Controlling Information to Influence Consumer Beliefs

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November 14, 2015

Abstract

Access to product information changes a consumer’s initial belief about the product’s value in ways that are often unobservable to the firm. In such situations, how much information does the firm want the consumer to have access to? This paper expands on Lewis and Sappington’s (1994) model of information supply between a seller and a buyer using the information environment developed by Kamenica and Gentzkow (2011) to answer this question. Unlike the results in Lewis and Sappington (1994) and Johnson and Myatt (2006) which state that a monopolist prefers either full information or no information, partial information revelation is generally optimal for a monopolist facing a rational Bayesian consumer. The monopolist allows the consumer to learn whether or not her valuation is higher than the outside option but no more.

1 Introduction

In many economic transactions, consumers are uncertain about their own valuation for a particular product due to lack of information. This is particularly true of new or complex products such as health insurance plans, credit card contracts, retirement plans, and financial securities. In such situations, firms can opt to supply consumers with the means to help consumers privately learn

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about their valuation for the product. Many papers have looked at the incentives of a firm to provide product information when the effects of its disclosure on consumer valuations are known to the firm. Examples include the seminal papers in the literature on vertical product information revelation by Grossman and Hart (1980), Grossman (1981), and Milgrom (1981). In this paper, I examine the incentives of a firm to grant consumers access to product information when such access can potentially increase or decrease the consumer's initial expected valuation for a product in ways unobservable to the firm. My main result shows that a monopolist never finds it optimal to provide the consumer with full information about the consumers valuation for the product and only allows the consumer to learn whether or not her actual valuation for the product is higher than that of the outside option.

Information asymmetry exists on both sides of the market. Sellers are more informed about the characteristics of their product and buyers are more knowledgeable about the potential uses of the product to them and their private tastes for particular product characteristics. A seller can provide product information to help consumer make better choices, but how that information maps to a consumers' preferences is often private information known only to the consumer. In effect, by choosing how much information to reveal to a consumer, the seller is designing an information environment through which individual consumers can learn about their own private valuations for a product. A firm that provides more information about its product allows consumers to learn their private valuation for the product with high accuracy while a firm that provides no information about its product does not allow consumers to learn anything about their valuation.

Consider a pharmaceutical company deciding whether or not to provide information about an over the counter drug to patients. The drug varies in effectiveness across patients. If the company offers patients a free trial, an individual patient will be able to privately assess the effectiveness of the drug. If the company does not offer a free trial, patients only know the overall effectiveness of the drug over the whole population from the results of prelaunch tests. The sample in this case provides all patients with a private and accurate signal of the valuation of the drug to that individual. The company could alternatively state on its product label that the drug is less effective for a certain group of patients informing patients in this group that their valuation for the product will be lower than that of the overall population. This is an example of a partially informative signal targeted towards a particular group of consumers. In the model, I allow the seller to have unrestricted freedom in designing the information environment through which the consumer gets her private information.
The problem that a seller faces in providing information to buyers about their private valuations was first formalized in a model by Lewis and Sappington (1994). Using a series of examples, they found that when the seller can choose the probability the buyer gets to perfectly learn her valuation, the seller optimally chooses this probability to be either zero or one. I expand on the model of Lewis and Sappington using an information framework developed by Kamenica and Gentzkow (2011). In my model, the sellers choice of information structure is unrestricted, only the buyer is assumed to be a rational Bayesian updater. Under this information design framework, I show that the buyer at best learns whether or not her valuation is higher than that of the outside option. Unlike the results established by Lewis and Sappington (1994), allowing the buyer to perfectly learn her private valuation for the product is never optimal for a monopolist seller. In addition, I also find competition encourages information revelation by sellers, reinforcing similar results established by Ivanov (2013).

To make this concrete, consider the example of a firm deciding how much information it would like a consumer to learn about its product. Let the value of the product to the consumer be $\theta_L$, $\theta_M$ or $\theta_H$ with equal probability where $0 < \theta_L < \theta_M < \theta_H$. Both the firm and the consumer know the distribution of the value of the product to the consumer but not the actual value. The firm however, can allow the consumer to interact with the product and privately learn its exact value to her. Since the firm does not observe the consumer’s value, the firm cannot condition the price it offers on the consumer’s value. Assume the firm posts one price for its product.

The consumer uses the information provided to her by the firm and observes the price before making her decision whether to buy the product or not. Assume she gets an outside option utility of $\bar{u}$ from not buying and that $\theta_L < \bar{u} < \theta_M$. The consumer will buy the product if her expected value less the price set by the firm is greater than the outside option value $\bar{u}$. If the consumer is indifferent, assume she buys.

Suppose the firm allows the consumer to fully interact with the product and learn the product’s true value to her. This is the case for example when the firm allows the consumer to have a free trial sample of the product. Given the consumer privately learns her exact value for the product, the ex ante probability the consumer buys the product is the probability the consumers actual value is higher than $P + \bar{u}$. The firm’s maximum expected profit when it provides full information is then:

$$\max \left\{ \frac{2(\theta_M - \bar{u})}{3}, \frac{\theta_H - \bar{u}}{3} \right\}$$
The firm achieves this by either charging \( P = \theta_M - \bar{u} \) and inducing types \( \theta_M \) and \( \theta_H \) to buy or by charging \( P = \theta_H - \bar{u} \) and inducing type \( \theta_H \) to buy.

On the other hand if the firm does not allow the consumer to interact with the product, the consumer learns nothing about her true value beyond the commonly known information given by the prior. The firm can then set the price equal to the average value less the outside option utility \( \bar{u} \) and make a profit of:

\[
\frac{\theta_L + \theta_M + \theta_H}{3} - \bar{u} = \frac{\theta_L + \theta_M + \theta_H - 3\bar{u}}{3}
\]

What if the firm can provide the consumer with partial information? For example, if the firm allows the consumer to learn whether or not her true value is higher than the outside option utility \( \bar{u} \) and charges \( P = \frac{\theta_M + \theta_H}{2} \), the firm makes an expected profit of:

\[
\frac{2}{3} \left( \frac{\theta_M + \theta_H}{2} - \bar{u} \right) = \frac{\theta_M + \theta_H - 2\bar{u}}{3}
\]

Since \( \theta_L - \bar{u} < \theta_M - \bar{u} < \theta_H - \bar{u} \) we have that,

\[
\frac{\theta_L + \theta_M + \theta_H - 3\bar{u}}{3} < \frac{\theta_M + \theta_H - 2\bar{u}}{3}
\]

and

\[
\max\left\{ \frac{2(\theta_M - \bar{u})}{3}, \frac{\theta_H - \bar{u}}{3} \right\} < \frac{\theta_M + \theta_H - 2\bar{u}}{3}
\]

Hence the firm will always prefer to provide partial information to the consumer rather than no information or full information.

In the paper, I show that this result holds more generally. When the seller’s information choice is unrestricted and the buyers is a rational Bayesian updater, the seller will always find it optimal to provide the buyer with partial information so long as the outside option utility is greater than the buyer’s lowest value for the product. In addition, I investigate information revelation when there is a second seller and the outside option value is determined under competition. I find that competition promotes information revelation by forcing firms to differentiate themselves from one another.

2 Related Literature

I now discuss the contribution of the paper to prior theoretical work. An important paper examining the incentive of a seller to allow potential buyers to
acquire private information about their tastes for the seller’s product is Lewis and Sappington (1994). The authors investigate the seller’s incentives to provide information in a simple monopolistic setting where buyer valuations are uniformly distributed. In their model the buyer observes a perfectly informative signal with probability $\gamma$ chosen by the seller. With probability $1 - \gamma$ the buyer receives no signal from the seller. Lewis and Sappington find that either buyers are supplied with the best available knowledge of their tastes or no information is supplied by the seller.

Johnson and Myatt (2006) reinforce the initial insight provided by Lewis and Sappington (1994). They consider a more general class of information structures in which signals can be ranked unambiguously in terms of informativeness. In both the monopolistic and Cournot oligopoly setting, the authors find that sellers maximize profit by using either the most informative or least informative signal.

Anderson and Renault (2006) interestingly showed that intermediate information structures can be optimal in this framework if the buyer has to pay a search cost. The optimal information structure in Anderson and Renault (2006) informs the consumer whether or not her valuation for the product is higher than the search cost. Similarly, Saak (2006) finds that when the consumer’s posterior distributions can be ranked by first order stochastic dominance, then to a monopolist, intermediate information structures are optimal among all information structures that generate the same marginal signal distribution.

Similar to Anderson and Renault (2006) and Saak (2006), I show that when consumers are rational Bayesian updaters, intermediate information structures are optimal among all possible choices of information structures. Unlike Saak (2006), this particular result does not depend on any restrictions on the seller’s choice of information structure. Unlike Anderson and Renault (2006), consumers in my model do not incur a search cost. This result is solely derived from the fact that sellers cannot observe the effect of the revealed information on the rational Bayesian consumer and is not an artifact of any other assumptions on the posterior distribution of beliefs. My finding also indicates that the extreme information choices found to be optimal in earlier papers by Lewis and Sappington (1994) and Johnson and Myatt (2006) are artifacts of the technical assumptions imposed on the class of information structures that can be chosen. This result carries important economic implications about firms’ voluntary incentives to disclose information.

I adopt the information design framework developed in Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2015). Kamenica and Gentzkow (2011) also show that a partially informative information structure is optimal
for a monopolist albeit in a simpler setting in which prices are exogenously
given. In the paper, I show that even when the seller chooses the price as well
as the information structure, this result continues to hold.

It should also be noted that in the economic literature on auctions, authors
have considered setups similar to the one used this paper. Gauza (2004),
Gauza and Penalva (2010), and Bergemann and Pesendorfer (2007) examine
an auctioneer’s incentive to allow bidders to learn their private valuations for
the object being sold. In their set up, aside from the differences in the class of
information structures analyzed, the prices in these models are determined by
an auction mechanism. Instead of looking at a set up with multiple buyers, I
further examine the seller’s incentives when he faces competition from another
seller.

Moscarini and Ottaviani (2001) investigate price competition when buyers
have private information. In their two-state, two-seller, and one-buyer setup
they show that sellers’ profits fall with the revelation of public information.
Damiano and Li (2007) extend Moscarini and Ottaviani’s setup by allowing the
seller’s to control the buyer’s private information and find that full information
disclosure is possible. Ivanov (2013) using a different setup shows that this
result need not hold with two sellers given a richer signal space. However, if the
number of sellers is significantly large full information disclosure is possible.
Ivanov uses the same class of information structures employed in Johnson and

My setup in the case of competition differs significantly from these pre-
vious papers, the most important difference being the class of information
structures accessible to the seller. In this section, I allow both sellers to dis-
close information about the rivals product. I use a two firm price competition
model similar to Moscarini and Ottaviani (2001) with a richer state and signal
space to show that competition promotes information disclosure. I find that
no information disclosure can never be an equilibrium of the model and full
information disclosure is always an equilibrium. This is similar to the findings
by Ivanov (2013) although Ivanov assumes sellers can only disclose information
about their own products. In comparison to Johnson and Myatt (2006),
Damiano and Li (2007) and Ivanov(2013), I find that intermediate information
structures can be chosen in equilibrium when sellers are able to disclose their
rival’s information.

Finally, my work is related to but distinct from the literature on vertical
product information disclosure started by Grossman and Hart (1980), Gross-
man(1981) and Milgrom (1981). In this literature, the seller always knows the
effect of disclosing his information on the buyer’s valuation. In my paper as in
all the papers mentioned prior to this paragraph, it is assumed that the seller
does not know the effect of the information disclosed on the buyer’s valuation. This leads papers in this strand of literature to produce a distinctly different result in the monopoly case from the robust “unraveling result” often found in papers in the literature on vertical product information disclosure.

3 The Baseline Model

A monopolist (he) sells an indivisible product to a buyer (she). Both seller and buyer are uncertain about the value of the product to the buyer, which I denote \( \theta \). Let \( \theta \) be drawn from a prior distribution \( G \) on \( \Theta = [0, 1] \). I will assume the prior distribution of buyer valuations \( G(\theta) \) is known to both seller and buyer.

In the game, the buyer receives signals allowing her to learn her true valuation. The seller, however, decides how informative the signals will be to the buyer. The timing of the game is as follows:

- The seller chooses the information structure which determines the probability the buyer receives a signal \( s \) conditional on the realization of \( \theta \). I describe the endogenous information structure in detail below. The seller also chooses a price for the product which I will denote by \( P \) where \( P \in \mathcal{P} = [0, +\infty) \).
- Nature selects the buyer’s true valuation \( \theta \in \Theta \) according to the prior \( G \). Neither seller or buyer observe nature’s move.
- A signal is sent to the buyer via the information structure chosen by the seller.
- The buyer observes the information structure chosen by the seller, the signal sent to her by the information structure and the price set by the seller.
- The buyer chooses whether to buy the product or not based on the chosen information structure, her signal and the price set by the seller. If she does not buy the product, she receives the outside option utility of \( \bar{u} \).
- Payoffs to buyer and seller are realized.

I assume the seller’s cost of producing the product is zero.\(^1\) The seller makes a profit of \( P \) if the buyer buys the product and he makes zero profit.

\(^1\)If the seller’s cost of producing the product is constant but positive, we can add this cost to the outside option utility when considering the seller’s incentives.
if the buyer does not. Since the seller does not observe the buyer’s actual valuation of the product, he chooses his information structure and price to maximize his ex ante expected profit. The seller’s ex ante expected profit is described by:

$$\pi(\alpha, P) = P \times \alpha$$

where $\alpha$ is the probability the buyer buys the product.

The buyer’s utility from consuming the product is the difference between her valuation for the product and the price she pays. I assume the buyer is risk neutral. Her utility from consuming the product is:

$$U(\theta, P) = \theta - P$$

The buyer buys the product if her expected utility from consuming the product is greater or equal to her outside option utility $\bar{u}$.

### 3.1 The Information Environment

The signal space is denoted by $S = [0, 1]$. Let $(\Theta \times S, \mathcal{B}(\Theta \times S))$ be a measurable space, where $\mathcal{B}(\Theta \times S)$ is the class of Borel sets of $\Theta \times S$. An information structure is defined as a distribution $F(\theta, s)$ where $S$ is the space of signal realizations and $F(\theta, s)$ is a joint probability distribution over the space of valuations $\Theta$ and the space of signal realizations $S$.

The joint probability distribution is defined in the usual way by

$$F(\tilde{\theta}, \tilde{s}) \equiv Pr(\theta < \tilde{\theta}, s < \tilde{s})$$

Let $F(\theta)$ and $F(s)$ denote the marginal distributions of $F(\theta, s)$. The conditional distribution functions derived from the joint distributions are defined in the usual way by

$$F(\tilde{\theta} | \tilde{s}) \equiv \frac{\int_{\tilde{\theta}}^{1} dF(\theta, s)}{\int_{0}^{1} dF(\theta, \tilde{s})}$$
and

\[ F(\tilde{s}|\tilde{\theta}) = \frac{\int_{\tilde{s}}^{1} dF(\tilde{\theta},.)}{\int_{0}^{1} dF(\tilde{\theta},.)} \]

I define \( F(\theta, s) \) to be a feasible information structure under the prior \( G(\theta) \) if and only if \( F(\theta) = G(\theta) \) for all \( \theta \in \Theta \). The seller’s information strategy is a choice of information structure from the set of feasible information structures under the prior distribution of \( \theta \).

This is a generalization of the Kamenica and Gentzkow (2011) class of information structures to a compact state space. As in their model, a choice of information structure in our model essentially pins down a set of conditional distributions of signals given each state \( \theta \). The seller is allowed to choose any information structure from the set of feasible information structures under the prior. All information structures are costless. Every information structure chosen by the seller generates a family of distributions \( \{F(s|\theta)\}_{\theta \in \Theta} \) over the set of signals \( S \).

The buyer observes the seller’s choice of information structure and a private signal \( s \). I assume the buyer is a rational Bayesian updater. Given a chosen information structure and signal, she forms a posterior belief about her valuation \( \theta \) according to Bayes rule. Her expected valuation given any signal is determined by:

\[ E(\theta|s) = \int_{0}^{1} \theta dF(\theta|s) \]

Every choice of information structure generates a distribution over posterior distributions of valuations \( \theta \). Since I require the buyer to be a rational Bayesian updater, any distribution over posteriors \( \tau \) must satisfy the condition.

\[ E_{\tau}(\mu_S) = \mu_0 \]

where \( \mu_S \) denotes a posterior in the support of \( \tau \) and \( \mu_0 \) is the prior distribution \( G \). Kamenica and Gentzkow (2011) refer to this as the Bayes plausibility assumption in their paper. This is the only restriction on the distribution over posterior expectations that can be chosen by the seller in equilibrium.

My approach to modeling information in this paper differs significantly from the approach used in most of the earlier papers mentioned in the literature review in that I do not directly impose conditions on the posterior distribution of valuations \( \theta \) conditional on the signals sent.
3.2 Strategies and Equilibrium Concept

Let the set of feasible information structures under the prior \( G(\theta) \) be denoted \( \mathcal{F}_G \) The seller’s strategy consists of the choice of an information structure \( F \in \mathcal{F}_G \) and a price \( P \). Each strategy uniquely determines a family of distributions \( \{ F(s|\theta) \}_{\theta \in \Theta} \) over the set of signals \( S \).

The buyer’s posterior belief \( \mu \) is:

\[
\mu : \mathcal{F}_G \times S \rightarrow \Delta \Theta
\]

The buyer’s posterior belief determines the buyer’s expected valuation given the seller’s choice of information structure and the signal she receives. Technically speaking, the buyer’s information strategy is also a function of the seller’s chosen price, but I omit this since given the information structure and the signal, the price provides no further information about the true state \( \theta \).

The buyer’s strategy \( \sigma_B \) gives her action to buy (\( a=1 \)) or not buy (\( a=0 \)) given the information choice of the seller and her signal.

\[
\sigma_B : \mathcal{F}_G \times S \times P \rightarrow \{0, 1\}
\]

Following Kamenica and Gentzkow (2011) I adopt the concept of Subgame Perfect Equilibrium.

- Given the seller’s chosen information structure and a signal realization \( s \), the buyer forms the posterior \( \mu \) using Bayes rule.

- The buyer’s action maximizes her expected utility given her posterior belief \( \mu \). If the buyer is indifferent between buying and not buying we assume she buys.

- Taking the buyer’s behavior as given, the seller chooses an information structure \( F \in \mathcal{F}_G \) and a price \( P \) that maximizes his expected profit.

In the remainder of the paper, I use the term “equilibrium” to refer to a Subgame Perfect Equilibrium.

Given a prior distribution on \( \Theta \), we define the optimal information structure to be any information structure that maximizes the seller’s expected profit. Formally, and information structure \( \mathcal{F} \) is optimal if there exists a price \( P \) such that for all \( \mathcal{F}' \in \mathcal{F}_G \) and for all \( P' \in [0, +\infty) \) the equilibrium expected profit satisfies:

\[
E(\pi | \mathcal{F}, P) \geq E(\pi | \mathcal{F}', P')
\]

Clearly, in equilibrium the seller selects an optimal information strategy.
3.3 Optimal Information Control

In this section, I characterize the set of equilibria of the baseline model and describe the seller’s optimal information structure. I say an information structure provides no information to the buyer if her posterior belief about $\theta$ after observing any signal sent via the information structure is identical to her prior belief. The first proposition establishes that when the product’s minimum value to the buyer is high enough, providing no information to the buyer maximizes the seller’s expected profit.

**Proposition 1.** When $\bar{u} \leq 0$, it is an equilibrium for the seller to provide no information to the buyer.

**Proof.** In any equilibrium, the buyer’s posterior belief is uniquely determined according to Bayes rule and her action given her posterior belief is uniquely determined given the posterior. We will show that providing no information to the buyer and setting a price $P^* = \bar{\theta}$ maximizes the seller’s expected profit.

If the seller provides no information to the buyer in equilibrium, the buyer’s expected valuation for the product given any realization of her equilibrium signal $s$ is then $E(\theta|F, s) = \bar{\theta}$ where $\bar{\theta}$ is the mean of the buyer’s valuation under the prior distribution $G$. The probability the buyer buys the product at price $P^* = \bar{\theta}$ is one and her expected profit is then $\pi^* = \bar{\theta}$.

Let $F'$ be any alternative information structure and $P'$ be any alternative price. We will show the seller cannot increase his expected profit under $F'$ and $P'$.

Let $S_1 = \{s \in S | E(\theta|s, F') \geq P'\}$ and let $S_0 = S \setminus S_1$. The expected profit to the seller under $(F', P')$ is

$$\pi(F', P') = P' \times Pr(s \in S_1|F')$$

where

$$Pr(s \in S_1|F') = \int_{s \in S_1} dF'(s)$$

Hence, we have that

$$\pi(F', P') = P' \int_{s \in S_1} dF'(s) \leq \int_{s \in S_1} E(\theta|s, F')dF'(s)$$

By the Law of Iterated Expectations

$$\int_{s \in S_1} E(\theta|s, F')dF'(s) + \int_{s \in S_0} E(\theta|s, F')dF'(s) = E(E(\theta|s, F')) = \bar{\theta}$$
So it must be that 
\[ \pi(F', P') \leq \bar{\theta} \]
Hence it is an equilibrium for the seller to provide no information to the buyer in equilibrium and charge \( P^* = \bar{\theta} \).

The next proposition establishes that when there is a positive probability the outside option is preferable to the consumer no matter what price the seller sets, then it is optimal for the seller to partially inform the consumer. Let \( Pr(s|\theta) \) be the probability signal \( s \) is sent given the state is \( \theta \).

**Proposition 2.** When \( 0 < \bar{u} \leq 1 \), it is optimal for the seller to choose the information structure \( F \) such that \( Pr(s_0|\theta \leq \bar{u}) = 1 \) and \( Pr(s = s_1|\theta > \bar{u}) = 1 \) for some \( s_0, s_1 \in S \) such that \( s_0 \neq s_1 \).

**Proof.** Let \((F', P')\) be any equilibrium strategy of the seller. When \( 0 < \bar{u} \leq 1 \), the consumer who receives signal \( s \) buys the product if and only if \( E(\theta|s, F') - \bar{u} \geq P' \). Hence, the seller can at best charge an equilibrium price of \( E(\theta|s, F') - \bar{u} \) if he wants the consumer who receives signal \( s \) to buy the product. If \( E(\theta|s, F') - \bar{u} < 0 \) or \( E(\theta|s, F') < \bar{u} \) then the consumer who receives signal \( s \) will never buy the product no matter what the price is.

Now let \( F \) be information structure such that \( F(s_0|\theta \leq \bar{u}) = 1 \) and \( F(s_1|\theta > \bar{u}) = 1 \) for some \( s_0, s_1 \in S \). Under this information structure, the consumer learns whether \( E(\theta|s, F') - \bar{u} \leq 0 \) or not. A consumer that receives signal \( s_0 \) will never buy. The price that maximizes the seller’s profit is \( P = E(\theta|s_1, F') - \bar{u} \) and the probability the consumer buys is the probability she receives signal \( s_1 \). The seller’s expected profit is then

\[
(E(\theta|\theta > \bar{u}) - \bar{u}) \times Pr(s = s_0|F) = (E(\theta|\theta > \bar{u}) - \bar{u}) \times Pr(\theta > \bar{u})
\]

I will show this is the highest expected profit the seller can achieve under any strategy choice.

Let \( S_1 \) be the set of messages sent under \( F' \) such that \( E(\theta|s) - \bar{u} < P' \) for all \( s \in S_1 \) and let \( S_2 \) be the set of messages sent under \( F' \) such that \( E(\theta|s) - \bar{u} \geq P' \) for all \( s \in S_2 \). Let \( S_0 = S \setminus (S_1 \cup S_2) \). If \( S_2 = \emptyset \) then the seller’s profit is zero and the proposition holds. Suppose \( S_2 \neq \emptyset \). The expected profit to the seller under \((F', P')\) is

\[
\pi(F', P') = P' \times Pr(s \in S_2|F')
\]

where

\[
Pr(s \in S_2|F') = \int_{s \in S_2} dF'(s)
\]
and \( P' \leq E(\theta|s) - \bar{u} \) for all \( s \in S_2 \). Hence, we have that
\[
\pi(F', P') = P' \int_{s \in S_2} dF'(s) \leq \int_{s \in S_2} (E(\theta|s; F') - \bar{u}) dF'(s)
\]
\[
= \int_{s \in S_2} [(E(\theta|s, \theta < \bar{u}; F') - \bar{u}) Pr(\theta < \bar{u}|s) + (E(\theta|s, \theta \geq \bar{u}; F') - \bar{u}) Pr(\theta \geq \bar{u}|s)] dF'(s)
\]
\[
\leq \int_{s \in S_2} (E(\theta|s, \theta \geq \bar{u}; F') - \bar{u}) Pr(\theta \geq \bar{u}|s) dF'(s)
\]
\[
\leq \int_{s \in S} (E(\theta|s, \theta \geq \bar{u}; F') - \bar{u}) Pr(\theta \geq \bar{u}|s) dF'(s)
\]
\[
= (E(\theta|\theta \geq \bar{u}) - \bar{u}) Pr(\theta \geq \bar{u})
\]

Hence when the outside option utility is high enough, the seller will find it beneficial to partially inform the consumer. However, the consumer only learns whether her valuation for the product is higher than the outside option utility or not and nothing more. Even when the consumer learns her valuation for the product is higher than the outside option utility, there is still a chance the valuation of the product to the consumer will be lower than the price she paid for the product.

The intuition for this result is straightforward. The seller ideally wants to extract all the consumer surplus by charging the buyer her willingness to pay. When there is an outside option, the buyer must be compensated for the outside option if she is going to buy from the seller. The net surplus, the surplus minus the outside option compensation, that can be extracted from some buyer types is then negative. The seller can maximize profit under the proposed information scheme by excluding these buyer types and leaving the remaining buyer types, on average, with no gains from trade.

This partial information revelation results differs from the results of Lewis and Sappington (1994) and Johnson and Myatt (2006) in an economically significant way. It implies that firms have no incentives to reveal full information to consumers to help them learn about their valuation. The seller is able to extract as much surplus as possible from the consumer by providing only partial information. This result contrasts with the “unraveling result” found in models of information disclosure where the firm knows the effect of disclosing information on consumer’s beliefs.
4 Information Control under Competition

In this section, I characterize the set of equilibria under competition. The outside option to the buyer in this case is not exogenously determined as in previous sections of the paper but will be determined endogenously. Ex post, the value of the outside option in this section is a function of the equilibrium prices set by sellers and the true state. Hence, the ex ante expected value of the outside option to the buyer will be a function of the equilibrium prices as well as the information provided to the buyer in equilibrium.

4.1 The Model

In this section, there are two sellers (he) each selling a differentiated product and one buyer (she). The buyer can purchase the product from either seller or not purchase the product at all, in which case, she gets a utility of zero. Let $\theta = (\theta_1, \theta_2)$ denote the buyer’s valuation for the product of seller one and two and let $\theta$ be drawn from the uniform prior distribution $G$ on $\Theta = [0, 1] \times [0, 1]$.

In the game, the buyer receives two signals, one from each seller, allowing her to learn her true valuation $\theta$ for both sellers’ products. The sellers design the information structure which determines how informative the signal that the buyer receives will be. The timing of the game is as follows:

- The sellers choose an information structure and a price for their product simultaneously. Each information structure determines a probability the receiver gets a certain signal about her true valuation for both firms products. That is, the signal can reveal to the receiver information about the rival’s product in addition to information about the seller’s product.

- Nature selects the buyer’s true valuation $\theta \in \Theta$ according to the prior $G$.

- Two signals are sent to the buyer via the chosen information structures chosen by the sellers.

- The buyer observes the information structures chosen by the seller, the signals sent, and the prices set by the sellers.

- The buyer chooses whether to buy the product from seller one or seller two or not buy at all based on her information.

- Payoffs to buyer and sellers are realized.
Again I assume each seller’s cost of producing their product is zero. A seller makes a profit of $P$ if the buyer decides to buy from him. Each seller chooses his information structure and price to maximize his ex ante expected profit. The buyer’s utility from consuming a product is the difference between her valuation for that product and the price she paid. Assume the buyer is risk neutral and chooses her action to buy from seller 1 ($a=1$), to buy from seller 2 ($a=2$) or to not buy the product ($a=0$) to maximize her expected utility.

$$U_B = \max_{a \in \{0,1,2\}} \{E(\theta_1) - P_1; E(\theta_2) - P_2; 0\}$$

I will assume the buyer buys if she is indifferent between buying and not buying. If her expected utility from buying from both sellers are the same, I assume she buys from each seller with equal probability.

### 4.2 The Information Environment

Let $S = [0,1] \times [0,1]$ be the set of signals a seller can send. Let $(\Theta \times S, \mathcal{B}(\Theta \times S))$ be a measurable space, where $\mathcal{B}(\Theta \times S)$ is the class of Borel sets of $\Theta \times S$. An information structure is defined as a distribution $F(\theta,s)$ where $S$ is the space of signal realizations and $F(\theta,s)$ is a joint probability distribution over the space of valuations $\Theta$ and the space of signal realizations $S$.

The joint probability distribution is defined in the usual way by

$$F(\tilde{\theta}, \tilde{s}) \equiv Pr(\theta < \tilde{\theta}, s < \tilde{s})$$

Let $F(\theta)$ and $F(s)$ denote the marginal distributions of $F(\theta,s)$. The conditional distribution functions derived from the joint distributions are defined in the usual way by

$$F(\tilde{\theta}|\tilde{s}) \equiv \frac{\int_{\tilde{s}}^{\tilde{\theta}} dF(\cdot, \cdot)}{\int_{\tilde{s}}^{1} dF(\cdot, \cdot)}$$

and

$$F(\tilde{s}|\tilde{\theta}) \equiv \frac{\int_{\tilde{\theta}}^{\tilde{s}} dF(\cdot, \cdot)}{\int_{\tilde{\theta}}^{1} dF(\cdot, \cdot)}$$

I define $F(\theta,s)$ to be a feasible information structure under the prior $G(\theta)$ if and only if $F(\theta) = G(\theta)$ for all $\theta \in \Theta$. Let $\mathcal{F}$ be set of feasible information structures under the prior distribution of $\theta$. A seller’s information strategy is a choice of information structure $F$ from $\mathcal{F}$.

This information structure allows each seller to reveal information regarding the consumer’s valuation for the other seller’s product. Although generally
restrictive, this assumption is reasonable in certain industries. For example, pharmaceutical companies regularly compare the efficacy of their product to rivals products, food manufacturers can easily test for and reveal the sugar content of competing products, and cellular service providers often compare the quality of their service to the service of rival companies.

4.3 Strategies and Equilibrium Concept

Each seller’s strategy is a pair consisting of a information structure $F_i$ and the chosen price $P_i$ where $F_i \in \mathcal{F}$. Sellers choose their strategies simultaneously.

The buyer observes the chosen information structures chosen by each seller and two signals sent by the information structures and forms a posterior $F(\theta | .)$ over the space of valuations $\Theta = [0, 1]^2$. The buyer then decides whether to buy from seller one, seller two or to not buy based on her expected valuations for the product and the prices posted. Her strategy is a mapping:

$$\sigma_B : \mathcal{F} \times \mathcal{F} \times S \times S \times \mathcal{P} \times \mathcal{P} \rightarrow \Delta\{0, 1, 2\}$$

I will only focus on pure strategy subgame perfect equilibria. A quintuple $(F_1, P_1, F_2, P_2, \sigma_B)$ and a posterior belief $\mu_B$ is an equilibrium if

- Given the sellers’ prices and information structures, the buyer forms her posterior belief $\mu_B$ using Bayes rule. Her posterior belief is derived from $\max\{F_1, F_2\}$
- The buyers strategy maximizes her expected payoff given her posterior.
- Given the buyer’s strategy and the other seller’s strategy, each buyer chooses his strategy to maximize his expected profit.

4.4 Optimal Information Provision under Competition

Competition between sellers encourages sellers to provide information to the consumer. As we establish in the first proposition, it is never an equilibrium for both firms to conceal information from the consumer. When both sellers choose to provide the consumer with no information, competition is fiercest since the consumer cannot distinguish between the sellers. Hence, it is never optimal for them to do so.

**Proposition 3.** It is never an equilibrium for both sellers to choose to not provide any information to the buyer.
Proof. Suppose by way of contradiction that both sellers choose to provide the consumer with no information in equilibrium, that is, both sellers choose $\sigma^N_i$ in equilibrium. Then in equilibrium the consumer infers her expected valuation for the sellers’ products is $(0.5, 0.5)$. In the eyes of the consumer the sellers are identical and hence we have the classic model of Bertrand competition with homogeneous sellers. Equilibrium prices for both sellers are $(P^*_0, P^*_1) = (0, 0)$. Profits for both sellers are zero. However, given seller 2’s strategy, by choosing to deviate to the information structure that fully informs the buyer, seller 1 can charge a positive price $0 < \tilde{P}_1 < 0.5$ and make a positive expected profit since the probability the consumer buys from seller 1 is positive and equal to the measure of $\theta$s for which $\theta_1 - \tilde{P}_1 > 0.5$.

The next proposition establishes the existence of a full revelation equilibrium in this model. This is the unique pure strategy equilibrium and in this equilibrium the consumer always learns her true type.

**Proposition 4.** A fully revealing equilibrium in which both sellers allow the buyer to learn her true valuation exists.

Proof. Suppose in equilibrium both sellers choose to reveal full information to the consumer. Then the consumer knows her exact valuation for each firm’s product after receiving the signals.

We first construct each sellers’ best response pricing strategy. Seller i’s profit given seller j’s price $P_j$ if seller i chooses $P_i$ and $P_i \geq P_j$ is

$$\frac{1}{2}(1 - P_i + 2P_j)(1 - P_j)P_i$$

From the first order condition we obtain:

$$P^*_i(P_j|P_i \geq P_j) = \frac{2(1 + P_j) - \sqrt{1 + 2P_j + 4P_j^2}}{3}$$

If seller i chooses $P_i$ and $P_i \leq P_j$ then his profit is:

$$[(1 - P_i) - \frac{1}{2}(1 - P_j)^2]P_i$$

From the first order condition we obtain:

$$P^*_i(P_j|P_i \leq P_j) = \frac{1 + 2P_j - P_j^2}{4}$$

So the posted price chosen in equilibrium is $P^*_1 = P^*_2 = p$ where $p$ is the solution to the equation:

$$p^2 + 2p - 1 = 0$$
So $P_1^* = P_2^* = \sqrt{2} - 1$. Neither firm can deviate to $F_L$ or $F_M$ and make a profit since the consumer can learn her true valuation from the other firm’s signal. Hence $(F_H, F_H, \sqrt{2} - 1, \sqrt{2} - 1)$ and the buyer’s strategy constitutes an equilibrium. 

As opposed to the monopoly case, the consumer benefits from lower prices and the full revelation of information under competition. Competition promotes the revelation of information to consumers by forcing sellers to disclose product information to avoid intense price competition. This result mirrors the results found in Ivanov (2013) under a different information structure in which firms disclosed information about the consumers valuation for only the firm’s product and not that of its rival. Ivanov finds under the information structure imposed by Johnson and Myatt (2006) as the number of sellers increase, there exists a unique equilibrium in which all firms fully disclose their information. Gentzkow and Kamenica (2015) also find that competition promotes the revelation of information when sellers are allowed to disclose their rival’s information, however, in their set up prices are not chosen by the sellers but exogenously determined.

5 Application to Horizontal Product Information Disclosure

Here, I show how the model can be used to explain a firm’s disincentives to disclose product information. Suppose a firm’s sells a single product which can be one of two types, A or B with equal probability. The firm faces a heterogeneous group of consumers which can be of type 1 or 2 with equal probability. Type 1 consumers prefer product A to product B and obtain a utility of $\theta_H$ and $\theta_L$ from consuming products A and B respectively where $\theta_H > \theta_L$. In contrast, type 2 consumers prefer product B to product A and obtain utility levels of $\theta_L$ and $\theta_H$ from consuming products A and B.

The firm does not observe the consumer’s types and hence cannot charge different prices to different types. Assume it chooses posts one price for the product. Each firm can commit to sending a message to each consumer type from the message space $M = \{h,l\}$. If the firm sends a distinct message to each type, I interpret this as the firm fully disclosing its product type to all consumers since the consumer will be able to learn her exact valuation given the message she receives. If the firm commits to sending one message to both
consumer types, consumers learn nothing about their valuation beyond what they already know. I interpret this as the firm concealing its product type. Both consumer types have an expected valuation of $\frac{\theta_L + \theta_H}{2}$ from consuming the product given no information disclosure from the firm.

The game proceeds as follows

- The firm’s type is drawn.
- The firm commits to an information strategy and chooses a price for its product.
- The consumer decides whether to buy the product or not. If she buys she obtains utility $\theta_{ij} - P$ she obtains a utility of zero otherwise.

Due to my assumptions on the distribution of consumers, each firm type faces the same distribution of valuations and hence the same demand function. Models of vertical product information disclosure allows the demand function of high quality firms to shift outwards, hence firms of a higher quality type would always want to disclose their type and we would obtain the unraveling result of Grossman (1981) and Milgrom (1981). By Proposition 1, I can conclude in the case of horizontal product disclosure the firm commits to revealing no information and charges an equilibrium price of $P = \frac{\theta_L + \theta_H}{2}$. Consumers cannot distinguish between firm types by the product information provided by the firm or the price charged.

This example illustrates why firms may have a disincentive to disclose a particular characteristic of a product when different types of consumers may value this characteristic differently.

6 Concluding Remarks

In reality, perfectly rational consumers are often persuaded to change their behavior through various information channels that influence their beliefs such as advertisements, brochures, and product demonstrations. It is important that we understand firms incentives to provide such information and how competitive forces affect the firms incentives to provide such information. This paper provides insights into the principles at work behind a firms’ decision to provide a certain type of information, in particular, information that allows a consumer to privately learn about her idiosyncratic valuation of a product. The results of the model suggest that when the effects of information disclosure are unobservable to the firm, firms have an incentive to conceal such information
revealing just enough to entice the consumer to buy the product. This allows the firm to fully extract all the consumer surplus and maximize profits.

Unlike papers in the earlier literature, the optimality of a partially informative information structure in the monopoly case in my model does not depend on assumptions on the information structure chosen by the seller. The only restriction to posterior beliefs that results in equilibrium arises from the assumption that buyers are rational Bayesian updaters. This result and its generality carries an important economic implication. In the model, sellers profits are never highest when the consumer is perfectly able to learn about her valuation in both the monopoly case and the competition case. This result may help explain why certain firms and industries are against mandatory product information disclosure regulation since these regulations allow the consumer to perfectly learn about the product characteristics and therefore their valuation for the product. Models of vertical product information disclosure often reach the conclusion that when consumers are perfectly rational and disclosure is costless, firms have strong incentives to disclose information on the quality of their product to differentiate themselves from firms who provide lower quality products and hence, regulation requiring firms to disclose information is at best redundant. However, my model suggests if the same information that allows consumers to learn about a product’s average quality allows the consumer to privately learn about their idiosyncratic preferences for the product then firms may be less than willing to disclose such information. Consider, for example, the disclosure of the sugar content of a product. Information about the sugar content affects consumers perceptions of how healthy as well as how sweet or tasty the product is to them. While a firm which produces a product with relatively low sugar content may be willing to let the consumer learn about the health attributes of their product, the model suggests it is not likely to want consumers to learn how sweet the product is to them. Further theoretical and empirical analysis, however, is required to be able to answer this inherently more complex question.
References


