficiently fast. In this case, the existence of a welfare-improving price cap cannot be guaranteed. We also show that a welfare-improving cap may not exist in the case where demand is stochastic. The fact that a welfare-improving cap cannot be guaranteed in these two cases points to a sharp difference in results between an endogenous entry model and oligopoly models with a fixed number of firms. Finally, we provide sufficient conditions that ensure the existence of a welfare-improving cap in the case of stochastic demand.

*University of Arizona, P.O. Box 210108, Tucson, AZ 85721, reynolds@eller.arizona.edu
†University of Arizona, P.O. Box 210108, Tucson, AZ 85721, rietzkdm@email.arizona.edu

1 Introduction

Price ceilings or caps are relevant in many areas including: wholesale electricity markets, interest on loans and credit, telecommunications services, taxi services, and housing in densely populated areas. In electricity markets, price caps are used in ERCOT (Texas), New England, and PJM (midwest/east coast) regional wholesale markets. Wholesale electricity prices can be extremely volatile and this volatility can be exacerbated by the exercise of market power (see, for example, Borenstein, Bushnell and Wolak, 2002). Price caps are one tool which regulators can use to limit the extent to which firms exercise market power. The classic justification for the use of price caps is well known in the case of a monopolist facing constant marginal cost in a perfect-information environment. A price cap increases marginal revenue in those situations where it is binding and incentivizes the monopolist to increase output. Total output, consumer surplus, and total welfare are decreasing in the price cap; as the cap approaches marginal cost, the welfare optimal (perfectly competitive) level of output is achieved.

Earle, Schmedders and Tatur (2007) (EST) analyze the impact of price caps in a Cournot oligopoly setting with constant marginal cost. They show that the classic monopoly results for price caps carry over to oligopoly when demand is certain. However, EST show that these results may break down under demand uncertainty. In particular, they demonstrate that when firms must make output decisions prior to the realization of demand, total output, welfare, and consumer surplus may be locally increasing in the price cap. This result raises into question the effectiveness of price caps as a welfare-enhancing policy tool. On the other hand, Grimm and Zottl (2010) (GZ) demonstrate that, within the framework of Cournot oligopoly with uncertain demand analyzed by EST, there exists an interval of prices such that any price cap in this interval increases both total market output and welfare compared to the no-cap case. Thus, while the standard comparative statics results of price caps do not obtain in EST's framework, there always exists a welfare-improving price cap.

A limitation of the analyses of EST and GZ is that the number of firms is assumed to be constant. This assumption may be problematic since imposing a price cap could reduce expected profit and thereby reduce the number of firms that choose to enter and compete in the product market. A possible reduction in the number of firms is precisely what Cottle and Wallace (1983) consider in their analysis of the imposition of a price ceiling in a perfectly competitive market subject to demand uncertainty. Our interest is in the impact of price caps in oligopoly markets in which entry is endogenous.

We modify the analyses of EST and GZ to introduce an initial market entry stage prior to a second stage of product market competition. Market entry requires a firm to incur a sunk cost in stage one, prior to stage two product market competition. The inclusion of a sunk entry cost introduces economies of scale into the analysis. This would seem to be a natural extension for an oligopoly analysis. After all, the rationale for an oligopoly market structure in a homogeneous product market is likely to be based on economies of scale.

Our model of endogenous entry builds on results and insights from Mankiw and Whinston (1986) (MW) and Amir and Lambson (2000) (AL). MW show that when total output is increasing in the number of firms but per-firm output is decreasing in the number of firms (MW call this latter effect the business-stealing effect), the socially optimal number of firms will be less than the free-entry number of firms when the number of firms, n is continous. For discrete n the free entry number of firms may be less than the socially optimal number of firms, but never by more than one. Intuitively, when a firm chooses to enter, it does not take into account decreases in per-firm output and profit of the other active firms. Thus, the social gain from entry may be less than the private gain to the firm. Amir and Lambson provide a taxonomy of the effects of entry on total market output in Cournot markets. In particular, they provide a very general condition under which total equilibrium output is increasing in the number of firms. Our analysis relies heavily on the approaches and results in AL. We demonstrate that AL's condition under which total output is non-decreasing in the number of firms, is also a sufficient condition for total output to be non-decreasing in the number of firms when their is a price cap.

Our objective is to analyze the impact of price caps in oligopoly markets in which entry decisions are endogenous. We show that when entry is endogenous, demand is deterministic, and marginal cost is constant, the standard comparative statics results continue to hold. In this case, a price cap may result in fewer firms, but the incentive provided by the cap to increase output overwhelms the incentive to withold output due to a decrease in competition. It follows that, regardless of the number of firms that enter the market, output will always increase as the cap is lowered. Welfare gains are realized on two fronts. First, the cap increases total output. Second, the cap may deter entry, and in doing so, reduce the total cost associated with entry.

We also consider the case of increasing marginal costs of production. Given our assumption of a sunk cost of entry, this case implies U-shaped average cost curves for firms. We show that when demand is deterministic and marginal cost is increasing, the standard comparative statics results do not hold when entry is endogenous. In contrast to results in GZ for a fixed number of firms, it may be the case that *any* price cap reduces total output and welfare (i.e., there does not exist a welfare improving cap). Moreover, we demonstrate that if marginal cost is increasing sufficiently fast, it may be the case that the standard comparative statics results are reversed; that is, welfare and output may be increasing in the price cap.

Finally, we demonstrate that a welfare-improving price cap may not exist when demand is uncertain and entry is endogenous (for constant marginal cost). Thus, the results of GZ do not generalize to the case of endogenous entry. We provide sufficient conditions for existence of a welfare-improving price cap. These conditions restrict the curvature of inverse demand, which in turn influences the extent of the business-stealing effect when an additional firm enters the market. We also consider a version of the model with disposal; firms do not have to sell the entire quantity they produced, but instead may choose the amount to sell after demand uncertainty has been resolved. We show that the sufficient condition for existence of a welfare improving price cap for the no-disposal model carries over to the model with disposal.

2 The Model

We assume there is an arbitrarily large number, $N \in \mathbb{N}$, of symmetric potential market entrants. Our formulation can be viewed as a two-stage game.[1] At the beginning of the first stage, we assume that the N potential entrants are randomly ordered in a queue and make sequential entry decisions. Each firm's entry decision is assumed to be observed by the other firms. We assume that there is a cost of entry $K > 0$ which is sunk if a firm enters. If a firm does not enter it receives a payoff of zero.

In the second stage, each of the n market entrants produces a homogeneous good. Each firm faces the cost function $C : \mathbb{R}_+ \to \mathbb{R}_+$. For the case of deterministic demand, we will examine both the case of constant marginal cost and the case of strictly increasing marginal cost. Output decisions are made simultaneously. The inverse demand function is given by $P(Q, \theta)$ which depends on total output, Q, and a random variable, θ. θ is continuously distributed according to CDF F with corresponding density f. The support of θ is bounded and given by $\Theta \equiv [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$. Each firm knows the distribution of θ but must make its output decision prior to its realization. We assume that a regulator may impose a price cap, denoted $\overline{p}$. We make the following assumptions.

Assumption 1.

(a) $P(Q, \underline{\theta}) > 0$ *for all* $Q \in \mathbb{R}_+$.

(b) P *is differentiable in* Q *with* $P_1(Q, \theta) < 0$[2] *for each* θ.

(c) P *is differentiable in* θ *with* $P_2 > 0$ *for each* Q

(d) $\lim_{Q \to \infty} P(Q, \overline{\theta}) - C'(Q) < 0$

[1]Since entry decisions are made sequentially, this is technically a $N+1$ stage game. However, for convenience throughout the paper we will refer to the sequence of entry decisions as the first stage.

[2]Throughout the paper, for any function, $G(x_1, x_2, \cdots, x_k)$, we write $G_j(x_1, x_2, \cdots, x_k)$ to indicate the partial derivative of G with respect to x_j.

(e) $\max_{Q \in \mathbb{R}_+} E[Q(P(Q,\theta) - C(Q))] > K$

Assumptions (1a) - (1c) are analogous to the assumptions imposed by Earle, Schmedders, and Tatur (2007) (EST) (with only differentiability added) and match the assumptions in Grimm and Zottl (2010) (GZ).[3] . Assumption (1c) is necessary to ensure that equilibrium outputs in the second-stage game are well defined. Assumption (1d) is not necessary but is included for ease of exposition.

EST assume that $E[P(0,\theta)] > c$ (where c is marginal cost). This assumption ensures that "production is gainful"; that is, given a fixed number, $n > 0$ of market participants, this assumption ensures that there exist price caps such that equilibrium market output will be strictly positive. Our assumption (1e) is a "profitable entry" condition which guarantees that there exist price caps such that at least one firm enters the market and that equilibrium output will be strictly positive. We let $\mathbb{P}$ denote the set of price caps which induce at least one market entrant. That is

$$\mathbb{P} \equiv \left\{ \overline{p} > 0 \mid \max_{Q \in \mathbb{R}_+} E\left[Q \min\{P(Q,\theta), \overline{p}\} - C(Q)\right] \geq K \right\}$$

In this paper we are only concerned with price caps $\overline{p} \in \mathbb{P}$. In the analysis that follows, we restrict attention to subgame-perfect pure strategy equilibria and focus on symmetric second-stage equilibria. Finally, for a fixed number of firms competing in the second-stage sub-game, there generally exist multiple subgame equilibria. We abstract away from this issue and assume that firms coordinate on the second-stage symmetric equilibrium with the lowest output and highest profit per firm (Henceforth, when a second-stage equilibrium is mentioned this means the subgame equilibrium of the second stage with the smallest total output).[4] For fixed number of firms, both EST and GZ prove the existence of the studied second-stage equilibrium. So, we take as given the

[3]Grimm and Zottl assume constant marginal cost and so our assumptions match theirs for this case. In the case of convex costs, our assumption is analogous.

[4]It is standard in the literature to examine the properties of both the smallest and largest equilibria. We focus on the equilibrium with the lowest total output as this is the Pareto-preferred equilibrium for all firms who have entered in stage-one.

existence of second-stage equilibrium for a fixed set of market entrants.

One other point to note. Imposing a price cap may require demand rationning. When rationning occurs, we assume rationning is efficient; i.e., buyers with the lowest WTPs do not receive output. This is the rationning assumption used in EST and GZ.

We denote by $Q_n^*(\overline{p})$, the second-stage equilibrium total output when n firms enter and the price cap is $\overline{p}$. We let $q_n^*(\overline{p})$ be the per-firm output in this equilibrium and let $\pi_n^*(\overline{p})$ denote each firm's expected second-stage profit in this equilibrium. We let $Q_n^\infty = Q_n^*(\infty)$ be the second-stage equilibrium total output when n firms enter with no price cap and let q_n^∞ denote the corresponding per-firm output. Let π_n^∞ denote each firm's expected second-stage profit in this equilibrium. Firms are risk neutral and make output decisions to maximize expected profit. That is, each firm i takes the total output of its rivals, y, as given and chooses q to maximize

$$\pi(q, y, \overline{p}) = E[q \min\{P(q + y, \theta), \overline{p}\} - C(q)]$$

After being placed in the queue, firms have an incentive to enter as long as their expected second-stage equilibrium profit is at least as large as the cost of entry. We assume that firms whose expected second stage profits are exactly equal to the cost of entry will choose to enter. As mentioned above, we restrict attention to subgame-perfect equilibria. For a fixed price cap, $\overline{p}$, subgame perfection in the entry stage (along with the indifference assumption) implies that the equilibrium number of firms, n^*, is the largest positive integer less than (or equal to) N such that $\pi_{n^*}^*(\overline{p}) \geq K$. Clearly, n^* exists and is unique. Moreover, for any $\overline{p} \in \mathbb{P}$ we also have $1 \leq n^* \leq N$, due to Assumption (1e).

For convenience and clarity, we will define $Q^*(\overline{p}) \equiv Q_{n^*}^*(\overline{p})$, $q^*(\overline{p}) \equiv q_{n^*}^*(\overline{p})$, and $\pi^*(\overline{p}) \equiv \pi_{n^*}^*(\overline{p})$. Similarly, we define $Q^\infty = Q_{n^*}^\infty$, $q^\infty = q_{n^*}^\infty$, and $\pi^\infty = \pi_{n^*}^\infty$.

3 Deterministic Demand

We begin our analysis by considering a deterministic inverse demand function. That is, the distribution of θ places unit mass at some particular $\tilde{\theta} \in \Theta$. In this section, we will supress the second argument in the inverse demand function and simply write $P(Q)$. For the case of deterministic demand, we examine both the case where marginal cost is constant and the case where marginal cost is strictly increasing. We begin by analyzing the case where marginal cost is constant, ie for all $x \in \mathbb{R}_+, C(x) = cx$.

3.1 Constant Marginal Cost

As previously mentioned, for a given number, $n \in \mathbb{N}$, of market participants both EST and GZ prove the existence of a second-stage equilibrium. We therefore do not address this issue. Our main result in this section demonstrates that the classic results on price caps continue to hold when entry is endogenous. We first state three lemmas that are used in the proof of this result; all proofs are in the Appendix. Before proceeding, we provide one final piece of notation: for each $\overline{p} \in \mathbb{P}$ we define $\hat{Q}(\overline{p})$ to equal $P^{-1}(\overline{p})$.

Lemma 3.1. *For fixed $\overline{p}$, extremal (minimal) subgame equilibrium total output, $Q_n^*(\overline{p})$ is non-decreasing in the number of firms, n. Moreover, extremal (maximal) equilibrium profit $\pi_n^*(\overline{p})$ is non-increasing in the number of firms, n.*

Lemma 3.2. *For fixed n, extremal (maximal) subgame equilibrium profit per firm $\pi_n^*(\overline{p})$ is non-decreasing in the price cap $\overline{p}$.*

Lemma 3.3. *The equilibrium number of firms is non-decreasing in the price cap, $\overline{p}$.*

Proposition 1. *Restrict attention to $\overline{p} \in \mathbb{P}$. Then in equilibrium, total output, total welfare, and consumer surplus are non-increasing in the price cap.*

Proposition 1 is similar to Theorem 1 in EST. However, our model takes into account the effects of price caps on firm entry decisions. This is an important consideration, given that Lemma 3.1 ensures that for a fixed price cap, total equilibrium output is non-decreasing in the number of firms. This fact, along with the fact that a lower price cap may deter entry, suggest that a reduction in the cap could have the effect of hindering competition and reducing total output. Our result shows that with constant marginal cost and non-stochastic demand, even if entry is reduced, the incentive for increased production with a cap will dominate the possible reduction in output due to less entry. There are two sources of welfare gains. First, total output is non-increasing in the price cap, so a lower price cap yields either constant or reduced deadweight loss. Second, a lower price cap may reduce the number of firms, and thereby decrease the overall cost of firm entry.

An obvious consequence of Proposition 1 is that, with constant marginal cost, the welfare-maximizing price cap is the lowest cap which induces exactly one firm to enter. Imposing such a cap both increases output and reduces entry costs. Since marginal cost is constant, the total cost of production does not depend on the number of firms that enter. Total production cost is a function of only total output produced. However, if marginal cost were increasing, the total production cost associated with a given level of output is decreasing in the number of firms. Thus, a price cap which increases output and reduces entry may result in a significant increase in production costs. Finally, note that with constant marginal cost, a shortage will never arise in equilibrium. This is driven solely by the fact that $\overline{p} > c$ for any level of production. However, if marginal cost is strictly increasing in output, then firms will produce no more than the quantity at which their marginal cost is equal to the price cap, and this could result in a shortage in the market. In the next sub-section we examine the impact of price caps in an environment in which firms have an increasing marginal cost of production.

3.2 Convex Costs

In this section, we assume that the cost function, $C : \mathbb{R}_+ \to \mathbb{R}_+$, is twice continuously differentiable and for all $x \in \mathbb{R}_+$, $C'(x) > 0$ and $C''(x) > 0$. We also assume $C(0) = 0$.

Neither EST nor GZ devote significant attention to the issue of increasing marginal cost. However, it is mentioned in GZ [p. 3, ft. 9] that, when the number of firms is fixed, under a general convex cost function there always exists a price cap which strictly increases total output and welfare. Moreover, EST state [p. 96, ft. 4] that their Theorems 2, 4, and 5 may be generalized to a convex cost function. These theorems all address the case of stochastic demand. Neither paper addresses formally whether the classical monotonicity results hold for a fixed number of firms when marginal cost is increasing.

As in the constant marginal cost case, a decrease in the price cap may decrease the incentive for firms to withold output (placing upward pressure on output), but this must be weighed against the fact that each additional unit of output will be more costly than the next (placing downward pressure on output). Our first result below demonstrates that when the number of firms is fixed, there still exists a range of caps under which equilibrium output (the smallest and largest equilibrium outputs) and associated welfare are monotonically non-increasing in the cap. This range of caps consists of all price caps above the n-firm competitive equilibrium price. Intuitively, price caps above this threshold are high enough that marginal cost in equilibrium is strictly below the price cap for each firm. A slight decrease in the price cap means the incentive to increase output created by a lower cap outweighs the fact that marginal cost has increased (since the cap still lies above marginal cost).

When entry is endogenous the welfare impact of a price cap is much less clear. Suppose, for instance, that inverse demand is log concave. Then for fixed n equilibrium subgame output is unique for each $\overline{p}$ and equilibrium output and profit are continuous in $\overline{p}$. If the entry constraint is not binding in the absence of a price cap (ie $\pi^{\infty} > K$), then a price cap slightly less than the equilibrium price with no cap would leave the equilibrium number of firms unchanged. Proposition 2 below would then imply that there exists a range of caps under which equilibrium welfare is non-increasing in the cap. On the other hand, imposing a price cap may reduce the number of firms; with fewer firms, total production cost for a given level of output will increase. The overall welfare impact of imposing a price cap will depend on the balance of its effects on entry, output incentives, and costs.

In this section, we first demonstrate that when the number of firms is fixed then there exists a range of price caps in which output and welfare are monotonically non-increasing in the cap; thus extending Theorem 1 in EST to the case of convex costs. We then provide an example to show that with strictly convex costs, a welfare improving price cap may fail to exist when entry is endogenous. In contrast to our result in Proposition 2 for a fixed number of firms, and GZ's reference to a similar result, in Proposition 3 we provide conditions for which output and welfare are strictly increasing in *all* binding price caps.

Proposition 2. *Fix $n \in \mathbb{N}$ and define the n-firm competitive price as the unique price satisfying $\hat{p} = P(nC'^{-1}(\hat{p}))$. Then:*

(i) The smallest and largest equilibrium outputs are non-increasing in the price cap for all $\overline{p} > \hat{p}$.

(ii) Equilibrium welfare for the smallest and largest equilibrium outputs is non-increasing in the price capfor all $\overline{p} > \hat{p}$.

(iii) A price cap equal to $\hat{p}$ maximizes welfare. Welfare at this cap is strictly greater than welfare in the absence of a cap.

In contrast to the result above with a fixed number of firms, we now provide an example which demonstrates that when entry is endogenous, a welfare-improving price cap may fail to exist.

Example 1. Consider the inverse demand, cost function and entry cost given below:

$$P(Q) = 54 - Q, \quad C(q) = \frac{1}{3}q^3 \text{ and } K = 180$$

In the absence of a price cap, it may easily be verified that two firms enter with each producing 6 units of output. This results in an equilibrium price of 42 and gives equilibrium second-stage profit equal to the entry cost, 180. Equilibrium welfare with no cap is therefore given by:

$$W^{nc} = \int_0^{12} (54 - t)\, dt - \left(\frac{2}{3}\right) 6^3 - 360 = 72$$

Any price cap less than the equilibrium price results in at most one entrant. In order to ensure entry of at least one firm, the price cap must be greater than or equal to minimum of average total cost (ATC^m); $ATC^m \approx 41.77$ for this example. So, consider any price cap $\overline{p} \in [41.77, 42)$. This results in the entry of exactly one firm. Let $q^*(\overline{p})$ be the solution to monopolist's problem below:

$$\max_q \{q \min\{54 - q, \overline{p}\} - \frac{1}{3} q^3\}$$

The solution to the monopolist's problem is given by $q^*(\overline{p}) = \sqrt{\overline{p}}$ for $\overline{p} \in [41.77, 42)$. Equilibrium welfare under any relevant price cap is given by:

$$W(\overline{p}) = \int_0^{\sqrt{\overline{p}}} (54 - t)\, dt - \left(\frac{1}{3}\right) \left(\sqrt{\overline{p}}\right)^3 - 180$$

Differentiating welfare with respect to the price cap yields:

$$W'(\overline{p}) = \left(\frac{1}{2\sqrt{\overline{p}}}\right) \left(54 - \sqrt{\overline{p}} - \overline{p}\right)$$

Note that the above expression is strictly positive since, $54 - \sqrt{\overline{p}} - \overline{p} > 0$. This implies that equilibrium welfare is strictly increasing in $\overline{p}$ for $\overline{p} \in [41.77, 42)$. Moreover, $W^{nc} > lim_{\overline{p} \to 42} W(\overline{p}) \approx 58.23$. As a consequence, there is no welfare-improving price cap for this example.

A key feature of Example 1 is that marginal cost rises sharply enough with output so that the output at which MC equals the price cap is less than the quantity demanded at the cap. So by reducing the number of firms from two to one, imposition of a price cap also results in a discrete reduction in total output. Note that this effect is absent when marginal cost is constant.

Proposition 3 below provides conditions under which output and welfare are monotonically increasing in a price cap. Prior to stating the Proposition, we state and prove two lemmas used in its proof. In what follows, we will let

ATC^m denote the minimum of average total cost and let $p^\infty = P(Q^\infty)$.

Lemma 3.4. *Fix $\overline{p} \in \mathbb{P}$. Then extremal (minimal) equilibrium total output is non-decreasing in the number of firms. Moreover, extremal (maximal) equilibrium profit is non-increasing in the number of firms.*

Lemma 3.5. *Suppose, $n^\infty \geq 2$ and $\pi^\infty = K$. Consider a price cap $\overline{p} \in [ATC^m, p^\infty)$. Then $1 \leq n^* < n^\infty$.*

Proposition 3. *Suppose $\pi^\infty = K$. If $C'\left(\frac{Q^\infty}{n^\infty - 1}\right) > p^\infty$, then for all $\overline{p} \in [ATC^m, p^\infty)$:*

(i) The equilibrium number of firms is $n^\infty - 1$

(ii) Total output is non-decreasing in $\overline{p}$

(iii) Total output under the price cap is lower than in the absence of a cap

(iv) Welfare is non-decreasing in $\overline{p}$

To illustrate the intuition for the result, first note that Q^∞ must satisfy $C'(\frac{Q^\infty}{n^\infty}) < p^\infty$; that is, each firm's marginal cost is less than the price in the no-cap equilibrium. Thus, it must be the case that, at q^∞, marginal cost is increasing sufficiently fast that $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$. Clearly, any binding cap also satisfies $C'(\frac{Q^\infty}{n^\infty - 1}) > \overline{p}$. Since marginal revenue is always less than or equal to the price cap, if production remained at the level Q^∞, marginal cost would exceed marginal revenue. Hence, the introduction of a price cap in this case must reduce total output since no firm will produce output in excess of the quantity q where $C'(q) = \overline{p}$. The result is demand rationing and a market shortage. This shortage creates deadweight loss which grows as the price cap decreases.

Still, comparing the change in welfare before-and-after the introduction of the price cap is difficult as the introduction of a cap may decrease the number of firms. Example One satisfies the hypotheses of Proposition 3 and illustrates a case in which any binding price cap reduces welfare. On the other hand, one can construct examples which satisfy the hypotheses of Proposition

3 and for which a welfare-improving price cap exists. Under the hypotheses of Proposition 3 there are three competing forces which affect welfare. First, a decrease in the number of firms results in welfare gains associated with entry-cost savings. Second, as described above, the introduction of a price cap creates deadweight loss and decreases welfare. Third, the combination of a decrease in output and a decrease in the number of firms has an ambiguous impact on firms' production costs. The effect on welfare depends on the interplay of these three forces.

4 Stochastic Demand

We now investigate the impact of price caps when demand is stochastic. We will once again assume that marginal cost is constant and given by $c \geq 0$. GZ demonstrate that under a generic distribution of demand uncertainty, there exists a range of price caps which strictly increases output and welfare as compared to the case with no cap. We begin this section by providing an example which demonstrates that this need not be true when entry is endogenous. In what follows, we let $\overline{\rho}^{\infty}$ denote the lowest price cap that does not affect prices, ie $\overline{\rho}^{\infty} = P(Q^{\infty}, \overline{\theta})$.

Example 2. Consider the following inverse demand, entry cost and distribution for θ:

$$P(Q,\theta) = \theta \exp(-Q), \quad K = E[\theta]\exp(-4), \quad f(\theta) = \begin{cases} \frac{1-\epsilon}{\theta_0} & \theta \in [0, \theta_0) \\ & \\ \frac{\epsilon}{\theta_1 - \theta_0} & \theta \in [\theta_0, \theta_1] \end{cases}$$

where $c = 0$, $\epsilon = \frac{1}{1{,}000}$, $\theta_0 = 990$ and $\theta_1 = 10,000$. With no cap, each firm has a dominant strategy in the stage 2 subgame to choose an output of 1. This leads to 4 market entrants; each earning second stage profit exactly equal to the cost of entry. This results in total output of 4 units, total welfare of approximately 454.2, and $\overline{\rho}^{\infty} \approx 183.16$.

Consider a price cap set just below $\overline{\rho}^{\infty}$; say $\overline{p} = 183$. Clearly such a cap will deter at least one firm from entering as the entry constraint is binding in the absence of a cap. Indeed it may be verified that the equilibrium number of firms under the cap falls to 3 while per-firm output increases to approximately 1.005. Total output falls to 3.015 and welfare decreases to approximately 448. It may be verified for this example that any binding price cap will deter entry, reduce total output, and reduce total welfare.

Example 2 demonstrates that when demand is stochastic and entry is endogenous, there need not exist a welfare improving price cap. There are two important features of this example. First, the exponential inverse demand function coupled with zero marginal cost implies that firms have a dominant strategy to choose an output of exactly one unit. Hence, the business-stealing effect is absent and total output increases linearly in the number of firms.[5] Thus, in the absence of a cap, a reduction in the number of firms leads to a significant decrease in output and welfare. The second feature is the role and nature of the demand uncertainty. As explained in EST, when demand is uncertain firms maximize a convex combination of profit when the cap is non-binding (low demand realizations) and profit when the cap is binding (high demand realizations). These two scenarios provide conflicting incentives for firms. When the cap is non-binding, firms have an incentive to withold output (as in the standard oligopoly model). However, the possibility of a binding cap (high demand realizations) creates an incentive for firms to increase production relative to the case with no cap. In the example above, the distribution of θ has a long, thin upper tail; there is a possibility that the demand realization will be extremely high, albeit with a very low probability. A price cap set just below $\overline{p}^{\infty}$ will therefore bind with very low probability, even if one less firm competes in stage 2. So, the incentive provided by the cap to increase production (relative to the no-cap case) is very weak.

The Proposition below provides sufficient conditions on demand that ensure the existence of a welfare-maximizing cap. Prior to the Proposition, we state a lemma used in its proof.

[5] It may be shown that the same result holds for numerical examples with a small marginal cost. In this case the business-stealing effect is present but small.

Lemma 4.1. *Suppose $f(\theta) > 0$ for all $\theta \in \Theta$. Then for fixed $\overline{p}$, extremal (minimal) subgame equilibrium total output, $Q_n^*(\overline{p})$ is non-decreasing in the number of firms, n. Moreover, extremal (maximal) equilibrium profit $\pi_n^*(\overline{p})$ is non-increasing in the number of firms, n.*

Proposition 4. *Suppose that inverse demand is additively separable in Q and θ with $P(Q, \theta) = \theta + p(Q)$ and that p is twice continuously differentiable and concave in Q. Moreover, suppose that $n^\infty \geq 2$. Then there exists a price cap which increases total welfare.*

Proposition 4 provides sufficient conditions for existence of a welfare improving price cap. When demand is concave in output, then the business-stealing effect is relatively strong. As demonstrated in Mankiw and Whinston (MW), when the business-stealing effect is present and n is continuous, the free-entry number of firms entering a market exceeds the socially optimal number of firms. [6] The proof of Proposition 4 first establishes that, when the entry constraint is not binding in the absence of a cap, then there is an interval of prices such that a price cap chosen from this interval will yield the same number of firms, but higher total output and welfare. This follows directly from Theorem 1 in GZ. The proof proceeds to show that when the entry constraint is binding (i.e., zero equilibrium profit in the endogenous entry game) in the absence of a cap, then a reduction in the number of firms by one will increase total welfare. Indeed, it may be shown that, in the absence of a cap under the hypotheses of Proposition 4, the socially optimal number of firms is strictly less than the free-entry number of firms. The imposition of a price cap in this case has two welfare-enhancing effects. First, the cap deters entry. As explained above, in this case entry deterrence (by exactly one firm) is welfare enhancing. Second, the cap increases total output and welfare relative to what output and welfare *would* be in the new entry scenario (ie with one less firm) in the absence of a cap.

[6]MW's result is for a model in which n is a continuous variable, whereas out analysis restricts n to whole numbers. Nonetheless, we are able to apply the intuition on excess entry from MW to prove our result.

4.1 Free Disposal

We now examine a variation of the game examined in the previous sections. This model is a three-stage game. In the first stage, firms sequentially decide whether to enter or not (again, with each firm's entry decision observed by the next). Entry entails some cost $K > 0$. In the second stage, before θ is realized, each firm, i, that entered in the first stage simultaneously chooses a level of capacity, $x_i \geq 0$ built at constant marginal cost $c > 0$. In the third stage, firms first observe θ and then each firm simultaneously chooses a level of output, $0 \leq q_i \leq x_i$, which is produced at zero cost.[7] This model has been analyzed (for a fixed number of firms) by Grimm and Zottl (2006) and has also been analyzed in the context of price caps by EST and GZ.

This model with free disposal may be interpreted as one in which the firms that have entered make long run capacity investment decisions prior to observing the level of demand, and then make output decisions after observing demand. Under this interpretation, c is the marginal cost of capacity investment, and the marginal cost of output is constant and normalized to zero.

GZ show that, for a fixed number of firms, in this model there always exists a price cap which increases total capacity and expected welfare. As with the proof of Proposition 4, we will demonstrate that in the absence of a price cap, if the free-entry constraint binds (ie $\pi_n^\infty = K$), then welfare is strictly higher if entry is reduced by one. For this section, we make the following additional assumptions on P and f:

Assumption 2.

(a) $P(Q,\theta) \geq 0$ *for all* $Q \in \mathbb{R}_+$ *and* $\theta \in \Theta$

(b) For all Q *and* θ *such that* $P(Q,\theta) > 0$*,* P *is additively separable in* Q *and* θ *with:* $P(Q,\theta) = \theta + p(Q)$

(c) p *is twice continuously differentiable with* $p' < 0$*, and* $p'' \leq 0$*.*

[7] In the version of the model examined by EST, output is produced at cost δ which may be positive or negative. Our results continue to hold in this case.

(d) $\underline{\theta} + p(0) = 0$

(e) $f(\theta) > 0$ *for all* $\theta \in \Theta$*. Moreover, the distribution of* θ *contains no mass points.*

The assumptions made above are similar to those made in Proposition 4, but we require that $\underline{\theta} + P(0) = 0$. This assumption is made in Grimm and Zottl (2006) and may be viewed as a normalization of the support of θ.

In the absence of a price cap in the third stage, each firm solves:

$$\max_{q_i} (\theta + P(q_i + y))q_i - cx_i \text{ such that } q_i \leq x_i$$

Let n denote the number of entrants in the first stage in the absence of a price cap. Under the assumptions made above, there exists a unique symmetric equilibrium level of capacity for any $n \in \mathbb{N}$ (Grimm and Zottl, 2006). Denote by $X_n > 0$ the total equilibrium capacity and let $x_n = \frac{X_n}{n}$ denote the equilibrium capacity per firm. Let $Q_n(\theta)$ denote the total equilibrium output in the third stage. Let $\tilde{\theta}(X_n)$ satisfy:

$$\tilde{\theta}(X_n) + p(X_n) + p'(X_n)x_n = 0$$

Assumption 2d above ensures that $\tilde{\theta}(X_n) > \underline{\theta}$. Also note that our assumptions are sufficient to ensure $\tilde{\theta}(X_n)$ is unique for any X_n. Then for $\theta \in [\underline{\theta}, \tilde{\theta}(X_n))$, it holds that no firm is constrained in equilibrium. In the unconstrained case, our assumptions above are sufficient to guarantee that a unique equilibrium exists in the third stage. let $\tilde{Q}_n(\theta)$ denote the total equilibrium output in this case and let $\tilde{q}_n(\theta) = \frac{\tilde{Q}_n(\theta)}{n}$ denote the equilibrium per-firm output. Note that for each $\theta \in [\underline{\theta}, \tilde{\theta}(X_n))$, $\tilde{Q}_n(\theta)$ is given by the first-order condition:

$$\theta + p(\tilde{Q}_n(\theta)) + p'(\tilde{Q}_n(\theta))\tilde{q}_n(\theta) = 0$$

So, total equilibrium output in the third stage is given by:

$$Q_n^*(\theta) = \begin{cases} \tilde{Q}_n(\theta), & \underline{\theta} \le \theta < \tilde{\theta}(X_n) \\ \\ X_n, & \tilde{\theta}(X_n) \le \theta \le \overline{\theta} \end{cases}$$

We may write equilibrium profit per firm as:

$$\pi_n^* = \int_{\underline{\theta}}^{\tilde{\theta}(X_n)} \tilde{q}_n(\theta)(\theta + p(\tilde{Q}_n(\theta)))\, dF(\theta) + \int_{\tilde{\theta}(X_n)}^{\overline{\theta}} x_n(\theta + p(X_n))\, dF(\theta) - cx_n$$

Grimm and Zottl (2006) prove that equilibrium capacity must satisfy the first-order condition given by[8]

$$\int_{\tilde{\theta}(X_n)}^{\overline{\theta}} [\, \theta + p(X_n) + x_n p'(X_n)\,]\, dF(\theta) = c$$

Since $c > 0$ and F contains no mass points, it must be that $\tilde{\theta}(X_n) < \overline{\theta}$.

Lemma 4.2. *Let $m, n \in \mathbb{N}$ and let X_m and X_n denote total equilibrium output when m and n firms enter, respectively. Then $\tilde{\theta}(X_m) = \tilde{\theta}(X_n)$.*

Lemma 4.3. *Fix $n \ge 2$. Then for all $\theta \in \Theta$*

(i) $Q_n^*(\theta) > Q_{n-1}^*(\theta)$

(ii) $q_n^*(\theta) < q_{n-1}^*(\theta)$

(iii) $\pi_n^* > \pi_{n-1}^*$

Proposition 5. *Under Assumptions 1 and 2, the statement of Proposition 4 remains valid in the version of the model with disposal.*

[8] It may easily be verified that our Assumption 2 is a special case of Assumption 1 made in Grimm and Zottl (2006)

5 Conclusion

This paper has analyzed the welfare impact of price caps, taking into account the possibility that a price cap may reduce the number of firms that choose to enter a market. First, we analyzed the welfare impacts of price caps when there is no uncertainty about demand when firms make their output decisions. For this case, we showed that when marginal cost is constant, the standard monotone comparative statics results remain true. That is, output, welfare, and consumer surplus all increase as the price cap is lowered. We then showed that when demand is known but marginal cost is strictly increasing, it may be the case that welfare is lower under any price cap than in the absence of a cap. We also demonstrated that if marginal cost increases sufficiently fast, welfare and output may be monotonically increasing in the price cap. Second, we analyzed the welfare impacts of price caps when demand is stochastic and firms must make output decisions prior to the realization of demand. We showed that, once again the existence of a welfare-improving price cap cannot be guaranteed. We then provided sufficient conditions on demand under which a range of welfare-improving price caps will always exist. The sufficient conditions restrict the curvature of the inverse demand function, which in turn influences the welfare impact of entry. We also extended this result to an environment with free disposal. This type of environment can be viewed as one in which there is endogenous entry, capacity investment decisions are made prior to observing demand, and output decisions are made after observing demand.

One limitation of the present analysis is that we restrict attention to Pareto-dominant symmetric equilibria of stage 2 subgames (except for Proposition 2). EST and GZ consider extremal equilbria in their analyses, and we plan to extend our analysis to consider subgame equilibria with both the highest profit per firm and the lowest profit per firm. A second area for future research would be to extend our results for free disposal to a model that comes closer to capturing key features of electricity markets. As noted in our introduction, regulatory price caps have played an important role in many wholesale electricity markets. Zottl (2011) examines a modified version of a model with disposal that allows for multiple production technologies. His multiple technologies model can be interpreted as a model of electricity generation that allows for baseload, mid-merit, and peaker generation technologies. Firms make long run investments in capacity for the various technologies, and short run decisions about production after observing demand shocks. Our interest is in extending this type of mulitple technologies model to allow endogenous entry in a setting with a price cap.

Appendix

Proof of Lemma 3.1

Proof. Since $\lim_{q\to\infty} \pi(q, y, \overline{p}) < 0$ there exists some $M > 0$ such that a firm's best response is bounded by M. As in Amir and Lambson (2000) (AL), we can express a firm's problem as choosing total output, Q given y.

$$\max\{\tilde{\pi}(Q, y, \overline{p}) \equiv (Q - y) \min\{P(Q), \overline{p}\} - c(Q - y) : y \leq Q \leq y + M\} \quad (1)$$

We claim that for any $\overline{p} > c$, the maximand in (1) satisfies the single-crossing property in $(Q; y)$ on the lattice

$$\Phi = \{(Q, y) : 0 \leq y \leq (n - 1)M, y \leq Q \leq y + M\}$$

To see this, let $Q' > Q$ and $y' > y$ such that the points (Q', y'), (Q', y), (Q, y'), and (Q, y) are all in Φ. We assume that $\tilde{\pi}(Q', y, \overline{p}) > \tilde{\pi}(Q, y, \overline{p})$. We will show that $\tilde{\pi}(Q', y', \overline{p}) > \tilde{\pi}(Q, y', \overline{p})$.

Now, since $\tilde{\pi}(Q', y, \overline{p}) > \tilde{\pi}(Q, y, \overline{p})$ we have:

$$(Q' - y)[\min\{P(Q'), \overline{p}\} - c] > (Q - y)[\min\{P(Q), \overline{p}\} - c] \quad (2)$$

Note that $Q' \geq y' \implies Q' > y$ (similarly for Q). Moreover, since P is strictly decreasing it is implied by (2) that $P(Q) - c \geq 0$. To see this suppose to the contrary $P(Q) < c$. Then, since P is strictly decreasing and $\overline{p} > c$ we must have $P(Q') < P(Q) < c < \overline{p}$. Hence, (2) may be re-written: $(Q' - y)[P(Q') - c] > (Q - y)[P(Q) - c]$. Which is a contradiction since $(Q' - y) > (Q - y) > 0$ and $[P(Q') - c] < [P(Q) - c] < 0$. Thus, we must have $P(Q) \geq c$.

It may easily be verified that that $\frac{(Q'-y')(Q-y)}{Q'-y} > (Q - y')$. Using this inequality, along with (2), the fact that $Q' > y$, and the fact that $\min\{P(Q), \overline{p}\} \geq c$ see that

$$
\begin{aligned}
\tilde{\pi}(Q', y', \overline{p}) &= (Q' - y')[\min\{P(Q'), \overline{p}\} - c] \\
&> \tfrac{(Q'-y')(Q-y)}{Q'-y}[\min\{P(Q), \overline{p}\} - c] \\
&\geq (Q - y')[\min\{P(Q), \overline{p}\} - c] \\
&= \tilde{\pi}(Q, y', \overline{p})
\end{aligned}
$$

This establishes that the maximand in (1) satisfies the single-crossing property in (Q, y) on Φ. Also note that the feasible correspondence Φ is ascending in y and $\tilde{\pi}$ is continuous in Q. Then as shown in Milgrom and Shannon (1994) it follows that the maximal and minimal selections of $Q(y, \overline{p}) \equiv \arg\max\{(Q - y)[\min\{P(Q), y\} - c] : y \leq Q \leq y + M\}$ are nondecreasing in y.

From this point forward, the proof is similar to the proof of Theorem 2.2 in Amir and Lambson (2000). Define

$$B_n : [0, (n-1)M] \to 2^{[0,(n-1)M]}$$

$$y \to \left(\frac{n-1}{n}\right)(q\,(y, \overline{p}) + y)$$

where $q(y, \overline{p}) \equiv \arg\max_q \pi(q, y, \overline{p})$

Note that $Q(y, \overline{p}) = q(y, \overline{p}) + y$. Then since the maximal and minimal selections of $Q(y, \overline{p})$ are non-decreasing in y, immediately we have that the maximal and minimal selections of B_n are non-decreasing in y.

By, Tarski's fixed point theorem, there exists at least one fixed point, y^*, of B_n. A fixed point of B_n corresponds to a symmetric Nash equilibrium. To see this, note that $B_n(y^*) = y^*$ means

$$y^* = \frac{n-1}{n}(q(y^*, \overline{p}) + y^*)$$

or

$$q(y^*, \overline{p}) = \frac{y^*}{n-1}$$

Now, Topkis' theorem guarantees that the minimal selection of B_n exists. Denote the minimal selection of B_n by B_L. The symmetric equilibrium with the lowest total output corresponds to the smallest fixed point of B_L. Denote this fixed point by y_L^*. Denote the corresponding equilibrium total output by Q_L^*. Then since $\frac{n-1}{n}$ is strictly increasing in n this means that $\underline{B}_n$ is strictly increasing in n. Theorem 3 in Milgrom and Roberts (1994) implies that y_L^* is non-decreasing in in n. Since the minimal selection of $Q(y, \overline{p})$ is non-decreasing in y, immediately we have that Q_L^* is non-decreasing in n.

To establish the second part of the lemma we will first show that the equilibrium with the lowest total output corresponds to the equilibrium with the largest per-firm profit. Fix n and let x be the per-firm output in any symmetric equilibrium and let $q_n^* = \frac{Q_n^*}{n}$. Then

$$\begin{aligned} \pi_n\left(q_n^*, (n-1)q_n^*, \overline{p}\right) &= q_n^*\left(\min\{P(Q_n^*), \overline{p}\} - c\right) \\ &\geq x\left(\min\{P(x + (n-1)q_n^*), \overline{p}\} - c\right) \\ &\geq x\left(\min\{P(nx), \overline{p}\} - c\right) \\ &= \pi_n\left(x, (n-1)x, \overline{p}\right) \end{aligned}$$

The first inequality follows from the fact that $\underline{q}_n^*$ is a best response to $(n-1)\underline{q}_n^*$. The second inequality follows from the fact that $x \geq \underline{q}_n^*$ and that $P(\cdot)$ is strictly decreasing. Hence, the equilibrium with the lowest total output corresponds to the equilibrium with the highest second-stage per-firm profit. Denote by $\pi_n^*(\overline{p})$ the equilibrium second-stage profit to some firm i in the equilibrium with the lowest total output when n firms are active and the price cap is $\overline{p}$. Following the notation above, let $y_n^* = (n-1)q_n^*$ be the output of the $n-1$ other firms in equilibrium. Then we have

$$\begin{aligned} \pi_n^*(\overline{p}) &= q_n^*(\min\{P(Q_n^*), \overline{p}\} - c) \\ &\geq q_{n+1}^*(\min\{P(q_{n+1}^* + y_n^*), \overline{p}\} - c) \\ &\geq q_{n+1}^*(\min\{P(q_{n+1}^* + y_{n+1}^*), \overline{p}\} - c) \\ &= \pi_{n+1}^*(\overline{p}) \end{aligned}$$

The first inequality follows from the definition of q_n^*. The second inequality follows from the fact that y_n^* is non-decreasing in n, as demonstrated above. This establishes the claim.

□

Proof of Lemma 3.2

Proof. Fix $n \in \mathbb{N}$. Let $\overline{p}_1 > \overline{p}_2$ and note that

$$\begin{aligned}
\pi_n^*(\overline{p}_1) &= q_n^*(\overline{p}_1)(\min\{P(Q_n^*(\overline{p}_1)), \overline{p}_1\} - c) \\
&\geq q_n^*(\overline{p}_2)(\min\{P(q_n^*(\overline{p}_2) + (n-1)q_n^*(\overline{p}_1)), \overline{p}_1\} - c) \\
&\geq q_n^*(\overline{p}_2)(\min\{P(q_n^*(\overline{p}_2) + (n-1)q_n^*(\overline{p}_1)), \overline{p}_2\} - c) \\
&\geq q_n^*(\overline{p}_2)(\min\{P(q_n^*(\overline{p}_2) + (n-1)q_n^*(\overline{p}_2)), \overline{p}_2\} - c) \\
&= \pi_n^*(\overline{p}_2)
\end{aligned}$$

The first inequality follows from the fact that $q_n^*(\overline{p}_1)$ is a best response to $(n-1)q_n^*(\overline{p}_1)$. The second inequality follows since $\overline{p}_1 > \overline{p}_2$. The third inequality follows from the fact that for fixed n EST prove (in Theorem 1) that minimal equilibrium output per firm is non-increasing in the price cap. This means $q_n^*(\overline{p}_2) \geq q_n^*(\overline{p}_1)$. Since $P(\cdot)$ is decreasing, the inequality holds. This establishes the lemma □

Proof of Lemma 3.3

Proof. Let $\overline{p}_1 > \overline{p}_2$. Let n_i be the equilibrium number of firms under $\overline{p}_i$, $i \in \{1, 2\}$. We will establish the claim by contradiction. So, suppose $n_2 > n_1$. By definition, n_i must satisfy $\pi_{n_i}^*(\overline{p}_i) \geq K$ and $\pi_m^*(\overline{p}_i) < K$ for $m > n_i$. Since $n_2 > n_1$, we must have $\pi_{n_2}^*(\overline{p}_1) < K$. But by lemma (3.2) we must have $\pi_{n_2}^*(\overline{p}_1) \geq \pi_{n_2}^*(\overline{p}_2) \geq K$ which is a contradiction.

□

Proof of Proposition 1

Part (i)

Proof. Let $\overline{p}_1 > \overline{p}_2$. Let n_i be the equilibrium number of firms under $\overline{p}_i$, $i \in \{1,2\}$. Let $\hat{Q}_i = \hat{Q}(\overline{p}_i)$. First note that we must have $Q^*_{n_i}(\overline{p}_i) \geq \hat{Q}_i$, otherwise any one firm could increase output slightly and increase profit. Moreover, since $\overline{p}_1 > \overline{p}_2$ we must have $\hat{Q}_2 > \hat{Q}_1$

We must show that $Q^*_{n_2}(\overline{p}_2) \geq Q^*_{n_1}(\overline{p}_1)$. From lemma (3.3) we know that $n_1 \geq n_2$. However, if $n_1 = n_2$ then EST prove in theorem 1 that the desired result holds. So, without loss of generality assume that $n_1 > n_2$. We will proceed by contradiction. That is, suppose that $Q^*_{n_2}(\overline{p}_2) < Q^*_{n_1}(\overline{p}_1)$. Immediately we have $Q^*_{n_1}(\overline{p}_1) > Q^*_{n_2}(\overline{p}_2) \geq \hat{Q}_2 > \hat{Q}_1$. Since $n_1 > n_2$, then by definition of n_2 it must be the case that

$$\pi_{n_1}(q^*_{n_1}(\overline{p}_2), (n_1 - 1)q^*_{n_1}(\overline{p}_2), \overline{p}_2) < K \tag{3}$$

Now, consider the subgame with price cap $\overline{p}_2$ and n_1 active firms. Then, from theorem 1 in EST we know that $Q^*_{n_1}(\overline{p}_2) \geq Q^*_{n_1}(\overline{p}_1)$. We will show that this in fact holds with equality.

Suppose that $n_1 - 1$ firms produce a total output of $(n_1 - 1)q^*_{n_1}(\overline{p}_1)$. We will show that $q^*_{n_1}(\overline{p}_1)$ is a best response. Let $q > 0$. We must show that $\pi_{n_1}(q^*_{n_1}(\overline{p}_1), (n_1 - 1)q^*_{n_1}(\overline{p}_1), \overline{p}_2) \geq \pi_{n_1}(q, (n_1 - 1)q^*_{n_1}(\overline{p}_1), \overline{p}_2)$. Now see that:

$$\begin{aligned}
\pi_{n_1}(q^*_{n_1}(\overline{p}_1), (n_1 - 1)q^*_{n_1}(\overline{p}_1), \overline{p}_2) &= q^*_{n_1}(\overline{p}_1)(\min\{P(Q^*_{n_1}(\overline{p}_1)), \overline{p}_2\} - c) \\
&= q^*_{n_1}(\overline{p}_1)(\min\{P(Q^*_{n_1}(\overline{p}_1)), \overline{p}_1\} - c) \\
&\geq q(\min\{P(q + (n_1 - 1)q^*_{n_1}(\overline{p}_1)), \overline{p}_1\} - c) \\
&\geq q(\min\{P(q + (n_1 - 1)q^*_{n_1}(\overline{p}_1)), \overline{p}_2\} - c) \\
&= \pi_{n_1}(q, (n_1 - 1)q^*_{n_1}(\overline{p}_1), \overline{p}_2)
\end{aligned}$$

The first equality follows from the fact that $Q^*_{n_1}(\overline{p}_1) > \hat{Q}_2 > \hat{Q}_1$ (ie, the price cap is not binding under either cap). The first inequality follows by definition of $q^*_{n_1}(\overline{p}_1)$. The second inequality clearly holds since $\overline{p}_1 > \overline{p}_2$. Immediately, we have that $Q^*_{n_1}(\overline{p}_1)$ is an equilibrium quantity in the subgame with cap $\overline{p}_2$ and n_1 firms. Since it must be the case that $Q^*_{n_1}(\overline{p}_2) \geq Q^*_{n_1}(\overline{p}_1)$, we know that $Q^*_{n_1}(\overline{p}_2) = Q^*_{n_1}(\overline{p}_1)$ (ie $Q^*_{n_1}(\overline{p}_1)$ is the minimal equilibrium output in the subgame with price cap $\overline{p}_2$ and n_1 firms). Now since $Q^*_{n_1}(\overline{p}_1) > \hat{Q}_2 > \hat{Q}_1$

this means that:

$$\begin{aligned}
\pi_{n_1}(q^*_{n_1}(\overline{p}_2),(n_1-1)q^*_{n_1}(\overline{p}_2),\overline{p}_2) &= q^*_{n_1}(\overline{p}_1)(\min\{P(Q^*_{n_1}(\overline{p}_1)),\overline{p}_2\}-c) \\
&= q^*_{n_1}(\overline{p}_1)(\min\{P(Q^*_{n_1}(\overline{p}_1)),\overline{p}_1\}-c) \\
&= \pi_{n_1}(q^*_{n_1}(\overline{p}_1),(n_1-1)q^*_{n_1}(\overline{p}_1),\overline{p}_1) \\
&\geq k
\end{aligned}$$

But this contradicts (3). Thus, we must have $Q^*_{n_2}(\overline{p}_2) \geq Q^*_{n_1}(\overline{p}_1)$

□

Part (*ii*)

Proof. Let $W(\overline{p})$ be total welfare in the equilibrium with the lowest output when the price cap is $\overline{p}$. Let $\overline{p}_1 > \overline{p}_2$. Must show that $W(\overline{p}_2) \geq W(\overline{p}_1)$. Let $Q^*_i = Q^*_{n_i}(\overline{p}_i)$, $i \in \{1,2\}$. Now note:

$$\begin{aligned}
W(\overline{p}_2) &= \textstyle\int_0^{Q^*_2} P(z)\,dz - cQ^*_2 - n_2k \\
&= \textstyle\int_0^{Q^*_2}[P(z)-c]\,dz - n_2k \\
&\geq \textstyle\int_0^{Q^*_2}[P(z)-c]\,dz - n_1k \\
&\geq \textstyle\int_0^{Q^*_1}[P(z)-c]\,dz - n_2k \\
&= W(\overline{p}_1)
\end{aligned}$$

The first inequality follows since $n_1 \geq n_2$. The second inequality follows from the fact that $Q^*_2 \geq Q^*_1$ and that $P(Q^*_2) \geq c$ (otherwise any firm could increase its second-stage profit by reducing output).

□

Part (*iii*)

Proof. Let $\overline{p}_1 > \overline{p}_2$. Let $CS(Q,\overline{p})$ denote consumer surplus when total production is Q and the price cap is $\overline{p}$. See that

$$CS(Q,\overline{p}) = \int_0^Q [P(z) - \min\{P(Q),\overline{p}\}]\,dz$$

Note that $CS(Q,\overline{p})$ is increasing in Q and is decreasing in $\overline{p}$. Since $Q^*_{n_2}(\overline{p}_2) \geq Q^*_{n_1}(\overline{p}_1)$ and $\overline{p}_2 < \overline{p}_2$, immediately we have that $CS(Q^*_{n_2}(\overline{p}_2),\overline{p}_2) \geq CS(Q^*_{n_1}(\overline{p}_1),\overline{p}_1)$.

□

Proof of Proposition 2

Proof. Let y denote the total output of all firms other than some firm i. Note that each firm i solves

$$\max_{q \geq 0}\{\pi(q, y, \overline{p}) = q \min\{P(q + y), \overline{p}\} - C(q)\}$$

Now, let $\hat{p}$ be as defined in the statement of the theorem and consider a slightly modified version of this game, where each firm i solves:

$$\max\{\pi(q, y, \overline{p}) = q \min\{P(q + y), \overline{p}\} - C(q) \text{ such that } 0 \leq q \leq C^{'-1}(\hat{p})\} \quad (4)$$

We then claim that for all $\overline{p}$, q^* is an equilibrium of the original game if and only if q^* is an equilibrium of the modified game. To see this, first let q^* be an equilibrium output choice of the modified game. Then q^* satisfies:

$$q^* \in \arg\max_{q}\{q \min\{P(q + (n-1)q^*), \overline{p}\} - C(q) \text{ such that } 0 \leq q \leq C^{'-1}(\hat{p})\}$$

Now, suppose that q^* is not an equilibrium output choice of the original game. This implies that there exists some $\tilde{q} > C^{'-1}(\hat{p})$ such that

$$\pi(\tilde{q}, (n-1)q^*, \overline{p}) > \pi(q^*, (n-1)q^*, \overline{p}) \quad (5)$$

Now, let q^{**} be an equilibrium output choice of the original game. Note that it must be that $q^{**} \leq C^{'-1}(\hat{p})$. To see this, suppose not. Then $q^{**} > C^{'-1}(\hat{p})$. But since q^{**} is an equilibrium of the original game, it must be that $\min\{P(nq^{**}), \overline{p}\} \geq C^{'}(q^{**})$. Hence we must have

$$\begin{aligned} P(nq^{**}) &\geq C'(q^{**}) \\ &> C'(C'^{-1}(\hat{p})) \\ &= \hat{p} \\ &= P(nC'^{-1}(\hat{p})) \end{aligned}$$

But this immediately implies that $q^{**} < C'^{-1}(\hat{p})$ which is a contradiction. Thus, $q^{**} \leq C'^{-1}(\hat{p})$. Hence, q^{**} is a feasible output choice of the modified game. By the optimality of q^* when the other firms choose $(n-1)q^*$, this means

$$\pi(q^*, (n-1)q^*, \overline{p}) \geq \pi(q^{**}, (n-1)q^*, \overline{p}) \tag{6}$$

Finally, by the optimality of q^{**} when the other firms choose output $(n-1)q^{**}$ we must have

$$\pi(q^{**}, (n-1)q^{**}, \overline{p}) \geq \pi(\tilde{q}, (n-1)q^{**}, \overline{p}) \tag{7}$$

Inequalities (5) and (6) imply that

$$\pi(\tilde{q}, (n-1)q^*, \overline{p}) > \pi(q^{**}, (n-1)q^*, \overline{p})$$

Combining this fact with (7) we see that:

$$\pi(\tilde{q}, (n-1)q^*, \overline{p}) - \pi(q^{**}, (n-1)q^*, \overline{p}) > 0 \geq \pi(\tilde{q}, (n-1)q^{**}, \overline{p}) - \pi(q^{**}, (n-1)q^{**}, \overline{p})$$

Equivalently,

$$\pi(\tilde{q}, (n-1)q^*, \overline{p}) - \pi(\tilde{q}, (n-1)q^{**}, \overline{p}) > 0 \geq \pi(q^{**}, (n-1)q^*, \overline{p}) - \pi(q^{**}, (n-1)q^{**}, \overline{p}) \tag{8}$$

Now note that for any q and any $\overline{p}$, $\pi(q, y, \overline{p})$ is non-increasing in y. Then the left-hand side of (8) implies that $(n-1)q^* < (n-1)q^{**}$, while the right-hand side of (8) implies that $(n-1)q^* \geq (n-1)q^{**}$ which is a contradiction.

Hence, it must be that q^* is an equilibrium output choice of the original game.

Now, suppose that q^* is an equilibrium of the original game. As shown above, it must be that $q^* \leq C^{'-1}(\hat{p})$. The fact that q^* is an equilibrium of the modified game is then immediate. Thus, q^* is an equilibrium output choice of the original game if and only if q^* is an equilibrium output choice of the modified game.

We claim that for all $\overline{p} > \hat{p}$ the maximand in (4) satisfies the single-crossing property in $(q; -\overline{p})$ for each y. To establish the claim, fix $y \geq 0$ and let $\overline{p}' > \overline{p} > \hat{p}$ and let $q < q' \leq C^{'-1}(\hat{p})$. Assume that $\pi(q', y, \overline{p}') > \pi(q, y, \overline{p}')$. We will show that this implies $\pi(q', y, \overline{p}) > \pi(q, y, \overline{p})$. To do this, we will seperately examine three cases.

(a) First, suppose that the price cap $\overline{p}$ binds for both quantities q and q'. Then since $\overline{p} > \hat{p}$ immediately it must be that $C^{'}(q) < C^{'}(q') \leq \hat{p} < \overline{p}$. Hence, $\pi(z, y, \overline{p}) = z\overline{p} - C(z)$ is strictly increasing in z for all $z \in [q, q']$. This immediately implies that $\pi(q', y, \overline{p}) > \pi(q, y, \overline{p})$

(b) Next, suppose that the price cap, $\overline{p}$ binds for q but not for q'. Since, the lower cap does not bind for output q', the higher cap must not have been binding either. So,

$$\begin{aligned} \pi(q', y, \overline{p}) &= \pi(q', y, \overline{p}') \\ &> \pi(q, y, \overline{p}') \\ &\geq \pi(q, y, \overline{p}) \end{aligned}$$

(c) If the cap does not bind for either quantity, then the profits are the same under $\overline{p}$ as they were under $\overline{p}'$

Hence, for all $\overline{p} > \hat{p}$ and for all y, the maximand in (4) satisfies the single-crossing property in $(q; -\overline{p})$. Let $q(y, \overline{p})$ denote the argmax of (4). Then since the maximand in (4) satisfies the single-crossing property in $(q; -\overline{p})$, the feasible correspondence is constant in $\overline{p}$, and profit is continuous in own output, then theorem 4 in Milgrom and Shannon implies that the minimal and maximal selections of $q(y, \overline{p})$ are non-increasing in $\overline{p}$.

As done in the proof of lemma (3.1), it will be useful to think of a firm choosing cumulative output, Q, of the modified game given the other firms produce y. Let $Q \equiv x + y$ and let $Q(y, \overline{p})$ be defined as follows

$$Q(y,\overline{p}) = \arg\max\{(Q-y)\min\{P(Q),\overline{p}\} - C(Q-y) \ \text{ s.t. } \ y \leq Q \leq y + C^{'-1}(\hat{p})\} \tag{9}$$

Note that $Q(y, \overline{p})$ solves (9) if and only if $q(y, \overline{p}) = Q(y, \overline{p}) - y$ solves (4). Now, using an identical argument as given in the proof of lemma 3.4, the maximand in (9) has strict increasing differences in (Q, y) on the feasible lattice. Finally, the feasible correspondence is ascending in y. Hence, every selection of $Q(y, \overline{p})$ is non-decreasing in y for fixed $\overline{p}$.

Using the definition of $q(y, \overline{p})$ given above, define the following correspondence:

$$B_{\overline{p}}(y) = \frac{n-1}{n}(q(y, \overline{p}) + y)$$

As argued in the proof of lemma 3.1, the maximal and minimal selections of B exist (denote these B_L and B_H, respectively) and the minimal and maximal equilibrium outputs (of the modified game) correspond to the smallest and largest fixed points of B_L and B_H, respectively.

Now, since every selection of $Q(y, \overline{p}) = q(y, \overline{p}) + y$ is non-decreasing in y, it holds that every selection of B is non-decreasing in y. Moreover, as argued above the smallest and largest selections of $q(y, \overline{p})$ are non-increasing in $\overline{p}$. Hence, B_L and B_H are non-increasing in $\overline{p}$.

Then, by theorem 3 in Milgrom and Shannon (1994), the smallest and largest fixed points of B (denote these $y_L^*(\overline{p})$ and $y_H^*(\overline{p})$, respectively) are non-increasing in $\overline{p}$.

Since every selection of $Q(y, \overline{p})$ is non-decreasing in y, and $y_L^*(\overline{p})$ and $y_H^*(\overline{p})$ are non-increasing in $\overline{p}$, it follows that the smallest and largest aggregate equilibrium outputs, $Q_L^*(\overline{p})$ and $Q_H^*(\overline{p})$, are non-increasing in $\overline{p}$. This means that the smallest and largest equilibrium outputs of the modified game are non-increasing in the price cap. But as shown previously, the equilibria of this modified game are the same as the equilibria of the original game. Hence, the smallest and largest equilibrium outputs of the original game must be non-increasing in the price cap (for caps above $\hat{p}$).

To establish part (ii), note that equilibrium welfare under some price cap $\overline{p}$ is given by:

$$\begin{aligned} W^*(\overline{p}) &= \int_0^{Q^*(\overline{p})} P(z)\,dz - nC\left(\frac{Q^*(\overline{p})}{n}\right) \\ &= \int_0^{Q^*(\overline{p})} \left[P(z) - C'\left(\frac{z}{n}\right)\right]\,dz \end{aligned}$$

Where $q^*(\overline{p})$ denotes either the smallest or largest equilibrium per-firm output and $Q^*(\overline{p}) = nq^*(\overline{p})$. Since it must be that $P(Q^*(\overline{p})) \geq C'(q^*)$ it must be that for all $z \in [0, Q^*(\overline{p})]$ we have $P(Q^*(\overline{p})) > C'(q^*)$. Then for all $\overline{p} > \hat{p}$ it follows that $Q^*(\overline{p})$ is non-increasing in $\overline{p}$. Thus, equilibrium welfare is non-increasing in $\overline{p}$ for all $\overline{p} > \hat{p}$. This establishes part (ii).

To establish part (iii), we will first show that output and welfare are non-decreasing in the cap for all $\overline{p} < \hat{p}$. So, fix $\overline{p} < \hat{p}$, let q^* denote an equilibrium output choice under the cap and let $Q^* = nq^*$. First, suppose that $P(Q^*) \leq \overline{p}$. Since, $\overline{p} < \hat{p}$ this means $\overline{p} < P(nC'^{-1}(\overline{p}))$. So, we must have $P(Q^*) < P(nC'^{-1}(\overline{p}))$. But this means $q^* > C'^{-1}(\overline{p})$ which implies $C'(q^*) > \overline{p} > P(Q^*)$ which contradicts the optimality of q^*. Thus, we must have $P(Q^*) < \overline{p}$. So, per-firm equilibrium profit must be given by $\pi(q^*, (n-1)q^*, \overline{p}) = q^*\overline{p} - C(q^*)$. Clearly, in this case any output choice other than $q^* = C'^{-1}(\overline{p})$ is sub optimal assuming profit at this level of output is positive (If profit is not positive at this level of output then each firm produces zero.)

Now, let $\overline{p} = \hat{p}$. By an argument similar to that made above, and using the fact that profit is positive if each firm produces $q = C'^{-1}(\hat{p})$, it may be verified that equilibrium output is strictly positive and satisfies$q^* = C'^{-1}(\overline{p}) = C'^{-1}(\hat{p})$. Hence, $P(Q^*) = \overline{p} = \hat{p}$

Since, C'^{-1} is strictly increasing, it must be that equilibrium output strictly decreases as we lower the cap below $\hat{p}$. An argument analogous to that given for part (ii) may establish that equilibrium welfare strictly decreases as we lower the cap below $\hat{p}$.

Finally, since it must be that $\hat{p} < p^{\infty}$, and by the argument given above it must be that $P(Q^*) = \hat{p}$. This means $Q^* > Q^{\infty}$. So, when the cap is equal to $\hat{p}$ equilibrium output must be strictly greater than equilibrium output in the absence of a cap. It holds that welfare must be strictly higher under the cap than in the absence of the cap.

$\square$

Proof of Lemma 3.4

Proof. From the proof of lemma (3.1) in the constant marginal cost case, it can be seen that the sufficient condition for the conclusion to hold[9] is for $\tilde{\pi}(Q, y, \overline{p}) \equiv (Q - y)\min\{P(Q), \overline{p}\} - C(Q - y)$ to satisfy the single crossing property in $(Q; y)$ on the lattice

$$\Phi \equiv \{(Q, y)|\ 0 \leq y \leq (n-1)M,\ y \leq Q \leq y + M\}$$

Here, we will demonstrate that $\tilde{\pi}$ has strictly increasing differences in (Q, y) which then implies that the single-crossing property is satisfied. The remainder of the argument follows from the previous proof.

Now let $Q' > Q$, $y' > y$ where $Q' > y'$ and $Q > y$. Then $\tilde{\pi}$ has strict ID in (Q, y) on Φ if and only if

$$\tilde{\pi}(Q', y') - \tilde{\pi}(Q', y) > \tilde{\pi}(Q, y') - \tilde{\pi}(Q, y)$$

Plugging in, and collecting like terms we see that the above expression is equivalent to

$$(y'-y)\left[\min\{P(Q), \overline{p}\} - \min\{P(Q'), \overline{p}\}\right] > C(Q'-y') - C(Q'-y) + C(Q-y) - C(Q-y') \tag{10}$$

Note that $Q' > Q$, $y' > y$ and the fact that $P(\cdot)$ is strictly decreasing, means the left-hand side of (10) is weakly positive. Now see that the right-hand side of (10) is strictly negative if and only if

[9] See Amir and Lambson (2000)

$$C(Q'-y')-C(Q'-y)<C(Q-y')-C(Q-y) \tag{11}$$

Define the function $H(Q,y)\equiv C(Q-y)$. Note that (11) is satisfied if and only if H has strict decreasing differences (DD) on Φ. But note that since H is differentiable on Φ, if the cross partial derivative of H with respect to Q and y is strictly negative, then H has strict DD. Then, see that the cross partial derivative of H with respect to Q and y is given by $-C''(Q-y)<0$. Hence, the right-hand side of (10) is strictly less than zero.

This establishes that $\tilde{\pi}$ has strict ID on Φ. The remainder of the proof is identical to the proof with constant marginal cost.

□

Proof of Lemma 3.5

Proof. First, note that since $\overline{p}\geq ATC^m$ it is profitable for at least one firm to enter. To establish the claim, lemma (3.4) implies that it suffices to show that in the subgame with n^∞ firms and a price cap $\overline{p}$, equilibrium profit is strictly less than the cost of entry.

Fix $\overline{p}\in[ATC^m,p^\infty)$. Let Q^* be the equilibrium total output in the subgame with n^∞ firms with price cap $\overline{p}$. Let $q^*=\frac{Q^*}{n^\infty}$. Finally, let π^* denote equilibrium second-stage profit with n^∞ firms and a price cap.

First, suppose $q^*>q^\infty$. Then we have:

$$\begin{aligned}
K &= q^\infty P(Q^\infty)-C(q^\infty)\\
&\geq q^*P(q^*+(n^\infty-1)q^\infty)-C(q^*)\\
&> q^*P(Q^*)-C(q^*)\\
&\geq q^*\min\{P(Q^*),\overline{p}\}-C(q^*)\\
&= \pi^*
\end{aligned}$$

The first inequality follows by definition of q^∞, while the strict inequality follows since $q^*>q^\infty$. Now suppose that $q^*\leq q^\infty$. See that:

$$
\begin{aligned}
K &= q^{\infty}P(Q^{\infty}) - C(q^{\infty}) \\
&\geq q^{*}P(Q^{\infty}) - C(q^{*}) \\
&> q^{*}\overline{p} - C(q^{*}) \\
&\geq q^{*}\min\{P(Q^{*}), \overline{p}\} - C(q^{*}) \\
&= \pi^{*}
\end{aligned}
$$

To see why the first inequality holds, note that Q^{∞} must satisfy $P(Q^{\infty}) > C'(q^{\infty})$. But this means $P(Q^{\infty}) > C'(x)$ for all $x \in [0, q^{\infty}]$. Hence, the function $xP(Q^{\infty}) - C(x)$ is strictly increasing in x for $x \in [0, q^{\infty}]$. Since $q^{*} \leq q^{\infty}$, the inequality holds. Finally, note that the strict inequality follows since $\overline{p} < P(Q^{\infty})$.

Hence, we must have $\pi^{*} < K$.

□

Proof of Proposition 3

Proof. First note that since the entry constraint is binding, by Lemma (3.5) any price cap, $\overline{p} \in [ATC^{m}, p^{\infty})$, results in the entrance of at least one firm and at most $n^{\infty} - 1$ firms. First, consider the subgame with $n^{\infty} - 1$ firms under a price cap $\overline{p} \in [ATC^{m}, p^{\infty})$. We will then show that maximal equilibrium profit in this subgame is at least as large as the cost of entry.

Let $\pi(Q, y, \overline{p}) = (Q - y)\min\{P(Q), \overline{p}\} - C(Q - y)$ be profit for a firm if total output is Q, rivals' output is y, and the price cap is $\overline{p} \in [ATC^{m}, p^{\infty})$. Initially we suppose that $y = (n-2)C'^{-1}(\overline{p})$; that is, each rival firm sets its output such that marginal cost is equal to the price cap. Note that $\pi(Q, y, \overline{p})$ is continuous in Q. Also

$$
\pi_{Q}(Q, y, \overline{p}) = \begin{cases} \overline{p} - C'(Q - y), & Q < P^{-1}(\overline{p}) \\ P(Q) - (Q - y)P'(Q) - C'(Q - y), & Q > P^{-1}(\overline{p}) \end{cases}
$$

For $Q > P^{-1}(\overline{p})$ we have the following inequalities:

$$
\pi_{Q}(Q, y, \overline{p}) < \overline{p} - C'(Q - y) < 0, \tag{12}
$$

where the first inequality follows because the inverse demand function is strictly decreasing in Q. The second inequality is due to the following argument. First, the assumption that $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$ implies that, $Q^\infty > (n^\infty - 1)C'^{-1}(p^\infty) \geq (n^\infty - 1)C'^{-1}(\overline{p}) = y + C'^{-1}(\overline{p})$. Second, $Q > P^{-1}(\overline{p})$ implies $Q > Q^\infty$ so, $Q - y > C'^{-1}(\overline{p})$. Third, this implies that $\overline{p} - C'(Q - y) < 0$, which is the second inequality in equation (12) above.

Since the payoff function is strictly decreasing in Q for $Q > P^{-1}(\overline{p})$, it must be that the best response Q to y is in the interval, $[y, P^{-1}(\overline{p})]$. Strict convexity of the cost function implies that the payoff function is strictly concave in Q in this interval. The unique best response to y satisfies, $\overline{p} = C'(Q - y)$; that is, it is optimal for a firm to set its output such that marginal cost is equal to the price cap when each of its rivals follows the same policy. The optimal choice of Q is in the interior of $[y, P^{-1}(\overline{p})]$ since (by assumption) $C'(\frac{Q^\infty}{n^\infty - 1}) > p^\infty$. $q* = C'^{-1}(\overline{p})$ is equilibrium output per firm in the subgame with $n^\infty - 1$ firms. A proof-by-contradiction can be used to show that this is the unique subgame equilibrium. Equilibrium profit is given by $\pi^*(\overline{p}) = q^*\overline{p} - C(q^*)$

To demonstrate (i), we show that in the subgame with $n^\infty - 1$ firms for all $\overline{p} \in [ATC^m, p^\infty)$ we have $\pi^*(\overline{p}) \geq K$. Let q^m solve $\min_x ATC(x)$. Then see that:

$$\begin{aligned}
\pi^*(\overline{p}) &= q^*\overline{p} - C(q^*) \\
&\geq q^m\overline{p} - C(q^m) \\
&\geq q^m ATC^m - C(q^m) = K
\end{aligned}$$

The first inequality follows by the optimality of q^* and the second follows since $\overline{p} \geq ATC^m$. Hence, the equilibrium number of firms is $n^\infty - 1$. Then since $C'' > 0$, we have that q^* is increasing in $\overline{p}$. Moreover, since $\overline{p} < p^\infty$, we must have $P\left((n^\infty - 1)C'^{-1}(\overline{p})\right) > P\left((n^\infty - 1)C'^{-1}(p^\infty)\right) > p^\infty$. So, $(n^\infty - 1)C'^{-1}(\overline{p}) < Q^\infty$. This establishes $(i) - (iii)$.

Finally, note that for any relevant cap, equilibrium welfare is given by:

$$W^*(\overline{p}) = \int_0^{(n^\infty-1)C'^{-1}(\overline{p})} P(z)\,dz - (n^\infty - 1)C(C'^{-1}(\overline{p})) - (n^\infty - 1)K$$

Now see that

$$W^{*'}(\overline{p}) = \frac{n^\infty - 1}{C''(C'^{-1}(\overline{p}))}\left[P\left((n^\infty - 1)C'^{-1}(\overline{p})\right) - \overline{p}\right] > 0$$

Which establishes (iv).

□

Proof of Lemma 4.1

Proof. As with lemma (3.4) we will demonstrate that for fixed $\overline{p}$,

$$\tilde{\pi}(Q, y, \overline{p}) = E\left[(Q - y)\min\{P(Q, \theta), \overline{p}\} - (Q - y)c\right]$$

has strict increasing differences in Q and y on the lattice

$$\Phi = \{(Q, y)|\ 0 \leq y \leq (n-1)M,\ y \leq Q \leq y + M\}$$

Define

$$\tilde{P}(Q, \overline{p}) = E[\min\{P(Q, \theta), \overline{p}\}] = \int_{\underline{\theta}}^{\theta^b(Q,\overline{p})} P(Q, \theta)\,dF(\theta) + \int_{\theta^b(Q,\overline{p})}^{\overline{\theta}} \overline{p}\,dF(\theta)$$

Note that since $f(\theta) > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$, and and$P_1(Q, \theta) < 0$ for all $Q \geq y$, $\theta \in \Theta$ we have[10]

$$\tilde{P}_1(Q, \overline{p}) = \int_{\underline{\theta}}^{\theta^b(Q,\overline{p})} P_1(Q, \theta)\,dF(\theta) < 0$$

[10]Note that any choice of total output Q such that $\theta^b(Q, \overline{p}) \leq \underline{\theta}$ is strictly dominated as this would imply $P(Q, \underline{\theta}) > \overline{p}$. In this case, the firm could increase output slightly and strictly increase profit since $\overline{p} > c$. Thus, we may without loss of generality assume $\theta^b > \underline{\theta}$

Now see that $\tilde{\pi}(Q, y) = (Q - y)\left(\tilde{P}(Q, \overline{p}) - c\right)$. Clearly, the cross partial derivative of $\tilde{\pi}$ with respect to Q and y on Φ is given by $-\tilde{P}_1(Q, \overline{p}) > 0$

The remainder of the proof is analogous to lemma (3.1).

□

Proof of Proposition 4

Proof. Concavity of $p(Q)$ implies the existence of a unique symmetric second-stage equilibrium. Now, if $\pi^\infty > K$ then a large enough binding cap will not change the equilibrium number of firms[11]. For a fixed number of firms, GZ establish that any price cap $\overline{p} \in [\overline{MR}_n, \overline{\rho}_n)$ both increases output and total welfare. Thus, a high enough price cap will leave the equilibrium number of firms unchanged and will increase both output and welfare.

If $\pi^\infty = K$ then there exists a range of price caps below $\overline{\rho}^\infty$ such that the equilibrium number of firms decreases by exactly one. Also, if $\pi^\infty = K$ then by Assumption (1e) we must have $n^\infty \geq 2$. For clarity, in this proof we let $n^\infty \equiv n$ and we let Q_m denote equilibrium total output in the absence of a price cap when m firms enter in the first stage. We also define per-firm output and profit analogously. We begin by demonstrating that $\pi_{n-1} > K$.

In a subgame with no cap and m firms, the symmetric equilibrium condition is given by:

$$\mu - c + p(Q_m) + \frac{Q_m}{m-1} p'(Q_m) = 0 \tag{13}$$

where $\mu = E[\theta]$. Given that $p(Q)$ is strictly decreasing and concave, equation (13) implies that $Q_{n-1} < Q_n$ and $q_{n-1} > q_n$. The inequality for total output implies that $p(Q_{n-1}) > p(Q_n)$, and this inequality combined with the inequality for output per firm, implies that $\pi_{n-1} > \pi_n = \pi^\infty = K$.

We now demonstrate that in the absence of a price cap, total equilibrium welfare is higher in the subgame with $n - 1$ firms than with n firms. Let

[11] It follows from the concavity of P that, for a fixed number of firms, equilibrium profit is continuous in the price cap. Hence, a high enough cap would leave equilibrium profit above the cost of entry when n^∞ firms enter

$\Delta Q \equiv Q_n - Q_{n-1}$. We claim that $\Delta Q \leq \frac{1}{n} q_n$. To establish the claim, we will proceed by contradiction. Suppose that $\Delta Q > \frac{1}{n} q_n$. Equivalently, $Q_{n-1} < g$ where $g \equiv Q_n - \frac{1}{n} q_n$.

Now , since $g > Q_n$ and since p is decreasing and concave, equation (13) implies that:

$$0 > \mu - c + p(g) + \frac{g}{n-1} p'(g) \tag{14}$$

Moreover, the concavity of p implies that:

$$p(g) \geq p(Q_n) - \left(\frac{1}{n} q_n\right) p'(g) \tag{15}$$

Then equations (14) and (15) together imply that

$$0 > \mu - c + p(Q_n) + q_n p'(g)$$

Note that $g < Q_n \implies p'(g) \geq p'(Q_n)$. So,

$$0 > \mu - c + P(Q_n) + q_n P'(Q_n)$$

But this contradicts (13) for $m = n$. So we must have $0 \leq \Delta Q_n \leq \frac{1}{n} q_n$.

Now, let

$$B(z) \equiv E\left[\int_0^z P(s, \theta)\, ds\right] = \mu z + b(z)$$

where

$$b(z) \equiv \int_0^z p(s)\, ds$$

Let $W(z, m) = B(z) - cz - mK$ denote total welfare when total production is z and m firms enter. Let $\Delta W \equiv W(Q_{n-1}, n-1) - W(Q_n, n)$ denote the change in equilibrium welfare when the number of firms decreases by one in the absence of a cap. Note that $\pi_n = K$ implies that

$$\Delta W = -\left[B(Q_n) - B(Q_{n-1}) - (\Delta Q_n)c\right] + \pi_n$$

Note that the term in square brackets above is given by

$$(\mu - c)\Delta Q + \int_{Q_{n-1}}^{Q_n} p(s)\, ds$$

Now, let $T(s;x) = p'(x)s + p(x) - p'(x)x$ denote the equation of the line tangent to p at the point x. Note that since p is concave and decreasing, for all $s \in [Q_{n-1}, Q_n]$, $p(s) \leq T(s, Q_n)$. This means

$$\int_{Q_{n-1}}^{Q_n} p(s)\, ds \leq \int_{Q_{n-1}}^{Q_n} T(s; Q_n)\, ds = \Delta Q p(Q_n) - \frac{1}{2}(\Delta Q)^2 p'(Q_n)$$

Plugging this back into the expression for ΔW we see that:

$$\Delta W \geq \frac{1}{2} p'(Q_n)(\Delta Q_n)^2 - (p(Q_n) + \mu - c)\, \Delta Q + \pi_n$$

Then, from (13), it follows that $p(Q_n) + \mu - c = -p'(Q_n)q_n$ and $\pi_n = -p'(Q_n)(q_n)^2$. Combining this with the fact that $\Delta Q \leq \frac{1}{n} q_n$, $p' < 0$ and $n \geq 2$ yields:

$$\begin{aligned}
\Delta W &\geq \tfrac{1}{2} p'(Q_n)(\Delta Q)^2 + p'(Q_n) q_n \Delta Q - p'(Q_n)(q_n)^2 \\
&\geq \tfrac{1}{2} p'(Q_n)(\tfrac{1}{n} q_n)^2 + p'(Q_m)\tfrac{1}{n}(q_n)^2 - p'(Q_n)(q_n)^2 \\
&= p'(Q_n) q_n^2 \left(\tfrac{1}{2n^2} + \tfrac{1}{n} - 1\right) \\
&> 0
\end{aligned}$$

Thus we see that, in the absence of a price cap total welfare increases when the number of firms that enter decreases by one. Let $\overline{MR}_m$ denote the maximal equilibrium marginal revenue when m firms enter and there is no price cap. Clearly, by assumptions placed on P,

$$\overline{MR}_m = \overline{\theta} + p(Q_m^\infty) + p'(Q_m^\infty) q_m^\infty$$

By the first order equilibrium conditions, it follows that $p(Q_m^\infty)+p'(Q_m^\infty)q_m^\infty = c-\mu$. Hence, independent of m it is clear that

$$\overline{MR}_m = \overline{\theta} + c - \mu$$

and since $p' < 0$:

$$\overline{MR}_m < \overline{\theta} + p(Q_m^\infty) = \overline{\rho}_m$$

Since $\overline{MR}_m$ is independent of m, we drop this subscript and write $\overline{MR}$. Let $\overline{\rho}_m = P(Q_m^\infty, \overline{\theta})$. Now, since $\pi^\infty = K$ and $\pi_{n-1} > K$, there exists a range of price caps strictly less than $\overline{\rho}_n$ such that the equilibrium number of firms will decrease by exactly one[12]. Let $\hat{p}_1$ denote the smallest of these price caps. Let $\hat{p} \equiv \max\{\hat{p}_1, \overline{MR}\}$. Then since $\overline{MR} < \overline{\rho}_n$ and $\hat{p}_1 < \overline{\rho}_n$ we have $\hat{p} < \overline{\rho}_n$. Also see that under any price cap $\overline{p} \in [\hat{p}, \overline{\rho}_n)$ the equilibrium number of firms is $n-1$. Finally note that by lemma (4.1) it is clear that $\overline{\rho}_n \leq \overline{\rho}_{n-1}$.

Choose $\overline{p} \in [\hat{p}, \overline{\rho}_n)$. Then since $\overline{p} \in [\overline{MR}, \overline{\rho}_{n-1})$ by GZ Theorem 1, it follows that equilibrium welfare under the price cap is strictly greater than equilibrium welfare in the subgame with $n-1$ firms and no cap. Since welfare in the absence of a cap when $n-1$ firms enter is strictly higher than welfare when n firms enter, the result follows.

□

Proof of Lemma 4.2

Proof. By the definition of $\tilde{\theta}(X_m)$ and $\tilde{\theta}(X_m)$ note that the first-order condition given in (**??**) may be re-written as:

$$\int_{\tilde{\theta}(X_m)}^{\overline{\theta}} \left[\theta - \tilde{\theta}(X_m)\right] dF(\theta) = \int_{\tilde{\theta}(X_n)}^{\overline{\theta}} \left[\theta - \tilde{\theta}(X_n)\right] dF(\theta) = c$$

Let $G(s) = \int_s^{\overline{\theta}} [\theta - s\,] dF(\theta)$ and note that $G'(s) = \int_s^{\overline{\theta}} -1\, dF(\theta)$. Note that $G'(s) < 0$ for all $s < \overline{\theta}$. Then, the first-order conditions imply that

[12] Once again, this follows since concavity of P implies the continuity of equilibrium profit in the price cap

$G(\tilde{\theta}(X_m)) = G(\tilde{\theta}(X_n))$. Since $\tilde{\theta}(X_m) < \overline{\theta}$ and $\tilde{\theta}(X_n) < \overline{\theta}$ it must be that $\tilde{\theta}(X_m) = \tilde{\theta}(X_n)$

□

Proof of Lemma 4.3

Proof. First, by lemma (4.2) it follows that $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1}) = \tilde{\theta}$. First, fix $\theta < \tilde{\theta}$. In this case we have $Q_n(\theta) = \tilde{Q}_n(\theta)$ and $Q_{n-1}(\theta) = \tilde{Q}_{n-1}(\theta)$. Both output choices must satisfy their respective first-order conditions:

$$\theta + p(\tilde{Q}_n(\theta)) + \tilde{q}_n(\theta)p'(\tilde{Q}_n(\theta)) = 0$$

and

$$\theta + p(\tilde{Q}_{n-1}(\theta)) + \tilde{q}_{n-1}(\theta)p'(\tilde{Q}_{n-1}(\theta)) = 0$$

From the first-order conditions, the concavity of p together with the fact that $p' < 0$ implies that $\tilde{Q}_n(\theta) > \tilde{Q}_{n-1}(\theta)$ and $\tilde{q}_n(\theta) < \tilde{Q}_{n-1}(\theta)$

Now, since $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1}) = \tilde{\theta}$ it follows from the definitions of $\tilde{\theta}(X_n)$ and $\tilde{\theta}(X_{n-1})$ that

$$\tilde{\theta} + p(X_n) + x_n p'(X_n) = 0$$

and

$$\tilde{\theta} + p(X_{n-1}) + x_{n-1} p'(X_{n-1}) = 0$$

Once again, these equations together with our assumptions on p ensure that $X_n > X_{n-1}$ and $x_n < x_{n-1}$

This establishes (i) and (ii)

Now see that

$$\pi_{n-1} = \int_{\underline{\theta}}^{\tilde{\theta}(X_{n-1})} \tilde{q}_{n-1}(\theta)[\theta + p(\tilde{Q}_{n-1}(\theta))]\, dF(\theta) + x_{n-1}\left(\int_{\tilde{\theta}(X_{n-1})}^{\overline{\theta}} [\theta + p(X_{n-1})\,]dF(\theta) - c\right)$$

Note that the first-order condition given in (**??**) implies that

$$\left(\int_{\tilde{\theta}(X_{n-1})}^{\overline{\theta}} [\theta + p(X_{n-1})\,] dF(\theta) - c\right) > 0$$

Then by parts (i) and (ii) above, the result is immediate.

□

Proof of Proposition 5

Proof. As in the proof of Proposition 4, we demonstrate our result by showing that welfare in the subgame with n firms and no price cap is strictly lower than welfare in the subgame with $n-1$ firms and no cap. For each $\theta \in \Theta$ let $\Delta Q(\theta) \equiv Q_n(\theta) - Q_{n-1}(\theta)$. We will first demonstrate that for each θ, $\Delta Q(\theta) \leq \frac{1}{n} q_n(\theta)$.

Using the fact that $\tilde{\theta}(X_n) = \tilde{\theta}(X_{n-1})$ we first examine the case when $\theta < \tilde{\theta}$. In this case, we use the first order conditions and the proof is identical to the proof in the previous section. For the case when $\theta > \tilde{\theta}$ we again use the definition of $\tilde{\theta}$ and follow a similar argument as in the previous section.

Thus, for each θ $\Delta Q(\theta) \leq \frac{1}{n} q_n(\theta)$

Now, let W_m denote equilibrium expected welfare in the subgame with m firms. Let $\Delta W \equiv W_{n-1} - W_n$. Note that

$$W_n = E\left[\int_0^{Q_n(\theta)} [\theta + p(s)\,] ds\right] - cX_n - n\pi_n$$

Which may be written:

$$W_n = \int_{\underline{\theta}}^{\tilde{\theta}} \left[\int_0^{\tilde{Q}_n(\theta)} [\theta + p(s)\,] ds\right] dF(\theta) + \int_{\tilde{\theta}}^{\overline{\theta}} \left[\int_0^{X_n} [\theta + p(s)]\, ds\right] dF(\theta) - cX_n - n\pi_n$$

Let $\Delta W = W_{n-1} - W_n$ and note that

$$\Delta W = -\int_{\underline{\theta}}^{\tilde{\theta}} \left[\int_{\tilde{Q}_{n-1}(\theta)}^{\tilde{Q}_n(\theta)} [\theta + p(s)\,] ds \right] \, dF(\theta) - \int_{\tilde{\theta}}^{\overline{\theta}} \left[\int_{X_{n-1}}^{X_n} [\theta + p(s)] \, ds \right] \, dF(\theta) + (\Delta X)c + \pi_n$$

Now, following a similar argument as in the previous section, using the concavity of p, we may show that for each $\theta \in [\underline{\theta}, \tilde{\theta}]$

$$\int_{\tilde{Q}_{n-1}(\theta)}^{\tilde{Q}_n(\theta)} [\theta + p(s)\,] ds \leq \Delta\tilde{Q}(\theta)(\theta + p(\tilde{Q}_n(\theta))) - \frac{1}{2}(\Delta\tilde{Q}(\theta))^2 p'(\tilde{Q}_n(\theta)) \equiv A(\theta)$$

Moreover, for each $\theta \in [\tilde{\theta}, \overline{\theta}]$

$$\int_{X_{n-1}}^{X_n} [\theta + p(s)\,] ds \leq \Delta X(\theta + p(X_n)) - \frac{1}{2}(\Delta X)^2 p'(X_n) \equiv B(\theta)$$

Now, using the first-order conditions we may write:

$$\pi_n = \left(-\int_{\underline{\theta}}^{\tilde{\theta}} (\tilde{q}_n(\theta))^2 p'(\tilde{Q}_n(\theta)) \, dF(\theta) \right) + \left(-\int_{\tilde{\theta}}^{\overline{\theta}} (x_n)^2 p'(X_n) \, dF(\theta) \right) \equiv \pi_n^A + \pi_n^B$$

Hence, it follows that

$$\Delta W \geq -\int_{\underline{\theta}}^{\tilde{\theta}} A(\theta) \, dF(\theta) - \int_{\tilde{\theta}}^{\overline{\theta}} B(\theta) \, dF(\theta) + (\Delta X)c + \pi_n^A + \pi_n^B$$

Now, note that

$$-\int_{\underline{\theta}}^{\tilde{\theta}} A(\theta) \, dF(\theta) + \pi_n^A = \int_{\underline{\theta}}^{\tilde{\theta}} \left[-\Delta\tilde{Q}(\theta)(\theta + p(\tilde{Q}_n(\theta))) + \frac{1}{2}(\Delta\tilde{Q}(\theta))^2 p'(\tilde{Q}_n(\theta)) - (\tilde{q}_n(\theta))^2 p'(\tilde{Q}_n(\theta)) \right] \, dF$$

Using the first-order conditions once again, we know that for each $\theta \in [\underline{\theta}, \tilde{\theta}]$ it holds that $\theta + \tilde{Q}_n = -\tilde{q}_n(\theta) p'(\tilde{Q}_n)$. This, combined with the fact that

$\Delta \tilde{Q}(\theta) \le \frac{1}{n}\tilde{q}_n(\theta)$ allow us to write

$$-\int_{\underline{\theta}}^{\tilde{\theta}} A(\theta)\, dF(\theta) + \pi_n^A \ge \int_{\underline{\theta}}^{\tilde{\theta}} (\tilde{q}_n(\theta) p'(\tilde{Q}_n(\theta)) \left(\frac{1}{n} + \frac{1}{2n^2} - 1\right) dF(\theta) > 0$$

Now also see that

$$-\int_{\tilde{\theta}}^{\overline{\theta}} B(\theta)\, dF(\theta) + (\Delta X)c + \pi_n^B$$

$$= -\Delta X \left[\int_{\tilde{\theta}}^{\overline{\theta}} (\theta + p(X_n))\, dF(\theta) - c\right] + \int_{\tilde{\theta}}^{\overline{\theta}} \left[\tfrac{1}{2}(\Delta X)^2 p'(X_n) - (x_n)^2 p'(X_n)\right] dF(\theta)$$

From the first-order condition, it follows that

$$\int_{\tilde{\theta}}^{\overline{\theta}} (\theta + p(X_n))\, dF(\theta) - c = \int_{\tilde{\theta}}^{\overline{\theta}} -x_n p'(X_n)\, dF(\theta)$$

This, combined with the fact that $\Delta X \le \frac{1}{n} x_n$ allows us to write:

$$-\int_{\tilde{\theta}}^{\overline{\theta}} B(\theta)\, dF(\theta) + (\Delta X)c + \pi_n^B \ge (x_n)^2 p'(X_n) \int_{\tilde{\theta}}^{\overline{\theta}} \left(\frac{1}{n} + \frac{1}{2n^2} - 1\right) dF(\theta) > 0$$

It follows immediately that $\Delta W > 0$. The remainder of the proof works identically to the previous section. The result follows.

□

References

[1] Rabah Amir and Val Lambson. On the effects of entry in cournot markets. *Review of Economic Studies*, 67, 2000.

[2] Severin Borenstein, James Bushnell, and Frank Wolak. Measuring market inefficiencies in california's restructured wholesale electricity market. *The American Economic Review*, 92(5), 2002.

[3] Rex Cottle and Myles Wallace. Economic effects of non-binding price constraints. *The Journal of Industrial Economics*, 31(4):469–474, 1983.

[4] Robert Earle, Karl Schmedders, and Tymon Tatur. On price caps under uncertainty. *Review of Economic Studies*, 74:93–111, 2007.

[5] Veronkia Grimm and Gregor Zottl. Capacity choice under uncertainty: The impact of market structure. *University of Cologne Working Paper Series in Economics*, (23), 2006.

[6] Veronkia Grimm and Gregor Zottl. Price regulation under demand uncertainty. *The B.E. Journal of Theoretical Economics*, 10(1), 2010.

[7] N. Gregory Mankiw and Michael Whinston. Free entry and social inefficiency. *The Rand Journal of Economics*, 17(1):48–58, 1986.

[8] Paul Milgrom and John Roberts. Comparing equilibria. *The American Economic Review*, 84(3), 1994.

[9] Paul Milgrom and Chris Shannon. Monotone comparative statics. *Econometrica*, 62(1), 1994.

[10] Gregor Zottl. On optimal scarcity prices. *International Journal of Industrial Organization*, 29(5), 2011.