Long-Run Backfire from Energy Policies*

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To compare the general equilibrium effects of common environmental policies, I develop a model of directed technical change with endogenous exploration for depletable fossil resources, endogenous innovation in technologies for accessing renewable resources, and endogenous innovation in technologies for converting fossil and renewable energy into energy services. I find that a supply restriction and a resource tax each nearly always reduce extraction of fossil resources. In contrast, an efficiency policy and a renewable energy policy each can increase contemporaneous extraction of fossil resources ("backfire") if those resources are sufficiently depleted. Interactions with innovation and depletion can make the cumulative backfire induced by a time t policy differ substantially from a static analysis.

JEL: H23, O13, O38, Q32, Q43

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In the absence of a federal price on U.S. carbon emissions, environmental activists have lobbied for a diverse set of policies. They protest against approving pipelines to Canadian oil sands and export terminals for western states' coal resources, they urge government subsidies for renewable energy research and deployment, and they call for stricter efficiency standards for buildings, appliances, and vehicles. As one result of these efforts, President Obama indicated the U.S. government would not approve the Keystone XL pipeline if it significantly increased carbon emissions, and the American Recovery and Reinvestment Act of 2009 provided nearly $20 billion for energy efficiency programs and over $25 billion for renewable power generation (Aldy, 2013). I show that such policies' nearer-term environmental consequences critically depend on aggregate economic relationships and that their longer-term environmental consequences depend on how they interact with the trajectories of fossil resource depletion and private-sector innovation. In particular, I show that even if conventional partial equilibrium analyses suggest that an efficiency policy successfully reduces energy use from particular firms or households, the policy may nonetheless actually increase energy use by changing the composition of economic activity and by strengthening the incentive to search for new fossil resources.

In order to better understand the implications of several types of policy, I integrate fossil resource extraction, energy efficiency innovation, and renewable energy development into an endogenous growth model. I adapt recent models of directed technical change (Acemoglu, 2002) to allow workers to decide between employment in each of these sectors or in final-good production. Policies affect contemporaneous resource extraction by altering the incentives to work in each sector. Policies affect future extraction by changing the quality of technology available at the beginning of future periods and by changing the degree to which the fossil resource has already been depleted at the beginning of future periods. These innovation and depletion channels determine whether a policy's nearer-term effects multiply or decay as time passes.

I find that proposed policies have quite different static implications. Both a tax on resource extraction and a restriction on access to resource deposits reduce contemporaneous fossil resource extraction as long as there is not a sufficiently advanced renewable energy sector. In contrast, both a policy to increase energy efficiency and a policy to increase renewable energy can induce more contemporaneous fossil resource extraction, a result called "backfire" in the energy efficiency literature and not previously applied to renewable energy policies. The intuition is that improving energy efficiency makes each unit of a fossil resource more productive: more efficient machines generate more energy services per unit of fossil resource input. Similarly, when fossil resources help mitigate problems posed by renewable resources' intermittency and variability, increasing renewable generation raises the marginal product of fossil resources. In either case, increasing the marginal product of fossil resources raises their price and attracts more workers to the extraction sector.

This marginal productivity channel favors backfire, but there is a competing effect. Both an efficiency policy and a renewable energy policy increase the supply of energy services.

The price of energy services falls, which makes the price of the fossil resource input to energy service production also fall. Because they can now purchase more energy services at a given price, final-good firms' demand for wage-labor (the non-energy input to final-good production) shifts out, and the higher wages attract workers away from fossil resource extraction. The smaller is the elasticity of substitution in final-good production, the greater is the outward shift in the demand for wage-labor and the greater is the decline in the price of energy services.

The first, marginal productivity channel dominates the second, energy substitution channel when energy services and non-energy inputs are sufficiently substitutable in final-good production. In that case, an efficiency or renewable energy policy backfires. In contrast, resource taxes and supply restrictions both generally reduce equilibrium resource extraction. A resource tax reduces expected profits from resource extraction directly through tax payments, and a supply restriction reduces those profits indirectly by displacing extraction to lower-quality resource deposits. Workers respond by shifting into other sectors until the rewards to employment are equalized. These policies can backfire only if they increase innovation in both renewable and efficiency technologies to such an extent that final-good firms substitute away from their non-energy inputs. These policies most efficaciously reduce extraction when an efficiency policy would backfire and are least efficacious when an efficiency policy would successfully reduce extraction.

The dynamic implications of these policies potentially compound their static implications. The effects of a time t policy can differ in future periods because fossil resources become more depleted and because technology becomes more advanced. First, in a static setting, greater resource depletion makes an efficiency or renewable energy policy backfire more strongly, or makes the policy's reduction in emissions smaller. Therefore as extraction proceeds, this channel makes backfire from a time t policy grow, or shrinks the emission benefits of a time t policy. Second, the policy-induced change in extraction itself affects future depletion. A backfiring policy increases future depletion, which reduces future extraction. If backfire is a small percentage of a period's extraction, the first depletion channel can dominate and backfire can grow over time.

Other channels for dynamic effects operate through innovation in energy efficiency technologies. At low elasticities of substitution, an efficiency policy crowds out private innovation by making wage-labor more attractive, but at high elasticities of substitution, it attracts further private innovation due to complementarities with fossil resource extraction. In addition, the standing-on-shoulders nature of innovation means that a policy-induced time t improvement in efficiency generates an even larger improvement in future efficiency by making future private innovation more effective. A policy that backfires in time t often also induces private-sector efficiency innovation, in which case both innovation channels make it backfire more strongly in the future. But if an efficiency policy succeeds in reducing time t emissions, then it is likely to also crowd out private innovation, which tends to amplify its emission reductions over time.

A long literature has debated the potential for greater energy efficiency to not only raise energy consumption relative to a counterfactual with no induced substitution or income effects ("rebound") but even to raise energy consumption beyond its original level ("backfire") (Jevons, 1865; Khazzoom, 1980; Brookes, 1990; Sorrell, 2009; van den Bergh, 2011; Borenstein, 2013; Gillingham et al., 2013).[1] Formal macroeconomic models have suggested that backfire is possible when the elasticity of substitution between energy inputs and other inputs is unity or larger (Saunders, 1992, 2000b). These Solow growth settings supply energy from outside the model and hold the real price of energy fixed.[2,3] Improving the productivity of fuel in an aggregate production function can increase energy use by increasing total output and by inducing substitution away from other inputs. However, the literature suggests that energy and non-energy inputs are in fact complements, with an elasticity of substitution less than unity (Prywes, 1986; Manne and Richels, 1992; Chang, 1994; Koetse et al., 2008; van der Werf, 2008; Hassler et al., 2012; Stern and Kander, 2012). Backfire thus might seem irrelevant to policy. But backfire has yet to be explored in an endogenous growth model (Dimitropoulos, 2007; Sorrell, 2009). I show that endogenizing fossil resource extraction makes backfire possible for arbitrarily small elasticities of substitution, and I also show that interaction with private innovation can amplify backfire in the period a policy is implemented and also in the periods after a policy's implementation.[4] In the numerical calibration, backfire occurs at elasticities substantially below unity in settings with endogenous innovation but not in settings without endogenous innovation. When assessing the effects of large-scale

[1]A smaller literature has considered supply restrictions. In particular, Harstad (2012) demonstrates that a policymaker's ability to restrict supply from other jurisdictions can eliminate free-rider problems. The present paper abstracts from the spatial considerations at the core of Harstad (2012) and instead focuses on a supply restriction's interactions with other sectors and on its dynamic implications.

[2]The supply of labor is also fixed, and capital is also held fixed in short-run analyses of the rebound effect. Contrary to common belief, these settings actually do demonstrate the possibility of short-run backfire for elasticities less than unity, but the condition is usually treated as equal to unity because the gap depends on fuel's share of total value, which is taken to be small (Saunders, 2008). In the present model with endogenous fossil resource extraction, I find that the critical elasticity is less than unity and decreases in the degree of resource depletion. I abstract from capital accumulation, but I nonetheless find dynamic impacts through interactions with innovation and resource depletion.

[3]The present setting endogenizes energy supply, with price signals connecting efficiency improvements to energy consumption. Wei (2007) also allows for flexible prices in a general equilibrium setting, but restricts attention to Cobb-Douglas production functions for energy and non-energy goods, with energy produced from capital and labor inputs. Wei (2010) uses a more general functional form for aggregate production but defines energy supply as an exogenous function of its price.

[4]The backfire result is similar to the green paradox literature in that a supply-side perspective shows how policies can perversely increase emissions. However, backfire (also called "Jevons' paradox") and the green paradox are distinct. The green paradox is a form of intertemporal leakage arising from resource owners' anticipation of more stringent policies (Sinn, 2008). In contrast, the present paper considers the dynamic implications of surprising the economy with a policy. Further, profits in the present setting do not depend on future policies because patents and property rights expire within each large timestep. A green paradox could arise only in the presence of longer-lived property rights or longer-lived patents.

efficiency policies on energy consumption and emissions, it is critical to consider large-scale relationships in energy provision and in final-good production.

Previous analyses of energy in models of directed technical change have treated fossil resources as directly available for production (Di Maria and Valente, 2008; Grimaud and Rouge, 2008; Pittel and Bretschger, 2010; Acemoglu et al., 2012; Gans, 2012).[5] In reality, the energy sector is a significant fraction of the U.S. economy, with employment fluctuating in line with prices. In 2011, the nonrenewable resource sector's value added constituted 2.6% of U.S. GDP according to the Bureau of Economic Analysis, similar to the World Bank's estimate of 2.8% of GDP spent on all types of research and development. Just as it has been important to endogenize scientists' decisions to search for new technologies that increase productivity, so too is it important to endogenize geologists' decisions to search for the new energy resources that offset the declining quality of existing mines and wells. The present setting endogenizes both the cost and the quantity of extraction. I use a Heal (1976)-type approach in which the cost of extracting a resource increases in its cumulative extraction. The incentives for current exploration therefore depend on how past exploration has raised the expected cost of producing from a potential discovery. Workers' decisions about whether to search for a new fossil resource deposit depend on the depletion of the resource and on the market size for the resource, which in turn depends on the quality of efficiency technology and on the number of contemporaneous efficiency engineers.

The present setting also introduces a distinction between energy resources and energy services.[6] Energy resources must be converted into energy services in order to affect production of the consumption good. Economic output depends directly on lighting and mechanical motion, not on lumps of coal, and these are delivered by technologies called light bulbs and motors. These are general-purpose technologies that increase the productivity of both fossil and renewable resources, and their improvement has dramatically expanded useful work from energy consumption (Ayres et al., 2003; Ayres and Warr, 2005; Fouquet, 2011). These technologies are also the ones targeted by advocates of energy efficiency policies, which are often promoted as a cheap—or even free—way of combating climate change. I endogenize the evolution of these energy transformation technologies by making innovation effort depend on how expected profits compare to the rewards from innovation in renewable technologies, from exploration for fossil resources, and from wage-labor.

The next section describes the theoretical setting. Section 2 analyzes the effect of policy interventions in a setting without endogenous innovation and without a renewable resource. Section 3 analyzes how each type of intervention affects later outcomes via altered fossil

[5]Other work introducing energy into endogenous growth models neither includes directed technical change nor represents the decision to explore for energy resources: Bretschger and Smulders (2012) model a directed labor force and innovations that are used equally by different sectors, Tahvonen and Salo (2001) develop a Romer (1986)-style model where learning-by-doing occurs through resource extraction, and others make the quantity (Smulders and de Nooij, 2003) or cost (André and Smulders, 2012) of energy evolve exogenously.

[6]Howarth (1997) and Saunders (2000a) emphasize this distinction in a Solow growth setting.

resource depletion pathways. Section 4 introduces endogenous innovation in efficiency technologies and analyzes its response to policy interventions. Section 5 considers how innovation changes each policy's effect on later outcomes. Section 6 shows that renewable energy policies can also backfire. Section 7 is a calibrated numerical example that demonstrates the conditions under which backfire may occur and simulates policies' dynamic effects. Section 8 discusses policy implications. All proofs are in the appendix.

1 Framework

Consider a discrete-time economy with a fixed measure of workers who choose to work as laborers in final-good production, as engineers seeking to improve energy transformation technologies, as engineers seeking to improve renewable energy technologies, or as geologists exploring for new fossil resources. The time t composite final good Y_t is produced competitively using labor L_t and energy services E_t via the aggregate CES production function:

$$Y_t = \left[(1-\kappa)L_t^{\frac{\sigma-1}{\sigma}} + \kappa E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The parameter $\kappa \in (0,1)$ is a distribution parameter determining the relative importance of the two inputs in final-good production, and the parameter $\sigma \in (0,+\infty)$ is the constant elasticity of substitution between the two inputs. We call the two inputs (gross) substitutes when $\sigma > 1$ and (gross) complements when $\sigma < 1$. Throughout we ignore the special Cobb-Douglas case in which $\sigma = 1$. To isolate intuition, we abstract from capital accumulation, though the labor variables can be interpreted as labor-capital composites with predetermined aggregate measure.

One can interpret L_t and E_t as inputs to a physical production function or as inputs to a utility function. In the first case, shifting towards energy services suggests replacing labor via mechanization. Previous work points towards the two inputs being complements (Prywes, 1986; Manne and Richels, 1992; Chang, 1994; Koetse et al., 2008; van der Werf, 2008; Hassler et al., 2012; Stern and Kander, 2012). In the second case, shifting towards energy services suggests increasing consumption of material goods in place of personal services. Here the two inputs are again plausibly complements. We will use the first interpretation, but all results apply under the second.

Energy services E_t are intermediates like lighting and mechanical motion that directly assist in final-good production. Energy services are produced using energy and a continuum of machines:

$$E_t = \left[e\left(F_t, R_t\right)\right]^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha}\, di,$$

where $\alpha \in (0,1)$, A_{it} is the quality of a machine of type i, and x_{it} is the quantity of machines of type i. Each machine costs ϕ_E units of the final good. The energy input $e(\cdot,\cdot)$ is produced from fossil resources F_t and, potentially, renewable resources R_t. In settings without renewable resources, we have $e(F_t, R_t) = F_t$. In settings with renewable resources, the function $e(\cdot, \cdot)$ reflects how the intermittency and variability of renewable resources can limit the value of the energy they produce and can increase the value of fossil energy (e.g., Baker et al., 2013). Energy production increases in the use of fossil resources and in the use of renewable resources. The marginal energetic product of each type of resource is weakly decreasing in its quantity ($\partial^2 e(\cdot, \cdot)/\partial F_t^2 \leq 0$, $\partial^2 e(\cdot, \cdot)/\partial R_t^2 \leq 0$). Whereas the marginal energetic product of fossil resources could plausibly be constant, the variability and intermittency of renewable resources tend to pose larger problems as those resources become a larger component of the energy system. In addition, when renewable resources' variability and intermittency are important concerns, the cross-partial $\partial^2 e(\cdot, \cdot)/\partial F_t \partial R_t$ is positive because fossil resources can be dispatched so as to smooth out those fluctuations.[7] Otherwise, the cross-partial is zero.

There are three crucial differences between fossil and renewable resources. First, the same renewable resources are available period after period, but fossil resources are irreversibly depleted by extraction. Second, renewable resources are known and freely accessible, but fossil resources must be discovered and extracted. Third, the quantity of fossil energy depends only on the number of workers engaged in exploration and extraction, whereas the quantity of renewable energy depends only on the quality of renewable technology and on the number of engineers working to improve it.

The pools or deposits of fossil resources differ in their quality. Lower-quality pools have higher costs per unit of energy extracted. Geologists discover the cheapest pools first, so that the cost of extracting a fossil resource rises over time (as in Heal, 1976; Solow and Wan, 1976).[8] Whereas coal seams used to lie exposed on the surface, we now remove mountaintops.[9] Whereas oil used to seep out of the ground, we now drill miles beneath the sea. And whereas natural gas was once oversupplied as a byproduct of oil extraction, we now extract it by hydraulically fracturing shale formations. Cumulative extraction of fossil pools at the end of time t is Q_t, which means $Q_t - Q_{t-1} = F_t$ units were discovered during time t exploration. The cost of extracting from pool k is $\Psi(k)$, where $\Psi(\cdot)$ is positive, strictly increasing, and differentiable.

Whereas each period's fossil resource must be discovered and exploited, the renewable resource is known, constant, and freely available. There is a continuum of renewable resource

[7] Gans (2012) uses a similar energy aggregator but restricts the cross-partial to be weakly negative. In the present setting, increasing one type of resource can decrease the marginal *energy service* product of the other, but it never decreases the marginal *energy* product of the other.

[8] In our discrete-time setting with both exploration and extraction, we can coherently express cost as a function solely of the cumulative extraction state variable because all discovered resources are extracted at each timestep. With respect to Swierzbinski and Mendelsohn (1989), cumulative extraction therefore also proxies discoveries in the present setting. The extraction cost function may be interpreted as including the cost of exploration.

[9] As Jevons (1865, p. 198) noted: "The coal is itself limited in quantity—not absolutely, as regards us, but so that each year we gain our supplies with some increase of difficulty."

types, index by i, each with its specialized technology Z_{it}. Total renewable energy production is

$$R_t = \int_0^1 Z_{it}\, di.$$

Each type of resource produces the same type of energy: wind, solar, and geothermal all produce homogeneous electricity. The technology levels control how much of each resource can be converted to useful energy in time t. As technology improves, more renewable energy is produced from the same resource base.

The innovation framework is similar to Acemoglu et al. (2012). The primary difference is researchers' freedom to work in non-innovating sectors.[10] Engineers target sectors, not machines. An engineer targeting energy transformation machines (or renewable technologies) is allocated randomly to a machine type (resource type), and the engineer successfully innovates with probability η_E (η_R). A successful efficiency engineer receives a one-period patent on that machine and becomes its sole entrepreneur, and a successful renewable engineer receives a one-period property right to access the resource. Innovation increases quality by factors $\gamma_E > 0$ and $\gamma_R > 0$:

$$A_{it} = (1 + \gamma_E)\, A_{i(t-1)}, \qquad Z_{it} = (1 + \gamma_R)\, Z_{i(t-1)}.$$

The number of engineers working to improve the efficiency of energy transformation is ℓ_t, and the number of engineers working to improve technologies for renewable generation is r_t. Define the average efficiency of energy service production as $A_t = \int_0^1 A_{it}\, di$ and the average efficiency of renewable energy production as $Z_t = \int_0^1 Z_{it}\, di$. The average quality of each type of technology therefore evolves as:[11]

$$A_t = (1 + \gamma_E \eta_E \ell_t)\, A_{t-1}, \qquad Z_t = (1 + \gamma_R \eta_R r_t)\, Z_{t-1}.$$

Workers choosing to serve as geologists g_t expect to discover a pool of size ω. Time t discoveries are thus $\omega\, g_t$. Geologists extract a resource deposit upon discovering it, and they then supply the extracted resources to energy service producers. Because we take each timestep to be long, property rights to a discovery do not carry over between periods. Recalling previous definitions, we have

$$Q_t = Q_{t-1} + \omega g_t, \quad F_t = \omega g_t.$$

The deposits of fossil resources are discovered in order of increasing extraction cost, but conditional on being successful in time t, a geologist is equally likely to have discovered any

[10] Also, whereas innovation in Acemoglu et al. (2012) is applied to dirty and clean inputs to final-good production, here innovation enters further up the energy supply chain.

[11] There is idiosyncratic uncertainty in innovation and exploration but no aggregate uncertainty.

of the new time t deposits.[12] A geologist's expected per-unit cost of extraction is

$$\psi(g_t; Q_{t-1}) = \frac{1}{\omega g_t} \int_{Q_{t-1}}^{Q_{t-1}+\omega g_t} \Psi(k)\, dk.$$

$\Psi'(k) > 0$ implies that $\psi(g_t; Q_{t-1})$ is increasing in both of its arguments: expected extraction cost increases in depletion and in the number of contemporaneous geologists.

The economy must obey two constraints. First, there is a fixed measure of workers, normalized to 1, with all workers employed in each period. This labor constraint is

$$1 = L_t + \ell_t + r_t + g_t.$$

Second, the economy's resource constraint is

$$Y_t \geq C_t + \phi_E \int_0^1 x_{it}\, di + \int_{Q_{t-1}}^{Q_t} \Psi(k)\, dk,$$

where $C_t \geq 0$ is the single composite consumption good. Workers' monotonically increasing utility for that consumption good makes them want to maximize expected earnings.

We consider laissez-fair equilibria. Denote equilibrium outcomes with an asterisk.[13]

Definition 1. *An* equilibrium *is given by sequences of wages (w_t^*), prices for energy services (p_t^{E*}), prices for fossil resources (p_t^{F*}), prices for renewable resources (p_t^{R*}), prices for machines (p_{it}^{x*}), demands for machines (x_{it}^*), demands for inputs ($L_t^*, E_t^*, F_t^*, R_t^*$), and labor allocations ($L_t^*, \ell_t^*, g_t^*, r_t^*$) such that in each period t: (i) (p_{it}^{x*}, x_{it}^*) maximizes profits by the producer of machine type i, (ii) (F_t^*, R_t^*) maximizes profits of energy service producers, (iii) (L_t^*, E_t^*) maximizes profits of final-good producers, (iv) ($L_t^*, \ell_t^*, g_t^*, r_t^*$) maximizes the expected profit of every worker at date t, and (v) the wage w_t^* and factor prices ($p_t^{E*}, p_t^{F*}, p_t^{R*}$) clear the labor and input markets.*

[12] Acemoglu et al. (2012) also combine a model of directed technical change with an increasingly costly energy resource. In their setting, the time t cost of extraction is independent of time t extraction, depending only on extraction prior to time t. Further, in their setting, extraction does not compete for workers with either research or final-good production. In contrast, in the present setting, the expected cost of time t extraction depends both on previous extraction and on total time t exploration (and so on total time t extraction), and because geologists have other employment options, the resource price and expected cost of extraction must together provide sufficient rents to attract workers from other sectors. The present setting generalizes to specialized workers as long as the marginal workers can switch between sectors. Because we take the timestep to be on the order of decades, that employment decision could be taken at the time of education.

[13] I analyze how policy interventions affect decentralized market outcomes, not how the social planner would optimally allocate labor and output. The social planner's solution generally differs from the competitive allocation for several reasons: market power in the production of machines, pollution from fossil resources, the inability of geologists and engineers to capture rents beyond the immediate period, and spillovers in extraction and innovation.

Given the levels of incoming efficiency technology (A_{t-1}), incoming renewable energy technology (Z_{t-1}), and cumulative previous extraction (Q_{t-1}), the time t equilibrium prices clear all factor markets, all firms are maximizing profits, and a worker has the same expected earnings in every sector having nonzero employment under the equilibrium labor allocation. Throughout, we assume the existence of an equilibrium with nonzero employment in all sectors.

We analyze the response of equilibrium employment and prices to four types of policy interventions. First, a time t *efficiency policy* increases the quality of efficiency technology at the beginning of time t, prior to private sector engineering efforts. This type of policy marginally increases A_{t-1}. Second, a time t *supply restriction* blocks off marginal fossil resource deposits and thereby, for a given number of geologists, increases the expected cost of fossil resource extraction. This type of policy marginally increases Q_{t-1}.[14] Third, a *resource tax* imposes a tax of τ on each unit of extracted fossil resource. We analyze the effects of a marginal increase in the resource tax.[15] Fourth, a time t *renewable energy policy* increases the quantity of energy generated from the fixed renewable resources, prior to private sector engineering efforts. This type of policy marginally increases Z_{t-1}. When analyzing the persistence of a policy intervention, we consider the effects of a one-time intervention at time t, with the policy in periods $s > t$ fixed at the new time t level.

2 Contemporaneous Backfire in the Absence of Innovation or Renewables

We now consider how fossil resource extraction responds to policies to increase the efficiency of energy transformation, to lock up marginal fossil reserves, or to tax fossil resource extraction. We begin by analyzing a setting in which there are no renewable energy resources and there is no innovation in efficiency technologies. This setting approximates a world in which renewable energy production is a small fraction of total energy and in which the fraction of the labor force working in the fossil exploration sector is large relative to the fraction working in energy efficiency innovation.[16]

The following proposition describes how prices and extraction respond to each policy intervention:[17]

[14] If a time t supply restriction blocks non-marginal fossil resources, then we analyze the effects beginning in the future time s at which the blocked resources become marginal. In this paper's setting, the policy would not affect extraction or innovation in the periods between t and s.

[15] If the extraction cost function $\psi(\cdot)$ is linear, then the resource tax is equivalent to a supply restriction that blocks off τ/ψ' units of the resource.

[16] This section's results for a setting without equilibrium engineering effort are equivalent to the results for a setting with an exogenously fixed, nonzero measure of workers employed as engineers.

[17] We consider changes in the neighborhood of an equilibrium. The appendix shows that all interior equilibria are locally asymptotically stable in a tâtonnement sense.

Proposition 1 (Policies' effect on extraction)**.**

1. *There exists* $\hat{\sigma}(Q_{t-1}) \in (0,1)$ *such that a time* t *efficiency policy increases time* t *extraction if and only if* $\sigma > \hat{\sigma}(Q_{t-1})$ *and such that* $\hat{\sigma}'(Q_{t-1}) < 0$.

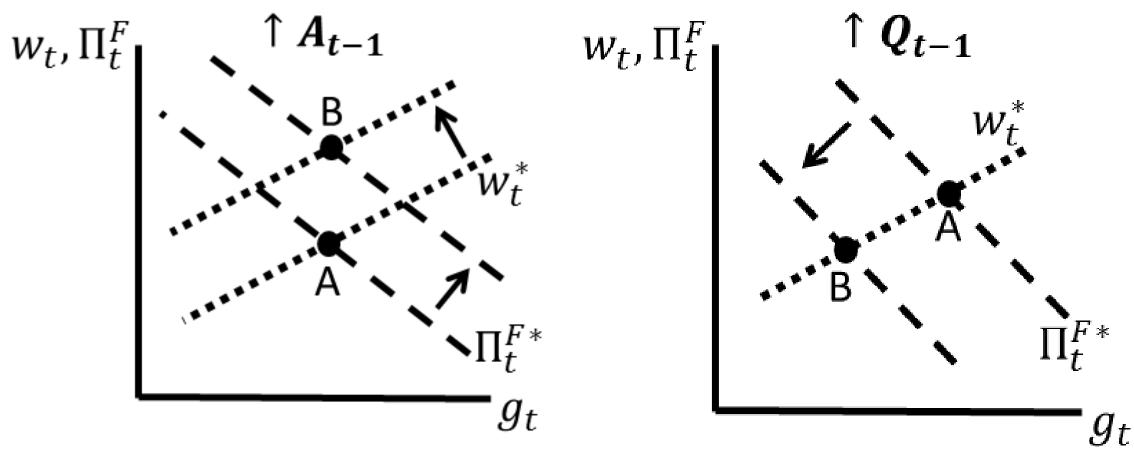
2. *A time* t *supply restriction and a time* t *resource tax each decrease time* t *extraction.*

Both the supply restriction and the resource tax succeed in reducing extraction. In contrast, the efficiency policy can increase extraction ("backfire") for sufficiently high elasticities of substitution. As extraction proceeds, the policy backfires for ever smaller elasticities of substitution, eventually backfiring in all cases.

To build intuition for the results, we analyze the change in equilibrium graphically and then algebraically. The graphical analysis constructs equilibrium profit and wage functions and imposes indifference between the two employment options. The downward-sloping dashed lines in Figure 1 (labeled Π_t^{F*}) plot the equilibrium expected profit from fossil exploration. It depicts the combinations of exploration activity and prices such that the wage and the price of energy services satisfy the final-good firm's zero-profit condition while also equating the wage to the reward from fossil exploration.[18] These dashed lines slope downward because the presence of additional geologists increases the expected cost of extraction: when more geologists are searching for resources, a given geologist becomes more likely to lose the race for the higher-quality remaining resources. The upward-sloping dotted lines (labeled w_t^*) plot combinations of wages and final-good labor such that the wage and the price of energy services satisfy the final-good firm's zero-profit condition while also paying labor its marginal product. These lines would slope downward in L_t as conventional demand curves, but we use labor-market clearing ($L_t + g_t = 1$) to express them in terms of geologists. These give the potential equilibrium rewards to wage-labor. When σ is small, these lines are nearly vertical because labor demand is price-inelastic, and when σ is large, these lines are nearly horizontal because labor demand becomes price-elastic. The equilibrium is at the intersection of the dashed and dotted lines, initially at point A.

The left panel shows how two competing effects determine whether an efficiency policy backfires. First, improving the efficiency of energy service production shifts the exploration reward curve upward because each unit of extracted resource now produces more energy services. This is a *marginal productivity channel* arising from energy service firms' willingness to pay a higher price for fossil resources. The appendix shows that the equilibrium price of

[18] To construct these sets of possible equilibria, plot the downward-sloping exploration reward curve as a function of geologists for several fixed prices of energy services. In equilibrium, the reward to exploration must equal the wage paid to final-good labor, and the final-good firm's zero-profit condition requires that the wage decrease in the price of energy services. The set of potential equilibria are constructed by varying the price of energy services in order to find the set of intersections between the wage corresponding to a fixed price of energy services and the exploration reward corresponding to the same fixed price of energy services. The equilibrium wage curve derives from applying similar steps to the final-good firm's demand for labor.



(a) Efficiency policy (b) Supply restriction or resource tax

Figure 1: Increasing A_{t-1} (left panel) or Q_{t-1} (right panel) shifts the equilibrium from point A to point B.

fossil resources is:

$$p_t^{F*} = (1-\alpha)\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}\left[p_t^{E*}\right]^{\frac{1}{1-\alpha}} A_t.$$

Better efficiency technology A_t increases the equilibrium number of energy service machines and also increases each machine's productivity. Both changes increase the marginal product of fossil resources.[19] Profit-maximization then requires that, for a given price of energy services, the energy service firm pay more for fossil resources.[20] The higher resource price increases geologists' expected revenue, which attracts more workers to fossil resource exploration.

A second, competing effect works against backfire and in favor of reducing extraction. Im-

[19] This marginal productivity channel in the context of resource extraction is similar to the weak absolute equilibrium bias described by Acemoglu (2007) in the context of directed technical change: due to market size effects, making energy services more abundant induces a change in extraction that increases the marginal product of energy efficiency technologies.

[20] Indeed, this is quite close to the original channel for backfire proposed by Jevons (1865, p. 141):

> Now, if the quantity of coal used in a blast-furnace, for instance, be diminished in comparison with the yield, the profits of the trade will increase, new capital will be attracted, the price of pig-iron will fall, but the demand for it increase; and eventually the greater number of furnaces will more than make up for the diminished consumption of each.

Jevons' pig-iron furnaces are our energy service machines. Fossil resource extraction becomes more attractive because the greater number of machines increases the price that the energy service firm is willing to pay for a unit of extracted resource. Fossil resource extraction increases even as the price of energy services ("pig-iron") falls.

proving the efficiency of energy service production shifts the labor demand curve out because of the greater availability of the other, energy service input. The dotted curve, expressed in terms of geologists rather than final-good labor, therefore shifts inward. This *energy substitution channel* tends to decrease extraction. The appendix shows that equilibrium labor demand is:

$$L_t^* = \left[\frac{1-\kappa}{\kappa}\right]^\sigma \left[\frac{p_t^{E*}}{w^*(p_t^E)}\right]^\sigma E_t^*, \text{ where } E_t^* = \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} [p_t^{E*}]^{\frac{\alpha}{1-\alpha}} A_t \; e\left(F_t, R_t\right).$$

The final-good firm's demand for labor (as a function of the price of energy services) shifts out when the equilibrium quantity of energy services E_t^* increases. For a given quantity of energy services, labor demand increases in the price of energy services directly, and it also indirectly increases in the price of energy services because, via the profit-maximizing final-good firm's zero-profit condition, the equilibrium wage must fall as the price of energy services rises. Labor demand increases more strongly in the price of energy services when the elasticity of substitution σ is larger. The equilibrium quantity of energy services E_t^* increases in fossil resource extraction (F_t), in the quality of efficiency technology (A_t), and in the price of energy services (p_t^E). Improving energy efficiency increases equilibrium final-good labor at a given price of energy services, which decreases the equilibrium number of geologists corresponding to that price of energy services.

The net effect of these two channels is to shift the equilibrium to a point such as B in the left panel of Figure 1. The new equilibrium clearly has a greater wage and a lower price of energy services (as demonstrated analytically in the appendix), but the effect on the equilibrium number of geologists is graphically ambiguous. The improved efficiency technology could raise the price of fossil resources even as it reduces the price of energy services.[21] For large elasticities of substitution, the labor demand curve is relatively flat and the shift in the expected profit curve dominates, leading to an increase in equilibrium extraction (i.e., backfire).[22] However, for small elasticities of substitution, the labor demand curve is relatively steep, making its inward shift dominate the shift in expected profit. In that case, equilibrium extraction decreases when energy efficiency improves.

Note that the equilibrium resource price is linear in the quality of efficiency technology. The revenue from resource extraction is then also linear in the quality of efficiency technology. For a given price of energy services, doubling the quality of efficiency technology also

[21] Borenstein (2013) and Gillingham et al. (2013) informally discuss how an efficiency-induced reduction in energy prices can in turn induce additional energy consumption (though not by enough to produce backfire), and Gillingham et al. (2013) further claim that efficiency policies reduce the incentive to extract energy resources. Our formal analysis shows that while the equilibrium price of *energy services* indeed decreases, the equilibrium price of *energy resources* may nonetheless increase and so strengthen incentives to extract energy resources.

[22] If we interpret the final-good production function as a utility function, then we may be interested in the implications of non-homothetic preferences. Non-homotheticity favors backfire if increasing income raises the elasticity of substitution between labor services and energy services.

doubles revenue. When the cost of resource extraction is high due to depletion, equilibrium revenue must also be high in order to maintain indifference with wage-labor. And when revenue is high, an improvement in efficiency more strongly increases the expected profit from exploration. Therefore, in states of high depletion (high Q_{t-1}), an improvement in efficiency technology shifts Π_t^{F*} out further and more strongly favors backfire. Intuitively, the equilibrium marginal product of fossil resources is greater when fossil resources are scarce (i.e., depleted), and improved efficiency technology more strongly increases marginal product when marginal product is already high.

The right panel of Figure 1 depicts the effect of an incremental increase in cumulative previous extraction. The reduction in resource quality imposes costs on geologists, shifting expected profit down. However, labor demand is unaffected. The shift to the new equilibrium reduces the wage, increases the price of energy services, and decreases the number of geologists. Final-good production tilts towards the non-energy, wage-labor input. The equilibrium reduction in extraction is greater the larger is σ (i.e., the flatter is the labor demand curve). A supply restriction or resource tax most effectively reduces extraction when an efficiency policy would backfire.

Before considering the dynamic effects of each policy, we algebraically analyze the competing channels determining the response of extraction to an efficiency policy. The proof of Proposition 1 shows that the effect of an efficiency policy on equilibrium geologists is proportional to the following expression:

$$\frac{\partial \frac{\Pi^{F*}(g_t, p_t^E; A_{t-1})}{w^*(p_t^E)}}{\partial A_{t-1}} \frac{\partial L^*(g_t, p_t^E; A_{t-1})}{\partial p_t^E} - \frac{\partial L^*(g_t, p_t^E; A_{t-1})}{\partial A_{t-1}} \frac{\partial \frac{\Pi^{F*}(g_t, p_t^E; A_{t-1})}{w^*(p_t^E)}}{\partial p_t^E},$$

where $\Pi^{F*}(\cdot)$ and $L^*(\cdot)$ are the equilibrium exploration reward and equilibrium labor demand as functions of the number of geologists and of the price of energy services. All of these partial derivatives are positive. The reward to exploration increases in both the quality of efficiency technology and the price of energy services because the price of fossil resources increases in these variables, via the energy service firm's first-order conditions. And we have already seen how the quantity of labor demanded increases in the quantity of energy services (and so in A_{t-1}) and in the price of energy services. The left-hand terms increase equilibrium exploration via the marginal productivity channel. The efficiency policy increases the expected revenue from fossil exploration, which increases the relative return Π_t^F/w_t to exploration and induces entry into exploration. The resulting decline in the price of energy services reduces the demand for wage-labor L_t, which increases the number of workers available to work as geologists. These terms favor backfire. In contrast, the right-hand terms work against backfire via the energy substitution channel described graphically. The improvement in energy efficiency increases the demand for wage-labor, which pulls workers away from fossil resource exploration. The improvement in efficiency also decreases the price of energy services, which reduces the relative return Π_t^F/w_t to exploration and so also pushes workers away from exploration.

Backfire occurs when the left-hand terms dominate. The left-hand wage-labor term is smaller the more inelastic is labor demand (i.e., the smaller is σ). If labor demand is highly inelastic, then few workers will shift into exploration, and the reequilibration of wages and expected profits from exploration must occur almost solely through price changes. But if labor demand is highly elastic, then workers respond strongly to changes in relative incentives. In that case, reequilibration between the exploration reward and wages primarily occurs from workers shifting out of wage-labor (thereby raising the wage) and into exploration (thereby reducing the reward to exploration by raising expected extraction costs).

Finally, observe that the rightmost term is the only one that depends on extraction costs. It is smaller in magnitude when expected extraction costs constitute a greater fraction of expected profit: a higher price for energy services increases the expected revenue from fossil resource exploration and decreases the wage paid to final-good labor, but it leaves the expected cost of fossil resource extraction unchanged. The ratio of expected profit to the wage is less responsive to the price of energy services when extraction costs are a larger fraction of revenues. Reequilibration between the exploration reward and wages then occurs primarily through labor shifts. Increasing depletion favors backfire.

3 Persistent Effects of a Policy Intervention, via Depletion

A time t policy also affects extraction in future periods. Differentiating a first-order Taylor approximation to time $t+N+1$ equilibrium geologists $g^*(A_{t+N}, Q_{t+N})$, we have the response to a time t efficiency policy:

$$
\begin{aligned}
\frac{dg^*(A_{t+N}, Q_{t+N})}{dA_{t-1}} \approx & \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1}} \\
& + \frac{\partial^2 g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1} \partial Q_{t-1}} \sum_{i=0}^{N} \omega\, g^*(A_{t-1+i}, Q_{t-1+i}) \\
& + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial Q_{t-1}} \sum_{i=0}^{N} \frac{\partial[\omega\, g^*(A_{t-1+i}, Q_{t-1+i})]}{\partial A_{t-1}}.
\end{aligned}
$$

A time t efficiency policy backfires at time t when $\partial g^*(A_{t-1}, Q_{t-1})/\partial A_{t-1} > 0$ and reduces time t extraction otherwise. The time t policy affects future extraction through three channels. The first line reflects that the technological improvement is permanent. This channel favors backfire in later periods if and only if there was backfire in time t. And greater time t backfire favors greater backfire in later periods. The second line controls how future backfire responds to the extraction that has occurred from time t onwards. The proof of Proposition 1 shows that, if backfire occurs, then the cross-partial is positive as long as $\psi(\cdot)$ is not too convex.[23] The summation gives the total amount of extraction that occurred since the policy's implementation. As long as the extraction cost function is not strongly curved, the greater resource depletion in future periods tends to make backfire stronger than in time t (and tends to make any future reduction in extraction smaller than in time t). The third line recognizes that even if the time t improvement in efficiency technology did not endure, the time t policy would still affect future extraction by having changed the incoming level of resource depletion. The summation gives the total change in extraction from periods t to $t+N$ due to the policy's implementation in time t. If the policy consistently backfires, then it increases future periods' extraction costs and so reduces future periods' extraction. And if the time t policy consistently reduces extraction, then it reduces future periods' extraction costs and so increases future periods' extraction.

If the time t policy backfires, then the first two lines often favor backfire in future periods whereas the third line works against further backfire: the persistent improvement in technology and the continual increase in extraction costs often favor future backfire, but backfire in early periods itself reduces the profit from future extraction. If backfire is only a small fraction of a period's extraction, then the second line becomes large relative to the third, tending to produce further backfire.

If the time t policy does not backfire, then the second and third lines both work to reduce the policy's effect over time—and potentially even reverse it to produce future backfire. The second line is as before, but now the third line recognizes that the policy has reduced future depletion. The second and third lines grow over time, whereas the first line is constant. Unless extraction ceases, the second and third lines would typically dominate the first at some point in the future. Any efficiency policy tends to backfire in some future period, though such a policy may still succeed in reducing both cumulative resource extraction and the present value of environmental damages associated with resource extraction.

Next, consider how a time t supply restriction affects extraction in future periods. A Taylor approximation shows that the response of time $t+N+1$ equilibrium extraction to a time t supply restriction is

$$\begin{aligned}\frac{dg^*(A_{t+N},Q_{t+N})}{dQ_{t-1}} \approx & \frac{\partial g^*(A_{t-1},Q_{t-1})}{\partial Q_{t-1}} \\ & + \frac{\partial^2 g^*(A_{t-1},Q_{t-1})}{\partial [Q_{t-1}]^2} \sum_{i=0}^{N} \omega\, g^*(A_{t-1+i},Q_{t-1+i}) \\ & + \frac{\partial g^*(A_{t-1},Q_{t-1})}{\partial Q_{t-1}} \sum_{i=0}^{N} \frac{\partial[\omega\, g^*(A_{t-1+i},Q_{t-1+i})]}{\partial Q_{t-1}}.\end{aligned}$$

As before, there are three channels. We know that the time t policy reduces time t extraction. The first line reflects the policy's permanence: blocking off the resource tends to decrease

[23] If the extraction cost function is strongly convex, then an initial increase in extraction raises future extraction costs to such an extent that future extraction falls.

extraction in future periods just as it does in time t. The second line recognizes that the policy's effect changes as depletion increases. The proof of Proposition 1 also shows that $\partial^2 g^*(\cdot,\cdot)/\partial Q_{t-1}^2 < 0$ if and only if $\psi(\cdot)$ is not too concave. In this case, the second line further reduces future extraction. The initial supply restriction raises the cost of extraction by more as the resource becomes more depleted. The third line reflects that by successfully decreasing extraction in earlier periods, a supply restriction bequeathes a less depleted resource base to later periods. The reduction in future extraction costs induces additional future extraction.

The second line increases the effectiveness of the supply restriction as time passes when $\psi(\cdot)$ is convex, but the third line makes past successes work against future success. The second line is relatively larger the more convex is the extraction cost function and the smaller is the induced reduction in extraction relative to a period's total extraction. A small supply restriction tends to become larger over time, whereas a large policy tends to become smaller over time.

The dynamic effects of a time t resource tax differ only through the second line: the analogous version of the second line increases the effectiveness of the resource tax over time if and only if $\psi(\cdot)$ is not too convex. Under a convex $\psi(\cdot)$, a blocked resource raises extraction costs more strongly over time, but a constant resource tax becomes a smaller fraction of extraction costs over time. Thus, even though a resource tax and a supply restriction appear interchangeable in a static setting, they have different dynamic implications. If the extraction cost function is convex, then a supply restriction acts like an increasing resource tax because it imposes increasingly large costs as depletion progresses.

4 Crowding Out vs Inducement of Private Innovation

The previous section held efficiency technologies constant in the absence of a policy intervention. In contrast, in a model with endogenous engineering, public support for innovation affects private innovation. This in turn affects the workers available for other activities and the incentives for resource extraction.

Engineering is qualitatively different from resource exploration in an important way: the expected profit from working as a geologist declines in the number of other geologists because the expected cost of extraction increases in the number of geologists, but, holding geologists and wage-labor fixed, the expected profit from working as an efficiency engineer is independent of the number of other efficiency engineers. And once geologists respond to the number of engineers, the expected profit from engineering actually increases in the number of engineers. The appendix shows that the equilibrium expected profit from efficiency engineering is:

$$\Pi_t^{E*} = \eta_E(1+\gamma_E)\phi_E \frac{1-\alpha}{\alpha}\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{1}{1-\alpha}} \left[p_t^E\right]^{\frac{1}{1-\alpha}} A_{t-1}\, e\left(F_t, R_t\right),$$

where the energy input $e(F_t, R_t)$ is just time t fossil resource extraction $(\omega\, g_t)$ in the present setting without a renewable resource. Increasing the number of geologists increases the profits from owning a machine that converts those resources into energy services, which in turn increases the return to efficiency engineering. We saw previously that the price of fossil resources increases in the realized efficiency technology A_t and so in the number of efficiency engineers. If the number of engineers increases, the resource price increases and so attracts more geologists. The additional geologists in turn raise the expected profit from efficiency engineering. Therefore, for a given price of energy services, the reward to engineering increases in the number of engineers.[24]

This upward-sloping profit curve can produce corner solutions in the labor market. Increasing the number of engineers increases the quantity of energy services, which are better substitutes for labor in final-good production when σ is large. In that case, demand for final-good labor is highly elastic. The wage offered to labor does not increase by much, but the profit from efficiency engineering increases due to the complementary resource extraction sector. The relative return to engineering may now be even greater than before, attracting further engineers. This is an example of strong absolute equilibrium bias (Acemoglu, 2007): increasing the abundance of energy services can actually increase the marginal product of energy services (i.e., greater supply of energy services can actually raise the price of energy services at the new equilibrium technology level). The appendix shows that there exists $\tilde{\sigma} > 1$ such that strong equilibrium bias arises only if $\sigma > \tilde{\sigma}$.

When strong absolute equilibrium bias arises, exogenously introducing geologists or efficiency engineers increases the relative return to exploration and engineering. Corner solutions with all labor engaged in either final-good production or in some combination of engineering and exploration therefore become attractive equilibria. Because these corner solutions are not realistic, and because empirical work suggests that energy and non-energy inputs to production are not strong substitutes, we ignore cases with strong absolute equilibrium bias:

Assumption 1. *Strong absolute equilibrium bias does not arise:* $\sigma < \tilde{\sigma}$.

The following proposition describes how efficiency, supply restriction, and resource tax

[24]Each engineer's decision generates a positive pecuniary externality on other engineers because of the response of geologists to the higher marginal product of fossil energy. These pecuniary externalities can have first-order effects because of the presence of imperfect competition in the machine-producing sectors (e.g., Murphy et al., 1989). See Young (1993) for another setting with tension between complementarity and substitution. Matsuyama (1995) discusses complementarities and multiplicity of equilibria in models with monopolistic competition.

policies affect equilibrium efficiency engineering and extraction:[25,26]

Proposition 2 (Endogenous efficiency engineering). *Let Assumptions 1 and 2 (given in the appendix) hold. Then:*

1. *There exists $\hat{\sigma}(Q_{t-1}) < 1$ such that a time t efficiency policy increases time t extraction if and only if $\sigma > \hat{\sigma}(Q_{t-1})$ and such that $\hat{\sigma}'(Q_{t-1}) < 0$.*
2. *A time t supply restriction and a time t resource tax each decrease time t extraction.*
3. *There exists $\hat{\sigma}(Q_{t-1}) \in (1, \tilde{\sigma})$ such that a time t efficiency policy increases time t efficiency engineering if and only if $\sigma > \hat{\sigma}(Q_{t-1})$. If $\psi(\cdot)$ is linear, then $\hat{\sigma}'(Q_{t-1}) < 0$.*
4. *There exists $\hat{\sigma} \in (1, \tilde{\sigma})$ such that a time t supply restriction and a time t resource tax each increases time t efficiency engineering if and only if $\sigma < \hat{\sigma}$.*

Endogenizing efficiency engineering does not change the qualitative effect of each policy on contemporary extraction. The effect of each policy on private-sector engineering depends on the elasticity of substitution. If energy services and final-good labor are complements or sufficiently weak substitutes ($\sigma < \hat{\sigma}$), then an efficiency policy reduces contemporaneous engineering whereas a supply restriction or tax policy increases contemporaneous efficiency engineering. The opposite pattern holds in the case of moderate substitutes ($\sigma \in (\hat{\sigma}, \tilde{\sigma})$).

The new results concern equilibrium engineering. As in Section 2, the upward-sloping dotted lines in Figure 2 plot labor demand (labeled w_t^*), here recognizing that $L_t = 1 - \ell_t - g_t$ and that, for a fixed price of energy services, g_t is increasing in ℓ_t. The upward-sloping dashed lines (labeled Π_t^{E*}) plot the equilibrium expected profit from efficiency engineering. These are combinations of engineering and prices such that the wage and the price of energy services satisfy the final-good firm's zero-profit condition while also equating the wage to the reward from efficiency engineering. This curve is upward-sloping because it allows geologists to respond to changes in engineering. For a fixed price of energy services, the energy substitution channel from efficiency improvements does not exist, so the marginal productivity channel makes the number of geologists increase in the number of engineers, which in turn makes

[25] The proposition also uses Assumption 2. That additional assumption restricts the responsiveness of the equilibrium wage function with respect to σ in order to ensure uniqueness of each cutoff $\hat{\sigma}$. The proof does demonstrate uniqueness in a range of cases without using the assumption. Numerical simulations suggest that the assumption usually holds in the cases for which it is invoked, and they also suggest that uniqueness always holds.

[26] The appendix shows that, in the presence of complementarities, all equilibria are unstable in a tâtonnement sense. Because engineering profits increase in the number of engineers, the natural tâtonnement-style price changes in response to excess demand for labor tend to induce entry into exploration and engineering, which reinforces the excessiveness of labor demand. Similarly, the natural tâtonnement-style employment changes in response to excess rewards to engineering and exploration tend to further amplify the relative rewards to engineering and exploration.

the profit from engineering increase in the number of engineers. As explained above, the complementarity between the two sectors generates an upward-sloping engineering profit curve. The plotted curve is constructed by equating the engineering reward to the wage implied by a given price of energy services (see footnote 18).

Equilibria occur at the intersections of these curves. The initial equilibrium in each panel of Figure 2 is at point A. Increasing the quality of incoming efficiency technology has two effects. First, the marginal productivity effect analyzed in Section 2 increases the number of geologists corresponding to any given number of engineers. This change increases the profit from engineering, which shifts the dashed curve upward and tends to increase the equilibrium number of engineers. Second, the improvement in efficiency technology increases the demand for final-good labor, which shifts the equilibrium wage curve (dotted lines) inward and tends to decrease the equilibrium number of engineers.

The net result of these effects is to shift the equilibrium to point B. For $\sigma < \hat{\sigma}$ (complements or weak substitutes, top panel), labor demand is not very elastic, which makes the equilibrium wage curve relatively steep. The energy substitution channel then dominates. The efficiency policy increases the wage and decreases the price of energy services. It also decreases the number of efficiency engineers. However, for $\sigma \in (\hat{\sigma}, \tilde{\sigma})$ (moderate substitutes, middle panel), the equilibrium wage curve is sufficiently flat that the shift in engineering profit dominates. Now, rather than crowding out private effort, an efficiency policy increases the equilibrium number of engineers.[27]

We saw previously that, for a given number of engineers, a time t supply restriction or resource tax reduces the equilibrium number of geologists. In Figure 2, a reduction in the number of geologists per engineer has the opposite effects as an efficiency policy: the reduced value of energy service machines shifts the equilibrium engineering reward curve down, and the reduction in the supply of energy services for a given number of engineers shifts the labor demand curve in (and so shifts the plotted equilibrium wage curve out). Engineering effort increases if energy services and final-good labor are complements or weak substitutes, and engineering effort decreases if they are moderate substitutes.

Finally, unlike the case of equilibrium extraction, the response of equilibrium engineering depends on the curvature of the extraction cost function $\psi(\cdot)$. The reason is that this function's curvature determines the steepness of the equilibrium engineering profit curves in Figure 2. When $\psi(\cdot)$ is linear, Q_{t-1} does not affect the rate of increase of $\psi(\cdot)$, and we are left with the clearly signed effects analyzed in the setting without engineering. However, the more convex is $\psi(\cdot)$, the more strongly a geologist's expected extraction cost increases in

[27] In a case with $\sigma > \tilde{\sigma}$, the equilibrium wage curve has rotated so far clockwise that it is flatter than the equilibrium engineering reward curve. The comparative statics for an interior equilibrium are opposite to those from the moderate substitute case because the equilibrium wage curve cuts the engineering reward curve from the opposite direction. However, for all disequilibrium employment levels, reshuffling workers towards the sector offering greater expected income moves towards a corner solution rather than towards the interior equilibrium.

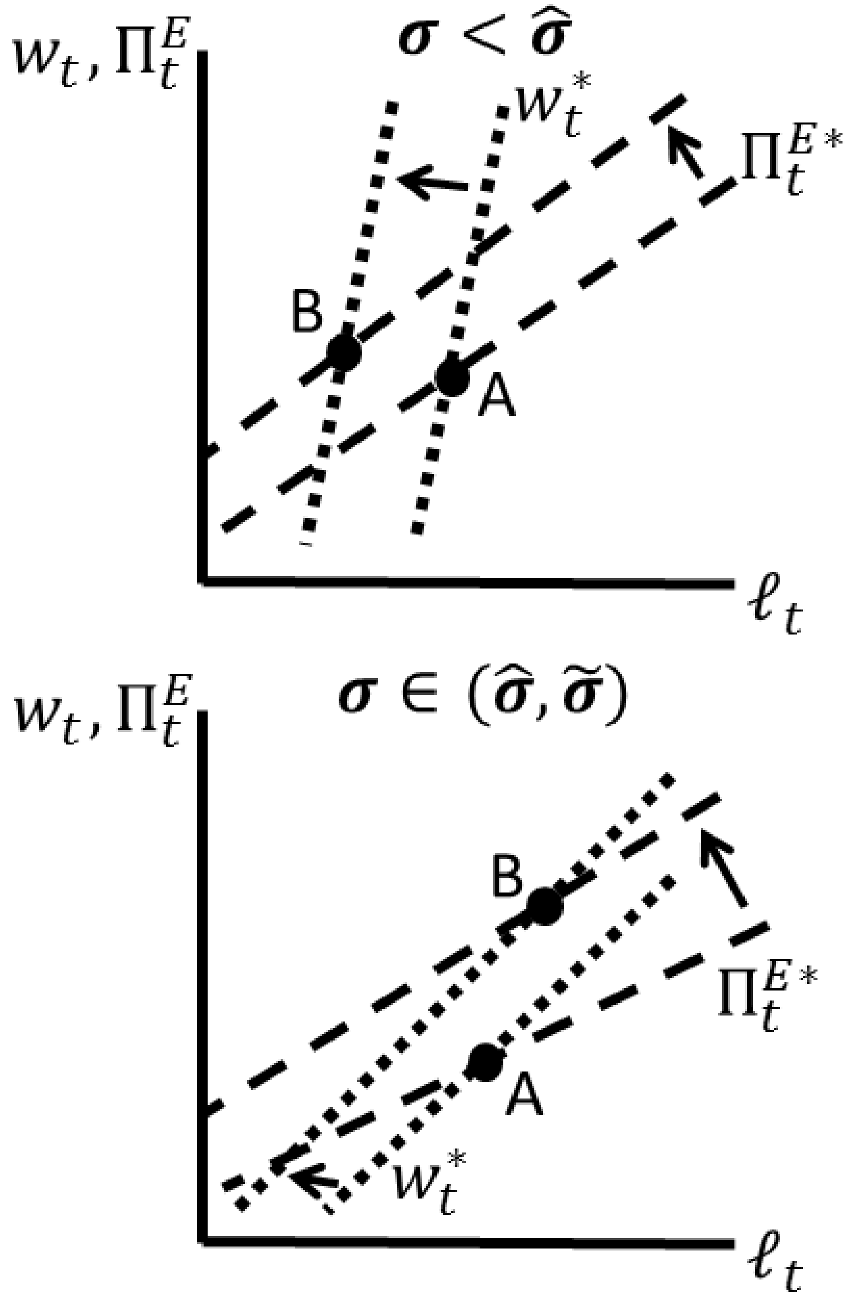


Figure 2: An efficiency policy shifts the equilibrium wage and number of efficiency engineers from point A to point B. A supply restriction or resource tax policy has the opposite effect.

the number of other geologists, in which case additional engineering induces relatively few additional geologists and the equilibrium engineering profit curve is relatively flat. Greater depletion thus tends to increase the range of σ for which an efficiency policy induces engineering. But by the same logic, a more convex $\psi(\cdot)$ also reduces the marginal productivity effect's implications for engineering profit. When depletion is greater, an efficiency policy shifts the equilibrium engineering profit curve out by less, reducing the range of σ for which an efficiency policy induces innovation. The direction in which an efficiency policy's effect on innovation changes as depletion progresses is therefore ambiguous when $\psi(\cdot)$ is nonlinear.

5 Persistent Effects of a Policy Intervention, via Innovation

Now consider how endogenous innovation alters the persistence of each policy's effect on extraction. Differentiating a first-order Taylor approximation to time $t+N+1$ geologists $g^*(A_{t+N}, Q_{t+N})$, we have the response to a time t efficiency policy in the presence of engineers:

$$\begin{aligned}
\frac{dg^*(A_{t+N}, Q_{t+N})}{dA_{t-1}} \approx & \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1}} \\
& + \frac{\partial^2 g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1} \partial Q_{t-1}} \sum_{i=0}^{N} \omega\, g^*(A_{t-1+i}, Q_{t-1+i}) \\
& + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial Q_{t-1}} \sum_{i=0}^{N} \frac{\partial[\omega\, g^*(A_{t-1+i}, Q_{t-1+i})]}{\partial A_{t-1}} \\
& + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1}} \prod_{i=0}^{N} (1 + \eta\gamma\ell^*(A_{t-1+i}, Q_{t-1+i})) \\
& + \frac{\partial^2 g^*(A_{t-1}, Q_{t-1})}{\partial [A_{t-1}]^2} \prod_{i=0}^{N} (1 + \eta\gamma\ell^*(A_{t-1+i}, Q_{t-1+i}))\, A_{t-1} \\
& + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1}} \eta\gamma\, A_{t-1} \sum_{i=0}^{N} \left(\frac{\partial \ell^*(A_{t-1+i}, Q_{t-1+i})}{\partial A_{t-1}} \prod_{j=0,\neq i}^{N} (1 + \eta\gamma\ell^*(A_{t-1+j}, Q_{t-1+j})) \right)
\end{aligned}$$

The first three lines are as in the discussion of the setting without engineering, where cumulative extraction was the only state variable that evolves over time. The remaining lines reflect the effect of endogenous innovation. The fourth line recognizes that future engineering effort has a multiplicative effect on future technology. Increasing the incoming time t efficiency technology increases the effectiveness of all future innovation effort. This amplifies the effects of the time t efficiency policy in all future periods. The fifth line recognizes that the propensity to backfire declines in the pre-existing quality of efficiency technology (the second derivative is negative).[28] As technology improves, backfire becomes less likely. The final line reflects how the efficiency policy itself redirects innovation. If $\sigma < \hat{\sigma}$, then the efficiency policy tends to decrease engineering, which mitigates the effect of the efficiency policy on future periods' extraction. However, if $\sigma > \hat{\sigma}$, then the efficiency policy tends to increase engineering, which amplifies its effect on future periods' extraction.

As long as there is continued engineering effort, the innovation channels become stronger as time passes. If there is backfire in time t, then the fourth and fifth lines should net out to a pro-backfire effect in future periods. And if $\sigma > \hat{\sigma}$, then the final line also increases backfire over time. In this case, innovation channels are likely to amplify cumulative backfire as time passes.

The analogous expression for a supply restriction is

$$\begin{aligned}\frac{dg^*(A_{t+N}, Q_{t+N})}{dQ_{t-1}} \approx & \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial Q_{t-1}} \\ & + \frac{\partial^2 g^*(A_{t-1}, Q_{t-1})}{\partial [Q_{t-1}]^2} \sum_{i=0}^{N} \omega\, g^*(A_{t-1+i}, Q_{t-1+i}) \\ & + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial Q_{t-1}} \sum_{i=0}^{N} \frac{\partial [\omega\, g^*(A_{t-1+i}, Q_{t-1+i})]}{\partial Q_{t-1}} \\ & + \frac{\partial^2 g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1} \partial Q_{t-1}} \prod_{i=0}^{N} (1 + \eta\gamma \ell^*(A_{t-1+i}, Q_{t-1+i}))\, A_{t-1} \\ & + \frac{\partial g^*(A_{t-1}, Q_{t-1})}{\partial A_{t-1}} \eta\gamma\, A_{t-1} \sum_{i=0}^{N} \left(\frac{\partial \ell^*(A_{t-1+i}, Q_{t-1+i})}{\partial Q_{t-1}} \prod_{j=0, \neq i}^{N} (1 + \eta\gamma \ell^*(A_{t-1+j}, Q_{t-1+j})) \right)\end{aligned}$$

The effects of a resource tax are qualitatively similar. Endogenous innovation introduces the final two lines. As noted before, the cross-partial is positive. The second-to-last line reflects how the policy increases the degree of backfire (or decreases the degree of resource savings) from future innovation by increasing the cost of resource extraction. This channel works to reverse the policy's effects over time. The final line recognizes that the time t policy redirects future innovation. If $\sigma > \hat{\sigma}$ and $\partial g^*/\partial A_{t-1} > 0$, or if $\sigma < \hat{\sigma}$ and $\partial g^*/\partial A_{t-1} < 0$, then the policy's effect on future innovation decreases future extraction. These combinations are particularly plausible because we have seen that high σ favors backfire. When σ is large, the equilibrium wage curves in Figures 1 and 2 are flat (i.e., demand for labor is elastic), and shifts in the engineering and exploration reward functions determine the change in equilibrium outcomes. Reduced extraction decreases the return to engineering, which decreases the return to exploration and so further decreases extraction. When σ is small, the equilibrium wage curve is steep, and shifts in that curve dominate shifts in the equilibrium reward

[28]The proof of Proposition 1 shows that $\partial^2 g_t/\partial A_{t-1}^2 < 0$ if $\partial g_t/\partial A_{t-1} > 0$.

curves. Reduced extraction decreases demand for wage-labor, which frees workers to become engineers, which shifts the demand for wage-labor back out and raises the opportunity cost of exploring for fossil resources. Endogenizing engineering now tends to amplify the degree to which a time t extraction policy decreases future extraction. It is not clear which of these two innovation channels dominates over time.

We have seen that endogenizing efficiency engineering does not reverse the main conclusions about contemporaneous backfire from an efficiency policy or about the contemporaneous benefit from a supply restriction or resource tax. As time passes after a policy's adoption, engineering responses tend to increase backfire from an efficiency policy, and they could either amplify or mitigate the benefits of a supply restriction or resource tax.

6 Backfire Induced by a Renewable Energy Sector

This section analyzes the implications of a renewable energy sector. The energy input to energy service production is now a function of both renewable and fossil energy, and workers can choose to work as engineers in the renewable sector. We will see that a renewable energy policy can backfire for similar reasons as an efficiency policy and that supply restrictions and resource taxes can now also backfire. Further, the potential for backfire now depends on the degree to which renewable resources are imperfect substitutes for fossil resources.

The following proposition describes how efficiency, supply restriction, resource tax, and renewable energy policies affect equilibrium engineering and extraction. As before, there exists $\tilde{\sigma} > 1$ such that strong equilibrium bias arises only if $\sigma > \tilde{\sigma}$, and, applying Assumption 1, we restrict attention to $\sigma < \tilde{\sigma}$ in order to avoid analyzing cases in which corner solutions are attractive equilibria:[29]

Proposition 3 (Renewable energy setting)**.** *Let Assumption 1 hold, and further assume that* $\partial J_1/\partial p_t^E$ *(defined in the appendix) has a unique root in* σ*. Then:*

1. *There exists* $\hat{\sigma} < \infty$ *such that a time* t *renewable energy policy increases time* t *extraction if and only if* $\sigma > \hat{\sigma}$ *and such that* $\hat{\sigma}$ *decreases in* Q_{t-1}*, in* $-\partial^2 e(F_t, R_t)/\partial R_t^2$*, and in* $\partial^2 e(F_t, R_t)/\partial F_t \partial R_t$*.*

2. *There exists* $\hat{\sigma} < \infty$ *such that a time* t *efficiency policy increases time* t *extraction if and only if* $\sigma > \hat{\sigma}$ *and such that* $\hat{\sigma}$ *decreases in* Q_{t-1}*. If* $\partial e(F_t, R_t)/\partial R_t$ *is constant, then a time* t *efficiency policy increases time* t *extraction for all* $\sigma \in (0, \tilde{\sigma})$*.*

3. *There exists* $\hat{\sigma} \in (1, \infty)$ *such that a time* t *supply restriction and a time* t *resource tax each decrease time* t *extraction if and only if* $\sigma < \hat{\sigma}$ *and such that* $\hat{\sigma}$ *increases in*

[29] As in the setting with efficiency engineering but no renewable energy sector, all equilibria are unstable in a tâtonnement sense due to complementarities between efficiency engineering and each type of energy resource production.

$-\partial^2 e(F_t, R_t)/\partial R_t^2$. If $\partial^2 e(F_t, R_t)/\partial R_t^2$ and $\partial^2 e(F_t, R_t)/\partial F_t \partial R_t$ are sufficiently small in magnitude and Z_{t-1} is sufficiently large, then $\hat{\sigma} < \tilde{\sigma}$.

Not only can a renewable energy policy backfire, but its potential for backfire depends on the degree to which the intermittent, variable nature of renewable resources diminishes their marginal product (or, equivalently, it depends on the quality of energy storage to offset this intermittency and variability). Supply restrictions and resource taxes can now induce backfire, but only if renewable resources are good substitutes for fossil resources and are available in sufficient quantities.

The intuition for how a renewable energy policy backfires is similar to that for an efficiency policy.[30] As in that case, a marginal productivity channel favors backfire while a energy substitution channel works against backfire, and the energy substitution channel weakens as the elasticity of substitution between energy services and final-good labor increases. The primary difference is the source of the marginal productivity channel. First, increasing renewable energy raises demand for energy service machines, which raises the return to efficiency engineering. Any resulting increase in engineering then acts like an efficiency policy in raising the expected return to exploration for fossil resources. Second, increasing the quantity of renewable energy also increases the marginal product of fossil energy via the positive cross-partial of $e(F_t, R_t)$: as renewable energy production increases, fossil resources become more valuable as a means of offsetting renewable resources' intermittency and variability. Absent some form of cheap energy storage, the potential for backfire increases as renewable resources grow in market share and become more challenging to integrate into the energy system.

The most surprising result is that introducing a renewable energy sector means that supply restrictions and resource taxes can backfire, but only if renewable resources provide substantial energy without having their value reduced by their intermittency and variability. The reason why supply restrictions and resource taxes can now backfire is that labor is endogenously redirected towards both types of engineering. We therefore begin by providing intuition for engineers' response to these policies. Figure 3 plots the equilibrium labor demand w_t^* (dotted lines) and the equilibrium return to renewable engineering Π_t^{R*} (dashed lines) against the number of renewable engineers r_t. The top panel depicts a case in which the intermittency and variability of renewable resources make the equilibrium return to renewable engineering decrease in the level of renewable engineering. As seen in previous sections, a supply restriction or resource tax raises the effective cost of fossil resource extraction and so tends to lower equilibrium exploration for fossil resources. This reduction

[30] In this section's setting, an efficiency policy always backfires if the marginal energetic product of renewable resources is constant in the quantity of renewable engineering. In this case, there are no diminishing returns to renewable engineering that can offset the complementarities between efficiency technology and renewable technology. These complementarities make the efficiency policy induce more renewable engineering, which induces more efficiency engineering. The additional efficiency engineering amplifies the policy's marginal productivity effect in fossil resource extraction.

in fossil resources reduces the marginal product of renewable resources, via both increased costs of intermittency and a reduction in the reward to efficiency engineering. The dashed line therefore shifts downward. In addition, as discussed in previous settings, the reduction in energy services from reduced extraction reduces demand for final-good labor, which shifts the dotted line out. Both changes tend to reduce the wage and raise the price of energy services, but the profit channel works to reduce the number of renewable engineers whereas the energy substitution channel works to increase the number of renewable engineers. The energy substitution channel dominates if and only if σ is sufficiently small (increasing the slope of the dotted line) and $\partial^2 e(F_t, R_t)/\partial F_t \partial R_t$ is sufficiently small (reducing the shift in the profit curve).

The bottom panel depicts a case in which energy storage is sufficiently cheap (or renewable resources are exploited at a sufficiently small scale) that intermittency and variability do not severely diminish the marginal energetic product of renewable resources as the level of renewable generation increases. Now the dashed equilibrium profit curve slopes upward due to complementarities with efficiency engineering: adding renewable engineers raises the return to efficiency engineering, and adding efficiency engineers raises the return to renewable engineering via the same marginal productivity channel as for fossil resources. Further, when the cross-partial of $e(\cdot,\cdot)$ is small, the shift in the labor demand curve tends to dominate the shift in the equilibrium profit curve. A supply restriction or resource tax thus increases the number of renewable engineers, increases the wage, and, by the final-good firm's zero-profit condition, decreases the price of energy services.[31] In the setting without renewable engineering, the equilibrium increase in efficiency engineers was always too small to offset the equilibrium decrease in fossil extraction, so a supply restriction or resource tax always increased the price of energy services. However, now the presence of the renewable energy sector and its complementarities with efficiency engineering mean that the combined increases in all types of engineering can actually increase the availability of energy services.

With these results in hand, we now develop intuition for how a supply restriction or a resource tax can backfire. Either policy directly reduces the return to extraction and so tends to reduce the quantity of extraction. In settings without renewable resources, the equilibrium price of energy services thus rises. However, this is not the end of the story when there are renewable resources. If these renewable resources are not overly hampered by intermittency and variability, the increase in Q_{t-1} or τ can induce so much additional efficiency engineering and renewable engineering that production of energy services actually increases and their price falls. The profit-maximizing final-good firm increases its demand for final-good labor, but the higher profits in the engineering sectors mean that it may nonetheless employ fewer workers at this higher wage than it would have in the absence of the policy. Some workers who would have been wage-laborers instead choose to work as engineers (raising the return

[31]The proof of Proposition 3 shows that, unlike in settings without a renewable energy sector, both a supply restriction and a resource tax can decrease the equilibrium price of energy services. It also shows that these policies tend to increase renewable engineering if the cross-partial of $e(\cdot,\cdot)$ is small and if σ is small.

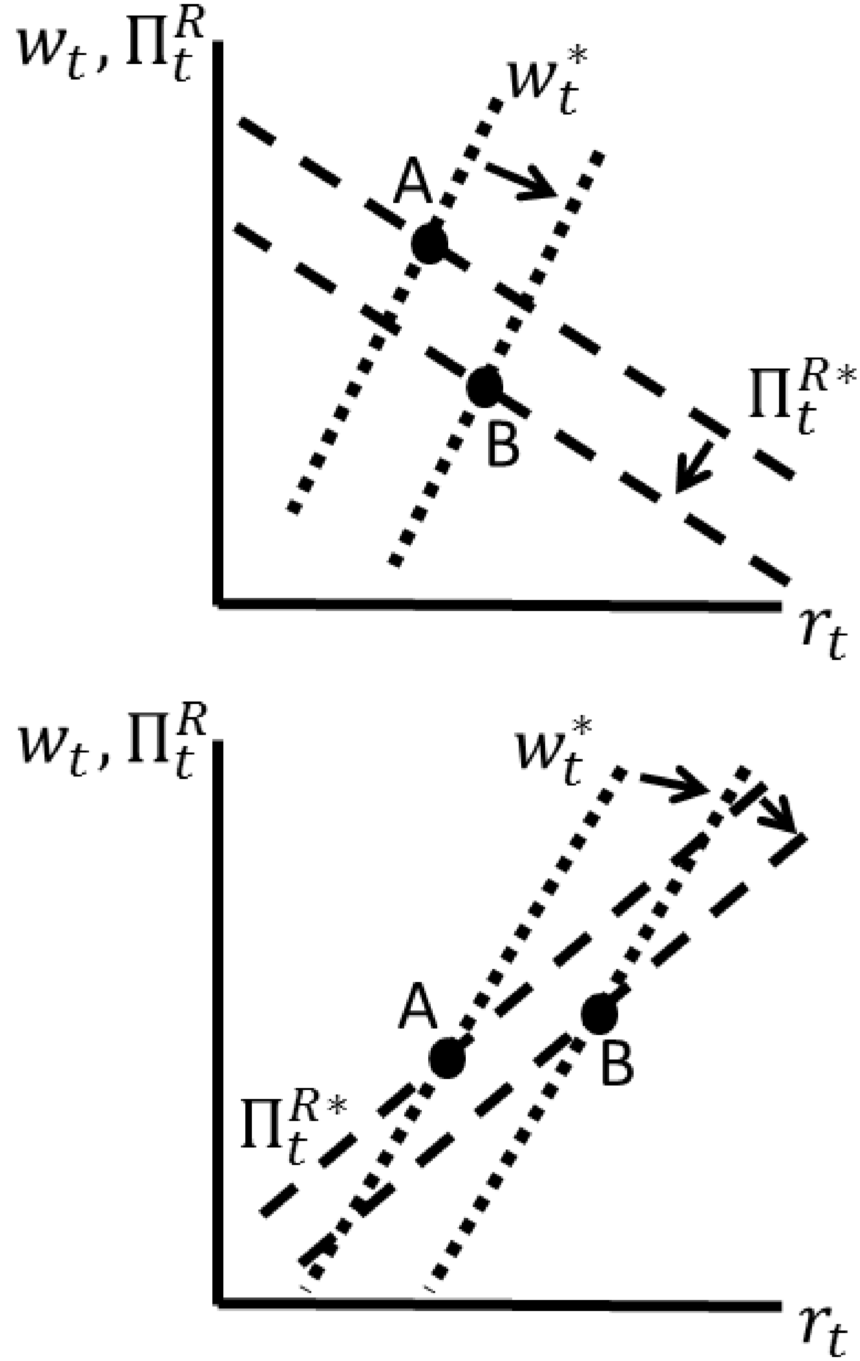


Figure 3: A supply restriction or resource tax shifts the equilibrium wage and number of renewable engineers from point A to point B. The top panel depicts a case in which the marginal energetic product of renewable resources decreases strongly in the quantity of renewable resources, for instance through problems posed by their intermittency and variability. The bottom panel depicts a case in which the marginal energetic product decreases less strongly, whether due to cheaper storage, greater reliability, or a lower quantity of generation.

to resource extraction), and some choose to work as geologists. The larger is the elasticity of substitution, the larger the reduction in final-good labor. For a sufficiently large elasticity of substitution, the increase in geologists from the displacement of final-good labor more than offsets the direct reduction in geologists from the supply restriction or resource tax.

Summing up, backfire from supply restrictions and resource taxes requires five ingredients. First, it requires complementarities between efficiency engineering and renewable engineering. These generate the upward-sloping profit curve in Figure 3 and allow a decrease in extraction to potentially increase overall energy service production.[32] Second, it requires that renewable resources are not overly hampered by problems of intermittency and variability; otherwise, their response to a reduction in fossil resource extraction is dampened or even reversed. Third, it requires that renewable resources be available at sufficient scale (i.e., that Z_{t-1} be sufficiently large) that they can compensate for a reduction in fossil extraction and provide strong complementarities for efficiency engineering. Fourth, it requires a sufficiently high elasticity of substitution between the energy and non-energy inputs to final-good production. If the elasticity of substitution is small, then the final-good firm is willing to pay a higher wage. Finally, it requires that workers shift freely between fossil resource extraction and final-good labor in response to relative rewards. Through these income-maximizing choices, the final-good firm's shift towards energy inputs redirects labor towards the energy sectors.

7 An Example

We now numerically solve the model in order to gain additional understanding of the potential for backfire at low elasticities of substitution and of the dynamic implications of each type of policy. Each timestep represents 20 years. In the final-good production function, the weight $\kappa = 0.7$ on energy services follows from Ayres and Warr (2005) finding that the marginal productivity of work from commercial energy has been around 0.7 in the U.S. over the last 100 years. For elasticities of substitution around unity, that marginal product translates directly into the parameter κ. In the energy service production function, we derive $\alpha = 0.85$ (the capital share, as opposed to the fuel share) from the weight on fossil resources in the calibrated dirty-good production function in Acemoglu et al. (2012), where we map their setting with a labor input into the present setting. The starting average efficiency of $A_0 = 0.2$ follows from the average conversion efficiency (across energy types) of around 20% in the year 2000 (Ayres et al., 2003).

In the extraction profit function, we must calibrate both the extraction cost function $\Psi(\cdot)$ and the expected discovery size ω. Rogner et al. (2012) estimate a supply function for

[32]Analysis similar to proofs in the appendix readily shows that a supply restriction and a resource tax cannot backfire in a setting that includes renewable energy but exogenously fixes the number of efficiency engineers.

the remaining world oil resource in 2005. An exponential function fits this projection well, leading to the following specification:

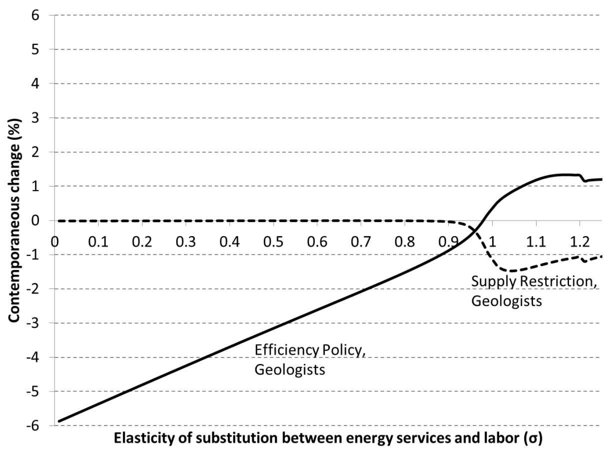
$$\Psi(k) = \psi_0 \exp\left[\psi_1 k\right].$$

Normalizing cumulative extraction so that it equals zero in 2005, a regression against the supply function produces $\psi_0 = 1.4$ and $\psi_1 = 0.00019$, where costs are measured in dollars per GJ and cumulative extraction is measured in EJ. Rogner et al. (2012) also state that global production of conventional oil was 166.68 EJ in 2009. We calibrate the model so that, in a baseline setting without engineering or a policy intervention, this annual production is matched in our initial timestep. Assuming this production is obtained using 1% of the workforce (in line with Bureau of Labor Statistics estimates for U.S. employment in mining, quarrying, and oil and gas extraction), we therefore have $\omega = 20 * 166.68 * 100$.

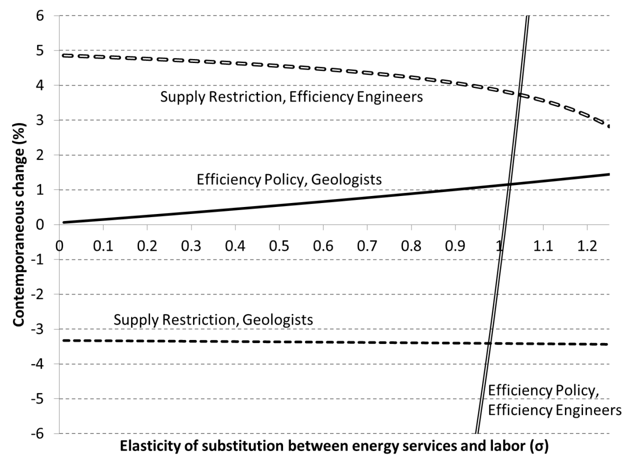
The remaining free parameters are the cost of energy service machines ϕ_E, the probability of an efficiency engineer successfully innovating η_E, and the quality increase γ_E from successful innovation. In the setting without efficiency engineering, we calibrate ϕ_E so that equilibrium geologists g_t^* are equal to 0.01 around $\sigma = 0.5$. This restriction implies $\phi_E = 0.01$. In the setting with efficiency engineering, we calibrate the three free parameters so that the equilibrium improvement in efficiency is 13% around $\sigma = 0.5$, which is in the ballpark of improvements in several sectors over the previous 20-year period (Ayres et al., 2003). This restriction yields $\eta_E = 0.5$, $\gamma_E = 2$, and $\phi_E = 3$, with g_t^* and ℓ_t^* both around 0.13.

Figure 4 plots the contemporaneous effects of a policy that increases the average efficiency of energy service technologies by 50% and of a supply restriction that blocks off fossil resources equivalent to half of the initial timestep's calibrated extraction (i.e., equal to $0.5*20*166.68$). The left panel plots the percentage change in geologists in the setting without endogenous engineering, and the right panel plots the percentage changes in geologists and efficiency engineers in the setting with endogenous engineering.

Begin by considering the setting without endogenous engineering (left panel). As predicted by the analytic setting, the efficiency policy backfires for elasticities above some critical value that is less than unity. However, the critical value in this calibration is very close to unity (around 0.98). Further, advancing the resource's depletion by even the equivalent of ten timesteps (not pictured) only lowers the critical value to around 0.90. The supply restriction indeed succeeds in reducing fossil resource extraction for all simulated elasticities of substitution, but its effect is quite small for elasticities sufficiently far below unity. In fact, the supply restriction begins having a stronger effect at almost precisely the same elasticity at which the efficiency policy begins to backfire. The efficiency policy fails to backfire when low elasticities of substitution prevent final-good firms from shifting strongly towards the relatively more abundant energy service input, and the supply restriction fails to have a strong effect when low elasticities of substitution prevent final-good firms from shifting strongly towards the relatively more abundant non-energy input. The elasticity at which the supply restriction's direct effect on extraction costs begins strongly dominating the final-



(a) No engineering



(b) With engineering

Figure 4: The effects of the time 0 efficiency policy and the time 0 supply restriction on equilibrium extraction and engineering in time 0.

good firms' substitution channels is approximately the same as the elasticity at which the efficiency policy's marginal productivity channel begins to outweigh the final-good firms' substitution channels. The supply restriction is most effective at reducing extraction in the same states of the world in which the efficiency policy tends to backfire.

Endogenizing energy efficiency innovation qualitatively changes the results (right panel). In particular, the efficiency policy now backfires even for elasticities of substitution as small as 0.01. For elasticities of substitution less than 1.01, the efficiency policy crowds out private sector innovation, but it induces further innovation for larger elasticities.[33] When private sector innovation is strongly crowded out at low elasticities of substitution, energy services do not become as relatively abundant as they would otherwise and final-good firms do not decrease their willingness to pay for energy services by as much. The marginal productivity channel can thereby dominate the energy substitution channel even at very low elasticities of substitution.

Endogenizing innovation not only makes the efficiency policy look like a less attractive tool to reduce resource extraction, but it also makes the supply restriction more attractive. We see that the supply restriction's effect on extraction is stronger than its effect in the setting without endogenous engineering and is also now insensitive to the elasticity of substitution. The reason for this change is its ability to induce additional efficiency engineering. The additional efficiency engineers work to offset the decrease in resource extraction so that

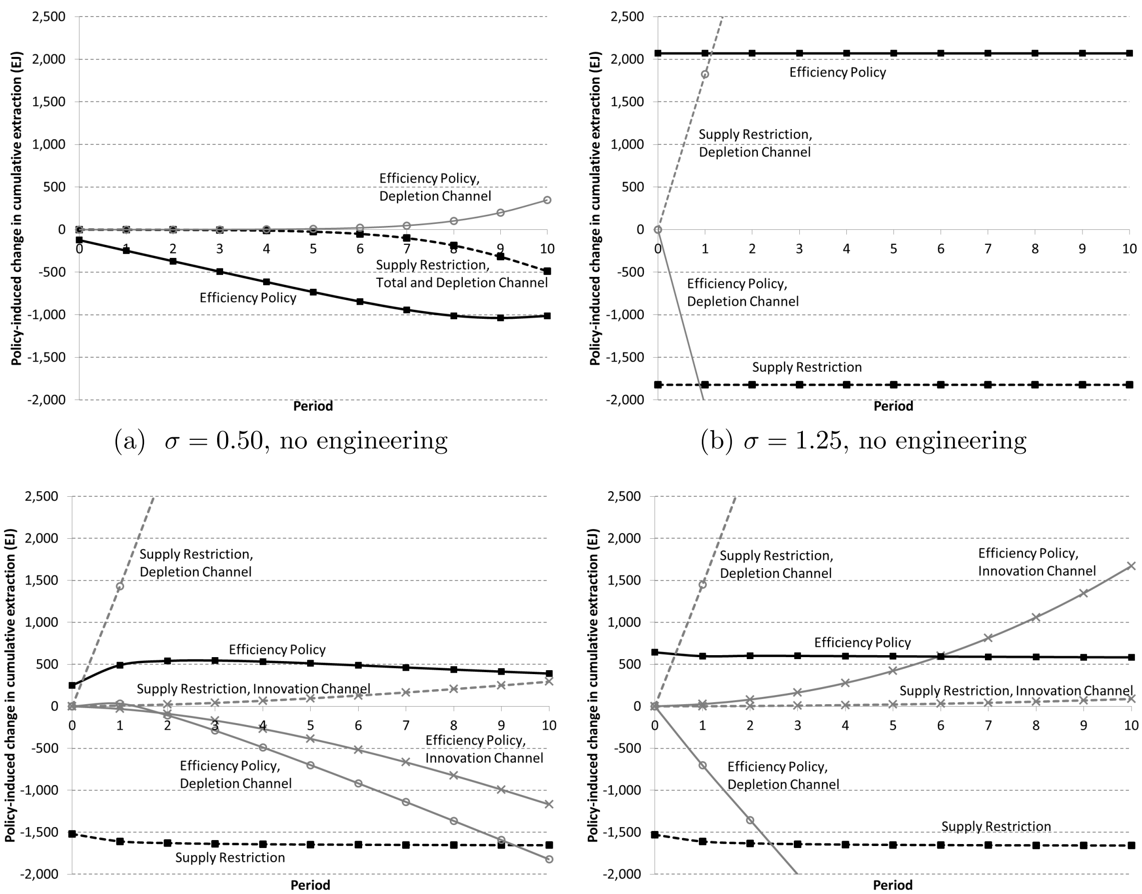
[33]The plot cuts off the most extreme values. At elasticities near zero, the efficiency policy reduces efficiency innovation by as much as 39%, and at elasticities near 1.25, it increases efficiency innovation by as much as 42%.

final-good firms do not see as large a decrease in the provision of energy services. Their willingness to pay for energy services therefore does not increase by as much as it would have in the absence of engineering, mitigating the price increase that otherwise opposes the supply restriction's effect on extraction costs.

In cases such as climate change, damages arise not from the flow of pollution but from the accumulated stock of pollution. The environmental implications of each type of policy then depend on how it affects extraction over time. Figure 5 plots the change in the path of cumulative resource extraction in response to a time 0 policy (black lines, with squares). The top panels plot cases without engineering, and the bottom panels plot cases with endogenous engineering. The left panels plot cases with $\sigma = 0.50$, and the right panels plot cases with $\sigma = 1.25$. The gray lines disentangle the depletion (circles) and innovation (crosses) channels. To calculate the depletion (innovation) channel, we solve the model while holding the quality of efficiency technology (level of depletion) constant between periods, regardless of the level of innovation (extraction). The desired channel is the difference between the resulting path of cumulative extraction and a case in which the static effect is replicated in each period. These channels include how the policy interacts with business-as-usual depletion (innovation) and also how they change the depletion (innovation) trajectory.

In the setting without efficiency engineering, both policies tend to reduce extraction more strongly as time passes when $\sigma = 0.50$ but tend not to have much effect on future extraction when $\sigma = 1.25$. In the latter case, the induced changes in the depletion pathway almost exactly offset the static effects of each policy. In contrast, with $\sigma = 0.50$, the efficiency policy reduces contemporaneous extraction by 124 EJ, but it reduces cumulative extraction by over 1,000 EJ over the first seven periods (140 years) by approximately replicating its static effect in each individual period. However, beyond the seventh period, the decrease in cumulative extraction has a sufficiently large effect on future extraction costs (reflected by the increased slope of the gray line) that per-period extraction eventually begins increasing relative to a no-policy counterfactual (reflected by the inflection point in the solid black line). The supply restriction also grows stronger over time: it reduces contemporaneous extraction by only 0.3 EJ, but it avoids 500 EJ of extraction over the first ten periods. Whereas with $\sigma = 1.25$ the depletion channel opposes the static effect, with $\sigma = 0.50$ it not only reinforces the static effect but is so strong that it is almost identical to the total effect of the policy. The reason for the difference was discussed in Section 3: a small supply restriction tends to become larger over time through the convexity of the extraction cost function, but a large supply restriction tends to become smaller over time by successfully reducing near-term extraction of the fossil resource and so leaving the future a less depleted resource base. As seen in Figure 4, the supply restriction acts like a small policy for σ near 0.50 and acts like a large policy for σ near 1.25. This difference is reflected not only in the size of the effect on contemporaneous extraction but also in how that effect accumulates over time.

In the setting with endogenous efficiency engineering, the policies' cumulative effects on extraction over the first ten periods are similar to their static effects, but that outcome masks



(a) $\sigma = 0.50$, no engineering (b) $\sigma = 1.25$, no engineering

(c) $\sigma = 0.50$, with engineering (d) $\sigma = 1.25$, with engineering

Figure 5: The impulse response of cumulative extraction to the time 0 efficiency policy and the time 0 supply restriction.

more complicated dynamic forces. Absent the effects on depletion and innovation, the static effect would hold in every subsequent period, magnifying any initial change in cumulative extraction as time passes. The gray lines with circles show that the depletion channel always works against the policy's static effect. The supply restriction reduces extraction, which works to undercut the supply restriction in future periods by making future extraction cheaper. The efficiency policy backfires, but this time 0 increase in extraction works to mitigate cumulative backfire by making future extraction more expensive. The gray lines with crosses show that the innovation channel always undercuts the supply restriction: as we saw in Figure 4, the supply restriction induces innovation, which in turn induces extraction in future periods just as an efficiency policy would do. In contrast, an efficiency policy's innovation channel works to mitigate backfire for $\sigma = 0.50$ but works to amplify backfire for $\sigma = 1.25$. We saw in Figure 4 that the efficiency policy crowds out innovation for elasticities less than unity and induces additional private sector innovation for elasticities sufficiently greater than unity. Crowding out innovation works to undercut the policy in future periods, but inducing innovation works to amplify the policy in future periods. In the case with $\sigma = 0.50$, the combined effect of the innovation and depletion channels actually begins reducing extraction in the fourth period and beyond, though not by enough to offset the initial periods' increase in extraction within the first 10 periods.

8 Conclusion

We have seen that aggregate relationships are crucial for policies that aim to reduce fossil resource extraction. First, contrary to conventional wisdom, policies to improve the efficiency with which the economy converts energy resources into energy services can increase consumption of energy resources for elasticities of substitution well below unity. Improving efficiency means that each unit of a fossil resource generates more energy services, which increases the reward to extracting fossil resources. On the other hand, improving efficiency also makes energy services more abundant, which lowers their marginal product in final-good production. The first effect dominates for sufficiently large elasticities of substitution between energy services and non-energy inputs to final-good production. Further, the first effect becomes stronger as fossil resources become scarcer through depletion. And by interacting with the pathways of depletion and innovation, policies' effects on cumulative extraction can differ substantially from their effects on nearer-term extraction.

In contrast, supply restrictions and resource taxes never backfire in the absence of a developed renewable energy sector, and even then they backfire only under special circumstances. These types of policies will almost always succeed in reducing fossil resource extraction. If the fossil resource depletion function is convex, then a constant supply restriction will have a more strongly persistent effect than will a constant resource tax. Supply restrictions and resource taxes tend to most effectively reduce extraction in those states of the world in which efficiency policies backfire, and they tend to least effectively reduce extraction when efficiency policies would succeed in reducing extraction.

Whether a renewable energy policy increases or decreases fossil resource extraction hinges on the extent to which renewable resources are scalable. If the value of renewable resources is limited by the difficulty of managing their intermittency and variability, then a policy to promote their use can induce more fossil resource extraction. This perverse outcome occurs when the energy system uses fossil resources to offset that intermittency and variability, as the electric grid currently uses natural gas to offset the intermittency and variability of wind and solar resources. In that case, a policy to promote renewable energy increases the marginal energetic product of fossil resources and potentially increases the incentive to explore for fossil resources.

Much economics literature has focused on the potential for "rebound" in particular partial equilibrium settings. For instance, a refrigerator efficiency policy may induce consumers to choose larger refrigerators, or a light bulb efficiency policy may induce consumers to leave their lights on longer. However, I have shown that we must better understand aggregate relationships in order to properly evaluate large-scale policy proposals like those needed to address climate change and like those implemented in the American Recovery and Reinvestment Act. In particular, I have shown that the long-run elasticity of fossil resource supply and the elasticity of substitution between energy and non-energy inputs to production are key. Without accounting for these relationships, well-intentioned policies can end up exacerbating market failures by changing the composition of economic activity and relative prices, even when these policies would not appear to backfire through more commonly analyzed partial equilibrium channels.

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A Appendix: Formal Analysis

We begin by expressing equilibrium engineering profit and final-good wages as functions of prices. Equilibrium requires that all firms are maximizing profits, firms in competitive sectors are receiving zero profits, workers receive the same expected reward in each sector, and all available workers are employed. In this introductory analysis, we focus on the firms' side of equilibrium. A lemma then establishes several useful facts about the equilibrium wage and equilibrium price of energy services. The proofs connect the labor market to firms' decisions. Because all of the analysis relies on equilibrium relations, I save notation by omitting the asterisk signifying equilibrium outcomes.

First consider final-good producers. The final good is the numeraire, so they receive one unit of revenue for each unit sold. They hire labor at rate w_t and buy energy services at price p_t^E. The representative final-good firm solves

$$\max_{L_t,E_t}\left\{\left[(1-\kappa)L_t^{\frac{\sigma-1}{\sigma}}+\kappa E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-w_tL_t-p_t^EE_t\right\}.$$

The first-order conditions are

$$w_t=(1-\kappa)Y_t^{\frac{1}{\sigma}}L_t^{\frac{-1}{\sigma}},$$
$$p_t^E=\kappa Y_t^{\frac{1}{\sigma}}E_t^{\frac{-1}{\sigma}}.$$

Zero profits and a final-good price of 1 then imply:

$$w_t=\left(\frac{1-\kappa^\sigma[p_t^E]^{1-\sigma}}{(1-\kappa)^\sigma}\right)^{\frac{1}{1-\sigma}}\triangleq w(p_t^E). \tag{1}$$

Now consider producers of energy services. This sector is also competitive. Each firm rents machines of type i at price p_{it}^x and buys fossil and renewable energy at prices p_t^F and p_t^R, respectively. The representative energy services firm's profit-maximization problem is

$$\max_{F_t,R_t,\{x_{it}\}_{i=0}^1}\left\{p_t^E\left[e(F_t,R_t)\right]^{1-\alpha}\int_0^1 A_{it}^{1-\alpha}x_{it}^\alpha\,di-p_t^FF_t-p_t^RR_t-\int_0^1 p_{it}^x x_{it}\,di\right\}.$$

The first-order conditions for an interior solution are:

$$\begin{aligned}
p_t^F =& p_t^E(1-\alpha)\left[e(F_t,R_t)\right]^{-\alpha}\frac{\partial e(F_t,R_t)}{\partial F_t}\int_0^1 A_{it}^{1-\alpha}x_{it}^{\alpha}\,di\\
p_t^R =& p_t^E(1-\alpha)\left[e(F_t,R_t)\right]^{-\alpha}\frac{\partial e(F_t,R_t)}{\partial R_t}\int_0^1 A_{it}^{1-\alpha}x_{it}^{\alpha}\,di\\
p_{it}^x =& p_t^E\alpha\left[e(F_t,R_t)\right]^{1-\alpha}A_{it}^{1-\alpha}x_{it}^{\alpha-1},\quad i\in[0,1].
\end{aligned}$$

Demand for machines of type i is

$$x_{it}=\left[\frac{\alpha p_t^E}{p_{it}^x}\right]^{\frac{1}{1-\alpha}}e(F_t,R_t)\,A_{it}.$$

Demand is isoelastic. Applying the standard monopolist markup rule, each secondary energy machine producer maximizes profits by choosing $p_{it}^x=\phi_E/\alpha$. Equilibrium demand is therefore

$$x_{it}=\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}}e(F_t,R_t)\,A_{it},$$

and the equilibrium supply of energy services is

$$\begin{aligned}
E_t =& \left[e(F_t,R_t)\right]^{1-\alpha}\int_0^1 A_{it}^{1-\alpha}\left(\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}}e(F_t,R_t)\,A_{it}\right)^{\alpha}di\\
=& e(F_t,R_t)\,A_t\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}.
\end{aligned} \tag{2}$$

The owner of machine type i receives equilibrium profits of π_{it}^E:

$$\pi_{it}^E=\phi_E\frac{1-\alpha}{\alpha}\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}}e(F_t,R_t)\,A_{it}.$$

Failed innovation leaves an engineer with nothing, while successful innovation produces a one-period patent. The expected profit Π_t^E of an engineer working on energy service machines is

$$\Pi_t^E=\eta_E(1+\gamma_E)\phi_E\frac{1-\alpha}{\alpha}\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}}e(F_t,R_t)\,A_{t-1}. \tag{3}$$

Next consider the renewable energy sector. We know the equilibrium price of renewable energy from the energy service producer's first-order condition. The equilibrium profits from owning a renewable resource of type i are:

$$\begin{aligned}\pi_{it}^R =& p_t^R \, Z_{it}\\ =& Z_{it} \, \frac{\partial e(F_t, R_t)}{\partial R_t} \, [p_t^E]^{\frac{1}{1-\alpha}} \, A_t \, (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}},\end{aligned}$$

where we substitute in for the equilibrium price of renewable energy from the energy service producer's first-order condition and for equilibrium production of energy services from equation (2). The expected profit from renewable energy engineering is thus:

$$\Pi_t^R = \eta_R(1+\gamma_R) Z_{t-1} \, \frac{\partial e(F_t, R_t)}{\partial R_t} \, [p_t^E]^{\frac{1}{1-\alpha}} \, (1+\eta_E \gamma_E \ell_t) A_{t-1} \, (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}. \tag{4}$$

Finally, consider fossil resource extraction. The equilibrium profits from owning resource pool k of unit size are:

$$\begin{aligned}\pi_{kt}^F =& p_t^F - \Psi(k) - \tau\\ =& [p_t^E]^{\frac{1}{1-\alpha}} (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} A_t \frac{\partial e(F_t, R_t)}{\partial F_t} - \Psi(k) - \tau,\end{aligned}$$

where we substitute in for the equilibrium price of fossil energy from the energy service producer's first-order condition and for equilibrium production of energy services from equation (2). The expected profit from fossil resource exploration is thus:

$$\Pi_t^F = \underbrace{\omega [p_t^E]^{\frac{1}{1-\alpha}} (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} A_t \frac{\partial e(F_t, R_t)}{\partial F_t}}_{\Pi_t^{F,Rev}} - \underbrace{[\omega \psi(g_t; Q_{t-1}) + \omega \tau]}_{\Pi_t^{F,Cost}}, \tag{5}$$

where $\partial e(F_t, R_t)/\partial F_t = 1$ in settings without renewable resources.

A.1 A lemma about equilibrium wages and prices

Define Θ as the negative of the elasticity of the wage with respect to the price of energy services:

$$\Theta \triangleq -\frac{w'(p_t^E)}{w(p_t^E)} \, p_t^E = \frac{\kappa^\sigma}{[p_t^E]^{\sigma-1} - \kappa^\sigma},$$

where the rightmost expression uses the definition of wages in equation (1).

Lemma 4. *Assume that $L_t > 0$.*

1. $w'(p_t^E) < 0$

2. *$w_t \leq 1-\kappa$ if and only if $p_t^E \geq \kappa$*

3. *$p_t^E < \kappa^{\frac{\sigma}{\sigma-1}}$ if and only if $\sigma < 1$*

4. *$\partial w_t / \partial \sigma > 0$*

5. *$\partial \Theta / \partial \sigma < 0$ if and only if $p_t^E > \kappa$*

6. *$\partial^2 \Theta / \partial \sigma^2 > 0$*

Proof. Using the definition of wages in equation (1), we have

$$w'(p_t^E) = -\frac{\kappa^\sigma}{(1-\kappa)^\sigma}[p_t^E]^{-\sigma} w_t^\sigma < 0.$$

This establishes the first part of the lemma. It also implies $\Theta > 0$. If $w_t \leq 1-\kappa$, then via equation (1) we have

$$\left[1-\kappa \geq \left(\frac{1-\kappa^\sigma[p_t^E]^{1-\sigma}}{(1-\kappa)^\sigma}\right)^{\frac{1}{1-\sigma}}\right] \Leftrightarrow \left[1-\kappa \geq 1-\kappa^\sigma[p_t^E]^{1-\sigma}\right] \Leftrightarrow \left[\kappa \leq p_t^E\right].$$

This establishes the second part of the lemma.

$L_t > 0$ implies $w_t > 0$. Again using equation (1),

$$\begin{aligned} w_t > 0 \Leftrightarrow & 0 < \left(\frac{1-\kappa^\sigma[p_t^E]^{1-\sigma}}{(1-\kappa)^\sigma}\right)^{\frac{1}{1-\sigma}} \\ \Leftrightarrow & 0 < \frac{1-\kappa^\sigma[p_t^E]^{1-\sigma}}{(1-\kappa)^\sigma} \\ \Leftrightarrow & \kappa^\sigma[p_t^E]^{1-\sigma} < 1 \\ \Leftrightarrow & 0 > \sigma\ln(\kappa) + (1-\sigma)\ln(p) \\ \Leftrightarrow & p < \kappa^{\frac{\sigma}{\sigma-1}} \text{ if and only if } \sigma < 1. \end{aligned}$$

This establishes the third part of the lemma.

Using equation (1), the derivative of wages with respect to the elasticity of substitution is

$$\frac{\partial w(p_t^E)}{\partial \sigma} = \frac{w(p_t^E)}{1-\sigma}\left[\ln\left(\frac{w(p_t^E)}{1-\kappa}\right) - \Theta\ln\left(\frac{\kappa}{p_t^E}\right)\right] \tag{6}$$

$$= \frac{w(p_t^E)}{1-\sigma}\underbrace{\left[\frac{1}{1-\sigma}\ln\left(\frac{1-\kappa^\sigma[p_t^E]^{1-\sigma}}{1-\kappa}\right) - \Theta\ln\left(\frac{\kappa}{p_t^E}\right)\right]}_{\chi(\sigma)}. \tag{7}$$

The sign of the left-hand term in brackets conflicts with the sign of the right-hand term in brackets. A zero of $\chi(\sigma)$ occurs at $\sigma = 1$:

$$\begin{aligned}\lim_{\sigma\to 1}\chi(\sigma) =&\frac{0}{0} - \frac{\kappa}{1-\kappa}\ln\left(\frac{\kappa}{p_t^E}\right)\\ =&\lim_{\sigma\to 1}\left\{\frac{\kappa^\sigma\,[p_t^E]^{1-\sigma}}{1-\kappa^\sigma[p_t^E]^{1-\sigma}}\ \ln\left(\frac{\kappa}{p_t^E}\right)\right\} - \frac{\kappa}{1-\kappa}\ln\left(\frac{\kappa}{p_t^E}\right)\\ =&0,\end{aligned}$$

where the second line follows by L'Hôpital's Rule. Differentiating yields

$$\chi'(\sigma) = \frac{\chi(\sigma)}{1-\sigma} - \left[\ln\left(\kappa/p_t^E\right)\right]^2 \frac{\kappa^\sigma[p_t^E]^{1-\sigma}}{\left[1-\kappa^\sigma[p_t^E]^{1-\sigma}\right]^2}. \tag{8}$$

It is easy to show, using L'Hôpital's Rule, that $\lim_{\sigma\to 1}\chi'(\sigma) < 0$. It is also readily apparent that $\chi'(\sigma) < 0$ for any other σ that is a zero of $\chi(\sigma)$. Because $\chi(\sigma)$ is decreasing at all of its zeros, it must have only one zero, which we know is at $\sigma = 1$.

$\chi(\sigma)$ is strictly positive for all $\sigma < 1$, which, by equation (7), means $\partial w_t/\partial\sigma$ is strictly positive at all $\sigma < 1$. $\chi(\sigma)$ is strictly negative for all $\sigma > 1$, which, by equation (7), means $\partial w_t/\partial\sigma$ is strictly positive at all $\sigma > 1$. Therefore $\partial w_t/\partial\sigma > 0$ for all allowed σ. This establishes the fourth part of the lemma.

Differentiate the definition of Θ, using the expression for w_t in equation (1):

$$\begin{aligned}\frac{\partial\Theta}{\partial\sigma} =&\frac{[p_t^E]^{\sigma-1}\kappa^\sigma\ln\left(\frac{\kappa}{p_t^E}\right)}{\left([p_t^E]^{\sigma-1}-\kappa^\sigma\right)^2} = \Theta\ [1+\Theta]\ \ln\left(\frac{\kappa}{p_t^E}\right)\\ <&0 \text{ iff } p_t^E > \kappa.\end{aligned}$$

This establishes the fifth part of the lemma. The second derivative simplifies to

$$\begin{aligned}\frac{\partial^2\Theta}{\partial\sigma^2} =&\frac{[p_t^E]^{\sigma-1}\kappa^\sigma\left(\kappa^\sigma+[p_t^E]^{\sigma-1}\right)}{\left([p_t^E]^{\sigma-1}-\kappa^\sigma\right)^3}\left[\ln\left(\frac{\kappa}{p_t^E}\right)\right]^2 = [1+2\,\Theta]\ \frac{\partial\Theta}{\partial\sigma}\ \ln\left(\frac{\kappa}{p_t^E}\right)\\ >&0.\end{aligned}$$

This establishes the sixth part of the lemma.

□

A.2 Proof of Proposition 1

This proposition fixes $\ell_t = 0$ and $R_t = 0$, so that workers are employed in either final-good production or fossil resource exploration. We will impose market-clearing in the fossil resource, energy service, and labor markets. By Walras' Law, the market for final goods must also clear. The approach is to reduce all equilibrium relations into expressions where the price of energy services p_t^E and the number of geologists g_t are the only endogenous variables and then use the implicit function theorem to consider how equilibrium extraction changes in the parameters controlled by each type of policy.

Begin by solving for the equilibrium demand for energy services. From the final-good good firm's first-order conditions, we have:

$$E_t = \left[\frac{\kappa}{1-\kappa}\frac{w_t}{p_t^E}\right]^{\sigma} L_t.$$

Recall that equation (1) expresses w_t in terms of p_t^E, via the final-good firm's zero-profit condition and a normalized final-good price of unity. Going forward, we write $w(p_t^E)$ from equation (1). Substitute into energy service demand:

$$E_t = \left[\frac{\kappa}{1-\kappa}\right]^{\sigma} [p_t^E]^{-\sigma}\,[w(p_t^E)]^{\sigma}\,L_t. \tag{9}$$

Equating energy service demand and supply (from equation (2)) yields equilibrium demand for final-good labor:

$$L_t = A_t\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} e\left(F_t, R_t\right)\left[\frac{1-\kappa}{\kappa}\right]^{\sigma} [p_t^E]^{\sigma}\,[w(p_t^E)]^{-\sigma}. \tag{10}$$

There are potentially five endogenous variables in this equation: L_t, p_t^E, g_t (via F_t), ℓ_t (via A_t), and r_t (via R_t). In a setting without endogenous engineering or renewable resources, we lose ℓ_t and r_t: we have $A_t = A_{t-1}$ and $e(F_t, R_t) = F_t$. We therefore need two additional relations to close the model. These are the labor constraint ($1 = L_t + g_t$) and equality between wages and the expected profit from fossil resource exploration ($w_t = \Pi_t^F$).

Substituting demand for final-good labor $L(p_t^E, g_t)$ from equation (10) into the labor constraint, using the expression for expected profit from fossil resource exploration from equation (5), and recognizing that $F_t = \omega\, g_t$, we have the following system of equations in p_t^E and g_t:

$$\begin{aligned}
1 =& g_t + \omega g_t A_{t-1}\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}\left[\frac{1-\kappa}{\kappa}\right]^{\sigma}[w(p_t^E)]^{-\sigma}[p_t^E]^{\sigma+\frac{\alpha}{1-\alpha}} \\
&\triangleq G_1(p_t^E, g_t), \\
1 =& \omega(1-\alpha)\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} A_{t-1}[p_t^E]^{\frac{1}{1-\alpha}}\,[w(p_t^E)]^{-1} - \omega\,\left[\psi(g_t; Q_{t-1}) + \tau\right]\,[w(p_t^E)]^{-1} \\
&\triangleq G_2(p_t^E, g_t).
\end{aligned}$$

The partial derivatives of each equation with respect to the endogenous variables are

$$\begin{aligned}
\frac{\partial G_1}{\partial p_t^E} =& \frac{\alpha}{1-\alpha}[p_t^E]^{-1} L_t + \sigma\, L_t \left[[p_t^E]^{-1} - [w(p_t^E)]^{-1}\, w'(p_t^E)\right] && > 0,\\
\frac{\partial G_1}{\partial g_t} =& 1 + g_t^{-1} L_t && > 0,\\
\frac{\partial G_2}{\partial p_t^E} =& \frac{1}{1-\alpha}[p_t^E]^{-1}\Pi_t^{F,Rev}\,[w(p_t^E)]^{-1} - [w(p_t^E)]^{-2}\, w'(p_t^E)\,\Pi_t^F && > 0,\\
\frac{\partial G_2}{\partial g_t} =& -\omega \frac{\partial \psi(g_t; Q_{t-1})}{\partial g_t}\,[w(p_t^E)]^{-1} && < 0.
\end{aligned}$$

Define the matrix

$$G \triangleq \begin{bmatrix} \frac{\partial G_1}{\partial p_t^E} & \frac{\partial G_1}{\partial g_t} \\ \frac{\partial G_2}{\partial p_t^E} & \frac{\partial G_2}{\partial g_t} \end{bmatrix}.$$

$det(G)$ is < 0, where det refers to the determinant. The partial derivatives of each equation with respect to the policy parameters are

$$\begin{aligned}
\frac{\partial G_1}{\partial A_{t-1}} =& A_{t-1}^{-1} L_t && > 0,\\
\frac{\partial G_2}{\partial A_{t-1}} =& A_{t-1}^{-1}\Pi_t^{F,Rev}[w(p_t^E)]^{-1} && > 0,\\
\frac{\partial G_1}{\partial Q_{t-1}} =& 0 && = 0,\\
\frac{\partial G_2}{\partial Q_{t-1}} =& -\omega \frac{\partial \psi(g_t; Q_{t-1})}{\partial Q_{t-1}}\,[w(p_t^E)]^{-1} && < 0,\\
\frac{\partial G_1}{\partial \tau} =& 0 && = 0,\\
\frac{\partial G_2}{\partial \tau} =& -\omega\,[w(p_t^E)]^{-1} && < 0.
\end{aligned}$$

Consider the effect of a marginal improvement in incoming energy services technology.

Using the implicit function theorem, we have:

$$\frac{\partial p_t^E}{\partial A_{t-1}} = -\frac{det\left(\begin{bmatrix}\frac{\partial G_1}{\partial A_{t-1}} & \frac{\partial G_1}{\partial g_t}\\ \frac{\partial G_2}{\partial A_{t-1}} & \frac{\partial G_2}{\partial g_t}\end{bmatrix}\right)}{det(G)} < 0,$$

$$\begin{aligned}\frac{\partial g_t}{\partial A_{t-1}} &= -\frac{det\left(\begin{bmatrix}\frac{\partial G_1}{\partial p_t^E} & \frac{\partial G_1}{\partial A_{t-1}}\\ \frac{\partial G_2}{\partial p_t^E} & \frac{\partial G_2}{\partial A_{t-1}}\end{bmatrix}\right)}{det(G)}\\
&\propto \left\{\frac{\alpha}{1-\alpha}[p_t^E]^{-1}L_t + \sigma\, L_t\left[[p_t^E]^{-1} - [w(p_t^E)]^{-1}\, w'(p_t^E)\right]\right\}\left\{A_{t-1}^{-1}\Pi_t^{F,Rev}[w(p_t^E)]^{-1}\right\}\\
&\quad - \left\{A_{t-1}^{-1}L_t\right\}\left\{\frac{1}{1-\alpha}[p_t^E]^{-1}\Pi_t^{F,Rev}\,[w(p_t^E)]^{-1} - [w(p_t^E)]^{-2}\, w'(p_t^E)\,\Pi_t^F\right\}\\
&\propto \Pi_t^{F,Rev}\,(\sigma-1)\,[p_t^E]^{-1} + [w(p_t^E)]^{-1}\, w'(p_t^E)\,\left[\Pi_t^F - \Pi_t^{F,Rev}\,\sigma\right]\\
&\propto \underbrace{\Pi_t^{F,Rev}\,(\sigma-1)}_{B} + \underbrace{\Theta\,[p_t^E]^{-1}\left[\Pi_t^{F,Rev}\,\sigma - \Pi_t^F\right]}_{C}. \qquad (11)\end{aligned}$$

$B > 0$ if and only if $\sigma > 1$. $C > 0$ if $\sigma \geq 1$. Therefore $\partial g_t/\partial A_{t-1} > 0$ for all $\sigma > 1$. As $\sigma \to 0$, we have $\partial g_t/\partial A_{t-1} < 0$ by the requirement that $\Pi_t^{F,Rev} > \Pi_t^F$ in equilibrium. Some $\sigma > 0$ must be a root of equation (11). At such a root, the term $\left[\Pi_t^{F,Rev}\,\sigma - \Pi_t^F\right]$ must be positive.

Differentiating equation (11) yields:

$$[1+\Theta]\;\Pi_t^{F,Rev} + \frac{\partial\Theta}{\partial\sigma}\left[\Pi_t^{F,Rev}\sigma - \Pi_t^F\right]. \qquad (12)$$

The left-hand term is positive. We know the right-hand term in brackets is positive at any σ that is a root of equation (11). If $\partial\Theta/\partial\sigma > 0$, then equation (12) is positive around any σ that is a root of equation (11). The σ that is a root of equation (11) is therefore unique if $\partial\Theta/\partial\sigma > 0$.

Now assume that $\partial\Theta/\partial\sigma < 0$ and that there is more than one σ that is a root of equation (11). Because $\partial g_t/\partial A_{t-1}$ is continuous in σ for $\sigma \in (0,1)$ and switches sign between the interval's endpoints, there must be at least three roots. From equation (11), the following must hold at a root:

$$\left[\Pi_t^{F,Rev}\,\sigma - \Pi_t^F\right] = \Pi_t^{F,Rev}\,(1-\sigma)\,p_t^E\,\Theta^{-1}.$$

Equation (12) must be negative at the second-largest root and positive at the roots on either side of it. It is positive at a root if and only if:

$$\begin{aligned}
&[1+\Theta]\ \Pi_t^{F,Rev} > -\frac{\partial \Theta}{\partial \sigma}\left[\Pi_t^{F,Rev}\sigma - \Pi_t^F\right] = -\frac{\partial \Theta}{\partial \sigma}\Pi_t^{F,Rev}\left(1-\sigma\right)p_t^E\,\Theta^{-1} \\
\Leftrightarrow &[1+\Theta]\ \Pi_t^{F,Rev} > -\Theta[1+\Theta]\ln\left(\frac{\kappa}{p_t^E}\right)\Pi_t^{F,Rev}\left(1-\sigma\right)p_t^E\,\Theta^{-1} \\
\Leftrightarrow &1 > -\ln\left(\frac{\kappa}{p_t^E}\right)\left(1-\sigma\right)p_t^E,
\end{aligned} \tag{13}$$

where the first line's equality substitutes the condition that must hold at a root and where the second line follows from the expressions for Θ and $\partial\Theta/\partial\sigma$. The right-hand side is monotonically decreasing in σ. If this inequality does hold at some root, then it cannot hold at any smaller root. The inequality must hold at the largest root because $\partial g_t/\partial A_{t-1} > 0$ for σ sufficiently large. Therefore equation (12) is strictly positive at all roots. The σ that is a root of equation (11) is unique if $\partial\Theta/\partial\sigma < 0$.

Finally, note that increasing Q_{t-1} decreases Π_t^F and that $\partial g_t/\partial A_{t-1}$ decreases in Π_t^F. Therefore increasing Q_{t-1} increases $\partial g_t/\partial A_{t-1}$.

Combining these results, we have that there exists $\hat{\sigma}(Q_{t-1}) \in (0,1)$ such that $\partial g_t/\partial A_{t-1} > 0$ if and only if $\sigma > \hat{\sigma}(Q_{t-1})$ and such that $\hat{\sigma}'(Q_{t-1}) < 0$.

Now consider the effects of a marginal increase in depletion and of a marginal increase in the resource tax.

$$\frac{\partial p_t^E}{\partial Q_{t-1}} = \frac{\partial p_t^E}{\partial \tau}\,\frac{\partial \psi(g_t;Q_{t-1})}{\partial Q_{t-1}} = -\frac{det\left(\begin{bmatrix}\frac{\partial G_1}{\partial Q_{t-1}} & \frac{\partial G_1}{\partial g_t}\\ \frac{\partial G_2}{\partial Q_{t-1}} & \frac{\partial G_2}{\partial g_t}\end{bmatrix}\right)}{det(G)} > 0,$$

$$\frac{\partial g_t}{\partial Q_{t-1}} = \frac{\partial g_t}{\partial \tau}\,\frac{\partial \psi(g_t;Q_{t-1})}{\partial Q_{t-1}} = -\frac{det\left(\begin{bmatrix}\frac{\partial G_1}{\partial p_t^E} & \frac{\partial G_1}{\partial Q_{t-1}}\\ \frac{\partial G_2}{\partial p_t^E} & \frac{\partial G_2}{\partial Q_{t-1}}\end{bmatrix}\right)}{det(G)} < 0.$$

This establishes the part of the proposition concerning a time t supply restriction and a time t resource tax.

The analysis of persistence involves second derivatives. Differentiating the implicit function theorem expression for $\partial g_t/\partial A_{t-1}$ with respect to A_{t-1}, we find that

$$\begin{aligned}
\frac{\partial^2 g_t}{\partial A_{t-1}^2} \propto &- A_{t-1}^{-2}\,L_t\,[w(p_t^E)]^{-2}\,w'(p_t^E)\,\Pi_t^{F,Cost}\,det(G) \\
&- A_{t-1}^{-1}\Bigg\{\left[2\frac{\partial G_2}{\partial p_t^E} - [w(p_t^E)]^{-2}\,w'(p_t^E)\,\Pi_t^{F,Cost}\right]g_t^{-1}L_t \\
&\qquad + \omega\frac{\partial \psi(g_t;Q_{t-1})}{\partial Q_{t-1}}[w(p_t^E)]^{-1}\frac{\partial L(p_t^E,g_t)}{\partial p_t^E}\Bigg\}\ det\left(\begin{bmatrix}\frac{\partial G_1}{\partial p_t^E} & \frac{\partial G_1}{\partial A_{t-1}}\\ \frac{\partial G_2}{\partial p_t^E} & \frac{\partial G_2}{\partial A_{t-1}}\end{bmatrix}\right).
\end{aligned}$$

The top line is negative. The bottom two lines are negative if and only if $\partial g_t/\partial A_{t-1} > 0$. Therefore, the whole expression is negative if $\partial g_t/\partial A_{t-1} > 0$. Similarly, we have:

$$\begin{aligned}\frac{\partial^2 g_t}{\partial A_{t-1}\partial Q_{t-1}} \propto & A_{t-1}^{-1}\, L_t\, [w(p_t^E)]^{-2}\, w'(p_t^E)\,\omega\, \frac{\partial\psi(g_t;Q_{t-1})}{\partial Q_{t-1}}\, det(G) \\ & - \left\{[w(p_t^E)]^{-2}\, w'(p_t^E)\,\omega\, \frac{\partial\psi(g_t;Q_{t-1})}{\partial Q_{t-1}}\left[1+g_t^{-1}\,L_t\right] + \omega\frac{\partial^2\psi(g_t;Q_{t-1})}{\partial Q_{t-1}^2}[w(p_t^E)]^{-1}\frac{\partial L(p_t^E,g_t)}{\partial p_t^E}\right\} \\ & \quad det\left(\begin{bmatrix}\frac{\partial G_1}{\partial p_t^E} & \frac{\partial G_1}{\partial A_{t-1}} \\ \frac{\partial G_2}{\partial p_t^E} & \frac{\partial G_2}{\partial A_{t-1}}\end{bmatrix}\right).\end{aligned}$$

The top line is positive. The term in braces on the second line is positive if and only if $\psi(\cdot;\cdot)$ is sufficiently convex. Assume backfire occurs, in which case the determinant on the third line is positive. Then there exists $x > 0$ such that the cross-partial is positive if $\partial^2\psi(g;Q)/\partial Q^2 < x$.

Analyzing the second derivative of the supply restriction's effect, we find:

$$\begin{aligned}\frac{\partial^2 g_t}{\partial Q_{t-1}^2} \propto & \frac{\partial L(p_t^E,g_t)}{\partial p_t^E}[w(p_t^E)]^{-1}\,\omega\,\frac{\partial^2\psi(g_t;Q_{t-1})}{\partial Q_{t-1}^2}\, det(G) \\ & - \left\{[w(p_t^E)]^{-2}\, w'(p_t^E)\,\omega\,\frac{\partial\psi(g_t;Q_{t-1})}{\partial Q_{t-1}} g_t^{-1} L_t \right. \\ & \left. \qquad + \omega\frac{\partial^2\psi(g_t;Q_{t-1})}{\partial Q_{t-1}^2}[w(p_t^E)]^{-1}\frac{\partial L(p_t^E,g_t)}{\partial p_t^E}\right\}\frac{\partial G_1}{\partial p_t^E}\,\frac{\partial G_2}{\partial Q_{t-1}} \\ = & -\frac{\partial L(p_t^E,g_t)}{\partial p_t^E}[w(p_t^E)]^{-1}\,\omega\,\frac{\partial^2\psi(g_t;Q_{t-1})}{\partial Q_{t-1}^2}\,\frac{\partial G_1}{\partial g_t}\,\frac{\partial G_2}{\partial p_t^E} \\ & - [w(p_t^E)]^{-2}\, w'(p_t^E)\,\omega\,\frac{\partial\psi(g_t;Q_{t-1})}{\partial Q_{t-1}} g_t^{-1} L_t\,\frac{\partial G_1}{\partial p_t^E}\,\frac{\partial G_2}{\partial Q_{t-1}}.\end{aligned}$$

This expression is negative if $\psi(\cdot)$ is convex or if $\psi(\cdot)$ is not too concave. We also have:

$$\begin{aligned}\frac{\partial^2 g_t}{\partial\tau\partial Q_{t-1}} \propto & -\left\{[w(p_t^E)]^{-2}\, w'(p_t^E)\,\omega\,\frac{\partial\psi(g_t;Q_{t-1})}{\partial Q_{t-1}} g_t^{-1} L_t \right. \\ & \left. \qquad + \omega\frac{\partial^2\psi(g_t;Q_{t-1})}{\partial Q_{t-1}^2}[w(p_t^E)]^{-1}\frac{\partial L(p_t^E,g_t)}{\partial p_t^E}\right\}\frac{\partial G_1}{\partial p_t^E}\,\frac{\partial G_2}{\partial\tau}.\end{aligned}$$

This expression is negative if $\psi(\cdot)$ is concave or if $\psi(\cdot)$ is not too convex.

Finally, we demonstrate the tâtonnement stability of the equilibrium. Consider a dynamic, tâtonnement-style process for describing the evolution of the economy in disequilibrium. If there is excess labor demand, then let the wage increase and the price of energy decrease. If expected profits from exploration exceed the wage paid to final-good labor, then let workers enter exploration. In notation, we are defining the following tâtonnement-style system:

$$\begin{aligned}\dot{p}_t^E =& g_t + L(p_t^E, g_t) - 1\\ &\triangleq \hat{G}_1(p_t^E, g_t),\\ \dot{g}_t =& \frac{\Pi^F(p_t^E, g_t)}{w(p_t^E)} - 1\\ &\triangleq \hat{G}_2(p_t^E, g_t),\end{aligned}$$

where dots indicate time derivatives. This system's steady state occurs at the equilibrium values, which we denote with stars. Linearizing around the steady state, we have

$$\begin{bmatrix}\dot{p}_t^E\\ \dot{g}_t\end{bmatrix} \approx \begin{bmatrix}\frac{\partial \hat{G}_1}{\partial p_t^E}(p_t^{E*}, g_t^*) & \frac{\partial \hat{G}_1}{\partial g_t}(p_t^{E*}, g_t^*)\\ \frac{\partial \hat{G}_2}{\partial p_t^E}(p_t^{E*}, g_t^*) & \frac{\partial \hat{G}_2}{\partial g_t}(p_t^{E*}, g_t^*)\end{bmatrix} \begin{bmatrix} p_t^E - p_t^{E*}\\ g_t - g_t^*\end{bmatrix}.$$

Label the matrix of partial derivatives as $\hat{G}$. Note that each entry in the top row of $\hat{G}$ is the negative of the corresponding entry in the matrix G, and each entry in the bottom row of $\hat{G}$ is the same as the corresponding entry in G. Therefore, $det(\hat{G}) = -det(G) > 0$ and $tr(\hat{G}) < 0$, where tr refers to the trace. The two eigenvalues are negative. The linearized system is globally asymptotically stable, and, by the Hartman-Grobman Theorem, the full nonlinear system is locally asymptotically stable around the equilibrium.

B Proof of Proposition 2

Requiring indifference between wage-labor and efficiency engineering, we have:

$$\begin{aligned} w(p_t^E) =& \Pi_t^E\\ =& \eta_E(1+\gamma_E)\phi_E \frac{1-\alpha}{\alpha}\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}} e(F_t, R_t)\, A_{t-1},\\ \Rightarrow g(p_t^E) =& \left[\eta_E(1+\gamma_E)\phi_E \frac{1-\alpha}{\alpha}\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}} \omega\, A_{t-1}\right]^{-1} w(p_t^E), \end{aligned} \tag{14}$$

where the second line follows from equation (3) and the third line recognizes that $e(F_t, R_t) = F_t$ in the absence of a renewable sector and substitutes $\omega\, g_t$ for F_t. We have written the equilibrium number of geologists as a decreasing function of the price of energy services.

Substitute $g(p_t^E)$ into demand for final-good labor from equation (10):

$$\begin{aligned}L(p_t^E,\ell_t) =&\omega\, g(p_t^E)\,(1+\eta_E\gamma_E\ell_t)\,A_{t-1}\,\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}\left[\frac{1-\kappa}{\kappa}\right]^{\sigma}[w(p_t^E)]^{-\sigma}[p_t^E]^{\sigma+\frac{\alpha}{1-\alpha}}\\ =&\frac{1}{\eta_E(1+\gamma_E)\alpha(1-\alpha)}\left[\frac{1-\kappa}{\kappa}\right]^{\sigma}(1+\gamma_E\eta_E\ell_t)\,[w(p_t^E)]^{1-\sigma}\,[p_t^E]^{\sigma-1}.\end{aligned} \tag{15}$$

Differentiating with respect to σ, we find that:

$$\begin{aligned}\frac{\partial L(p_t^E,\ell_t)}{\partial\sigma} =&\ln\left(\frac{p_t^E}{\kappa}\frac{1-\kappa}{w(p_t^E)}\right)\,L(p_t^E,\ell_t)+(1-\sigma)\frac{\partial w(p_t^E)}{\partial\sigma}[w(p_t^E)]^{-1}\,L(p_t^E,\ell_t)\\ =&L(p_t^E,\ell_t)\,\left[\ln\left(\frac{p_t^E}{\kappa}\frac{1-\kappa}{w(p_t^E)}\right)+\ln\left(\frac{w(p_t^E)}{1-\kappa}\right)-\Theta\ln\left(\frac{\kappa}{p_t^E}\right)\right]\\ =&L(p_t^E,\ell_t)\,\ln\left(\frac{p_t^E}{\kappa}\right)\,[1+\Theta]\\ >&0 \text{ iff } p_t^E>\kappa,\end{aligned}$$

where the second line substitutes for $\partial w(p_t^E)/\partial\sigma$ from equation (6).

Substituting equation (15) into the labor constraint, using the expression for expected profit from fossil resource exploration from equation (5), substituting $g(p_t^E)$ for g_t, and substituting $A_t=(1+\eta_E\gamma_E\ell_t)\,A_{t-1}$, we have the following system of equations in p_t^E and ℓ_t:

$$\begin{aligned}1 =&\ell_t+\left[\eta_E(1+\gamma_E)\phi_E\frac{1-\alpha}{\alpha}\left[\frac{\alpha^2 p_t^E}{\phi_E}\right]^{\frac{1}{1-\alpha}}\omega\,A_{t-1}\right]^{-1} w(p_t^E)\\ &+\frac{1}{\eta_E(1+\gamma_E)\alpha(1-\alpha)}\left[\frac{1-\kappa}{\kappa}\right]^{\sigma}(1+\gamma_E\eta_E\ell_t)\,[w(p_t^E)]^{1-\sigma}\,[p_t^E]^{\sigma-1}\\ &\triangleq H_1(p_t^E,\ell_t),\\ 1 =&\omega(1-\alpha)\left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}(1+\eta_E\gamma_E\ell_t)\,A_{t-1}\,[p_t^E]^{\frac{1}{1-\alpha}}\,[w(p_t^E)]^{-1}-\omega\,\left[\psi(g(p_t^E);Q_{t-1})+\tau\right]\,[w(p_t^E)]^{-1}\\ &\triangleq H_2(p_t^E,\ell_t).\end{aligned}$$

We next analyze the partial derivatives of each equation with respect to the endogenous variables.

$$\frac{\partial H_1}{\partial p_t^E} =[p_t^E]^{-1}\left\{-g(p_t^E)\,\left[\frac{1}{1-\alpha}+\Theta\right]+L(p_t^E,\ell_t)\,(\sigma-1)\,(1+\Theta)\right\}.$$

The first term in braces is negative and the second is positive if and only if $\sigma>1$. The whole expression is strictly negative for $\sigma\leq 1$. It is positive as $\sigma\to\infty$. There is at least one σ that is a root of this expression, and the expression must be increasing in σ at the smallest and the largest roots. If there is more than one root of $\partial H_1/\partial p_t^E$, then there must be at least two roots of $\partial^2 H_1/\partial p_t^E \partial\sigma$. The partial derivative with respect to σ is:

$$\frac{\partial^2 H_1}{\partial p_t^E \partial \sigma} =[p_t^E]^{-1}\Bigg\{\frac{\partial \Theta}{\partial \sigma}\left[-g(p_t^E)+L(p_t^E,\ell_t)\,(\sigma-1)\right]+\frac{\partial L(p_t^E,\ell_t)}{\partial \sigma}\,(\sigma-1)\,(1+\Theta) \\ -\frac{\partial g(p_t^E)}{\partial \sigma}\left[\frac{1}{1-\alpha}+\Theta\right]+L(p_t^E,\ell_t)\,[1+\Theta]\Bigg\}.$$

At a root of $\partial H_1/\partial p_t^E$, this becomes:

$$\left.\frac{\partial^2 H_1}{\partial p_t^E \partial \sigma}\right|_{\frac{\partial H_1}{\partial p_t^E}=0} \propto 1+\chi(\sigma)+(\sigma-1)\left\{-\frac{\partial \Theta}{\partial \sigma}\frac{1}{\frac{1}{1-\alpha}+\Theta}+\ln(p_t^E/\kappa)\right\}, \tag{16}$$

where we substitute from previous expressions in this proof and in the proof of Lemma 4 and factor out $p_t^E\,(1+\Theta)\,L(p_t^E,\ell_t)$. For there to be multiple roots of $\partial H_1/\partial p_t^E$, this expression must be positive at the first and last roots and negative at some root in between those.

First consider the case with $\kappa > p_t^E$ and $\sigma > 1$. In this case, the term in braces in equation (16) is negative. It is easy to show that the term in braces in equation (16) is decreasing in σ. Consider a root of $\partial H_1/\partial p_t^E$ at which equation (16) is negative. There must be an additional root at some larger σ for which equation (16) is positive. Therefore, $\chi(\sigma)$ must be increasing for some σ in between these two roots. We know from the proof of Lemma 4 that $\chi(\sigma)$ is decreasing at $\sigma = 1$. Therefore, in order for there to be two roots of $\partial H_1/\partial p_t^E \partial\sigma$, the second derivative of $\chi(\sigma)$ must be positive at some σ for which the first derivative is zero. From equation (8), we have:

$$\chi''(\sigma) =\frac{\chi'(\sigma)}{1-\sigma}+\frac{\chi(\sigma)}{(1-\sigma)^2}-\ln(\kappa/p_t^E)\frac{\partial^2\Theta}{\partial\sigma^2}. \tag{17}$$

For $\kappa > p_t^E$, the rightmost expression (including the negative sign) is negative. And we know from the proof of Lemma 4 that $\chi(\sigma) < 0$ for $\sigma > 1$. So at any σ for which $\chi'(\sigma) = 0$, $\chi''(\sigma) < 0$. Therefore $\chi(\sigma)$ is weakly decreasing for all $\sigma > 1$. Therefore $\partial^2 H_1/\partial p_t^E \partial\sigma$ is decreasing in σ for $\sigma > 1$. If equation (16) is negative for some root of $\partial H_1/\partial p_t^E$, it cannot be positive at some larger root. But we know $\partial H_1/\partial p_t^E$ is increasing at its largest root. Therefore, $\partial H_1/\partial p_t^E$ can have at most one root in σ if $\kappa > p_t^E$.

Now consider the case with $\kappa < p_t^E$ and $\sigma > 1$. Substituting from previous expressions, we have:

$$\frac{\partial^2 H_1}{\partial p_t^E \partial \sigma} \propto g(p_t^E)\left\{\frac{\chi(\sigma)}{\sigma-1}\left[\frac{1}{1-\alpha}+\Theta\right]-\frac{\partial\Theta}{\partial\sigma}\right\}+L(p_t^E,\ell_t)\,[1+\Theta]\,\left\{(\sigma-1)\ln(p_t^E/\kappa)+1\right\}.$$

This is positive if and only if

$$g(p_t^E)\,\left[\frac{1}{1-\alpha}+\Theta\right] < -\frac{\partial\Theta}{\partial\sigma}g(p_t^E)\frac{1-\sigma}{\chi(\sigma)} + L(p_t^E,\ell_t)\,[1+\Theta]\,\left\{(\sigma-1)\ln(p_t^E/\kappa)+1\right\}\frac{1-\sigma}{\chi(\sigma)}.$$

If $\partial H_1/\partial p_t^E > 0$, then

$$g(p_t^E)\,\left[\frac{1}{1-\alpha}+\Theta\right] < L(p_t^E,\ell_t)\,(\sigma-1)\,(1+\Theta).$$

Combining these expressions, we have that $\partial H_1/\partial p_t^E > 0$ and $\partial^2 H_1/\partial p_t^E\partial\sigma < 0$ only if

$$-\frac{\partial\Theta}{\partial\sigma}g(p_t^E)\frac{1-\sigma}{\chi(\sigma)} + L(p_t^E,\ell_t)\,[1+\Theta]\,\left\{(\sigma-1)\ln(p_t^E/\kappa)+1\right\}\frac{1-\sigma}{\chi(\sigma)} < (\sigma-1)\,(1+\Theta)\,L(p_t^E,\ell_t),$$

which holds if and only if

$$\frac{\chi(\sigma)}{1-\sigma} > \frac{\Theta\,\ln(p_t^E/\kappa)\,g(p_t^E)}{(\sigma-1)\,L(p_t^E,\ell_t)} + \ln(p_t^E/\kappa) + \frac{1}{\sigma-1} \triangleq D.$$

All terms are positive for $\sigma > 1$ and $p_t^E > \kappa$. Recalling from the proof of Lemma 4 that $\chi(\sigma)/(1-\sigma)$ is the fractional change in the equilibrium wage due to a change in σ, we have that $\partial H_1/\partial p_t^E > 0$ implies $\partial^2 H_1/\partial p_t^E\partial\sigma > 0$ if

$$\frac{\partial w(p_t^E)/\partial\sigma}{w(p_t^E)} < D.$$

If this condition holds, then there is a unique root of $\partial H_1/\partial p_t^E$ in σ. This condition always holds for σ sufficiently large, and the following assumption ensures that it holds more generally:

Assumption 2. *Let $\hat{\sigma}$ be the σ that is the smallest root of $\partial H_1/\partial p_t^E$. If $p_t^E > \kappa$ and $\sigma > \hat{\sigma}$, then $[w(p_t^E)]^{-1}\,\partial w(p_t^E)/\partial\sigma < D$.*

When this assumption holds, $\partial H_1/\partial p_t^E$ can have at most one root in σ if $p_t^E > \kappa$. Numerical simulations suggest that this assumption usually holds and also that it is always true that $\partial H_1/\partial p_t^E$ has at most one root in σ when $p_t^E > \kappa$.

Combining these results, we have seen that there exists $\hat{\sigma}_{H,1} > 1$ such that

$$\frac{\partial H_1}{\partial p_t^E} < 0 \text{ iff } \sigma < \hat{\sigma}_{H,1}.$$

Now consider the other partial derivatives.

$$\frac{\partial H_1}{\partial \ell_t} = 1 + \frac{\gamma_E\eta_E}{1+\gamma_E\eta_E\ell_t}L(p_t^E,\ell_t) > 0.$$

$$\frac{\partial H_2}{\partial p_t^E} = - \left[w(p_t^E)\right]^{-2} w'(p_t^E)\, \Pi_t^F(p_t^E, \ell_t) + \frac{1}{1-\alpha}[p_t^E]^{-1} \left[w(p_t^E)\right]^{-1} \Pi_t^{F,Rev}(p_t^E, \ell_t)$$
$$- \omega \frac{\partial \psi(g(p_t^E); Q_{t-1})}{\partial g(p_t^E)} g'(p_t^E) \left[w(p_t^E)\right]^{-1}$$
$$>0.$$

$$\frac{\partial H_2}{\partial \ell_t} = [w(p_t^E)]^{-1} \frac{\eta_e \gamma_E}{1 + \eta_E \gamma_E \ell_t} \Pi_t^{F,Rev}(p_t^E, \ell_t) > 0.$$

Define
$$H \triangleq \begin{bmatrix} \frac{\partial H_1}{\partial p_t^E} & \frac{\partial H_1}{\partial \ell_t} \\ \frac{\partial H_2}{\partial p_t^E} & \frac{\partial H_2}{\partial \ell_t} \end{bmatrix}.$$

If $\sigma < \hat{\sigma}_{H,1}$, then $det(H)$ is negative. Define $\hat{\sigma}_{H,2}$ as the smallest σ for which $det(H) = 0$. Clearly $\hat{\sigma}_{H,2} > \hat{\sigma}_{H,1} > 1$. It is easy to show that $\hat{\sigma}_{H,2}$ is unique if $\partial H_1 / \partial p_t^E$ is increasing in σ for $\sigma > \hat{\sigma}_{H,1}$. Also, note that if we define a tâtonnement-style adjustment process as in the setting without engineering, then the equilibrium is unstable for all σ. If $det(H) < 0$, then the determinant of the tâtonnement matrix is positive and its trace is also positive (implying two positive eigenvalues), and if $det(H) > 0$, then the determinant of the tâtonnement matrix is negative (implying one positive eigenvalue and one negative eigenvalue).

Consider the effect of exogenously increasing the number of engineers. As described in the text, we will restrict the analysis to those equilibria for which introducing engineers from outside the model reduces the incentive to undertake engineering instead of wage-labor. Because the engineers enter from outside the model, this increase affects the labor constraint only through $L(p_t^E, \ell_t)$ (where it acts like an increase in ℓ_t). Its effect on H_2 is proportional to the effect of increasing ℓ_t. Using the implicit function theorem, the equilibrium restriction requires that

$$-\frac{det\left(\begin{bmatrix} \frac{\partial H_1}{\partial \ell_t} - 1 & \frac{\partial H_1}{\partial \ell_t} \\ \frac{\partial H_2}{\partial \ell_t} & \frac{\partial H_2}{\partial \ell_t} \end{bmatrix}\right)}{det(H)} < 0,$$

which holds if and only if $det(H) < 0$. In turn, we have seen that $det(H) < 0$ if $\sigma < \hat{\sigma}_{H,2}$. We therefore only consider cases with $\sigma < \hat{\sigma}_{H,2}$.

The partial derivatives of each equation with respect to the policy parameters are:

$$\frac{\partial H_1}{\partial A_{t-1}} = -\, A_{t-1}^{-1}\, g(p_t^E) \qquad < 0,$$

$$\frac{\partial H_2}{\partial A_{t-1}} = A_{t-1}^{-1}\, \Pi_t^{F,Rev}(p_t^E, \ell_t)\, [w(p_t^E)]^{-1} + \omega\, \frac{\partial \psi(g(p_t^E); Q_{t-1})}{\partial g(p_t^E)}\, A_{t-1}^{-1}\, g(p_t^E)\, [w(p_t^E)]^{-1} \qquad > 0,$$

$$\frac{\partial H_1}{\partial Q_{t-1}} = 0 \qquad = 0,$$

$$\frac{\partial H_2}{\partial Q_{t-1}} = -\, \omega \frac{\partial \psi(g(p_t^E); Q_{t-1})}{\partial Q_{t-1}}\, [w(p_t^E)]^{-1} \qquad < 0$$

$$\frac{\partial H_1}{\partial \tau} = 0 \qquad = 0,$$

$$\frac{\partial H_2}{\partial \tau} = -\, \omega\, [w(p_t^E)]^{-1} \qquad < 0.$$

Consider the effect of a marginal improvement in incoming energy services technology. Using the implicit function theorem, we have:

$$\frac{\partial p_t^E}{\partial A_{t-1}} = -\frac{det\left(\begin{bmatrix} \frac{\partial H_1}{\partial A_{t-1}} & \frac{\partial H_1}{\partial \ell_t} \\ \frac{\partial H_2}{\partial A_{t-1}} & \frac{\partial H_2}{\partial \ell_t} \end{bmatrix}\right)}{det(H)}.$$

The numerator is negative, so the comparative static is negative if and only if $det(H) < 0$. We also have:

$$\frac{\partial \ell_t}{\partial A_{t-1}} = -\frac{det\left(\begin{bmatrix} \frac{\partial H_1}{\partial p_t^E} & \frac{\partial H_1}{\partial A_{t-1}} \\ \frac{\partial H_2}{\partial p_t^E} & \frac{\partial H_2}{\partial A_{t-1}} \end{bmatrix}\right)}{det(H)}.$$

Similar analysis as to the analysis establishing uniqueness of $\hat{\sigma}_{H,1}$ establishes that there is a unique σ at which the numerator switches sign. Therefore, there exists $\hat{\sigma}_{H,3} \in (1, \hat{\sigma}_{H,1})$ such that $\partial \ell_t / \partial A_{t-1} < 0$ if $\sigma \in (0, \hat{\sigma}_{H,3})$ and $\partial \ell_t / \partial A_{t-1} > 0$ if $\sigma \in (\hat{\sigma}_{H,3}, \hat{\sigma}_{H,2})$. Further, if $\psi(\cdot)$ is linear, then only $\partial H_2 / \partial p_t^E$ depends on Q_{t-1}, which makes $\hat{\sigma}_{H,3}$ decline in Q_{t-1}.

Now consider the effect of a marginal increase in depletion. Using the implicit function theorem, we have:

$$\frac{\partial p_t^E}{\partial Q_{t-1}} = \frac{\partial p_t^E}{\partial \tau}\, \frac{\partial \psi(g(p_t^E); Q_{t-1})}{\partial Q_{t-1}} = -\frac{det\left(\begin{bmatrix} \frac{\partial H_1}{\partial Q_{t-1}} & \frac{\partial H_1}{\partial \ell_t} \\ \frac{\partial H_2}{\partial Q_{t-1}} & \frac{\partial H_2}{\partial \ell_t} \end{bmatrix}\right)}{det(H)}.$$

Under the requirement that $det(H) < 0$, both comparative statics are positive. We also have:

$$\frac{\partial \ell_t}{\partial Q_{t-1}} = \frac{\partial \ell_t}{\partial \tau} \frac{\partial \psi(g(p_t^E); Q_{t-1})}{\partial Q_{t-1}} = -\frac{det\left(\begin{bmatrix} \frac{\partial H_1}{\partial p_t^E} & \frac{\partial H_1}{\partial Q_{t-1}} \\ \frac{\partial H_2}{\partial p_t^E} & \frac{\partial H_2}{\partial Q_{t-1}} \end{bmatrix}\right)}{det(H)}.$$

If $\sigma \in (0, \hat{\sigma}_{H,1})$, then $\partial \ell_t / \partial Q_{t-1} > 0$ and $\partial \ell_t / \partial \tau > 0$. If $\sigma \in (\hat{\sigma}_{H,1}, \hat{\sigma}_{H,2})$, then $\partial \ell_t / \partial Q_{t-1} < 0$ and $\partial \ell_t / \partial \tau < 0$.

Finally, consider the effects of these policies on equilibrium extraction. Using equation (14), we have:

$$\begin{aligned}
\frac{\partial g_t}{\partial A_{t-1}} =& - A_{t-1}^{-1} g(p_t^E) + \frac{\partial p_t^E}{\partial A_{t-1}} g(p_t^E) \left[w'(p_t^E) [w(p_t^E)]^{-1} - \frac{1}{1-\alpha} [p_t^E]^{-1} \right] \\
\propto & \frac{\partial H_2}{\partial \ell_t} \left[A_{t-1}^{-1} \frac{\partial H_1}{\partial p_t^E} - [p_t^E]^{-1} \left[\frac{1}{1-\alpha} + \Theta \right] \frac{\partial H_1}{\partial A_{t-1}} \right] \\
& - \frac{\partial H_1}{\partial \ell_t} \left[A_{t-1}^{-1} \frac{\partial H_2}{\partial p_t^E} - [p_t^E]^{-1} \left[\frac{1}{1-\alpha} + \Theta \right] \frac{\partial H_2}{\partial A_{t-1}} \right] \\
\propto & \frac{\partial H_2}{\partial \ell_t} L(p_t^E, \ell_t) (\sigma - 1) (1 + \Theta) + \frac{\partial H_1}{\partial \ell_t} \Theta \, \omega \, \psi(g(p_t^E); Q_{t-1}) [w(p_t^E)]^{-1}.
\end{aligned}$$

This expression is positive if $\sigma \geq 1$. There exists $\hat{\sigma}_{H,3} < 1$ such that $\partial g_t / \partial A_{t-1} > 0$ if and only if $\sigma > \hat{\sigma}_{H,3}$. Further, $\hat{\sigma}_{H,3}$ decreases in Q_{t-1}.

Again using equation (14), we also have:

$$\begin{aligned}
\frac{\partial g_t}{\partial Q_{t-1}} =& \frac{\partial p_t^E}{\partial Q_{t-1}} g(p_t^E) \left[w'(p_t^E) [w(p_t^E)]^{-1} - \frac{1}{1-\alpha} [p_t^E]^{-1} \right] < 0, \\
\frac{\partial g_t}{\partial \tau} =& \frac{\partial p_t^E}{\partial \tau} g(p_t^E) \left[w'(p_t^E) [w(p_t^E)]^{-1} - \frac{1}{1-\alpha} [p_t^E]^{-1} \right] < 0.
\end{aligned}$$

C Proof of Proposition 3

Requiring indifference between wage-labor and efficiency engineering, we have:

$$w(p_t^E) = \eta_E (1 + \gamma_E) \phi_E \frac{1-\alpha}{\alpha} \left[\frac{\alpha^2 p_t^E}{\phi_E} \right]^{\frac{1}{1-\alpha}} A_{t-1} \, e\left(\omega \, g_t, (1 + \eta_R \gamma_R r_t) \, Z_{t-1}\right). \tag{18}$$

This equation implicitly defines the number of renewable engineers r_t as a function of p_t^E and g_t. Via the implicit function theorem, we have:

$$\frac{\partial r(p_t^E, g_t)}{\partial p_t^E} = e(F_t, R_t) \frac{-\frac{1}{1-\alpha}[p_t^E]^{-1} + [w(p_t^E)]^{-1} \, w'(p_t^E)}{\frac{\partial e(F_t,R_t)}{\partial R_t} \eta_R \gamma_R \, Z_{t-1}} < 0,$$

$$\frac{\partial r(p_t^E, g_t)}{\partial g_t} = - \frac{\frac{\partial e(F_t,R_t)}{\partial F_t}}{\frac{\partial e(F_t,R_t)}{\partial R_t}} \frac{\omega}{\eta_R \gamma_R Z_{t-1}} < 0.$$

Now requiring indifference between wage-labor and renewable engineering, we have from equation (4):

$$w(p_t^E) = \eta_R (1+\gamma_R) Z_{t-1} \frac{\partial e(F_t, R_t)}{\partial R_t} [p_t^E]^{\frac{1}{1-\alpha}} (1 + \eta_E \gamma_E \ell_t) \, A_{t-1} \, (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}}.$$

Rearranging yields:

$$\ell(p_t^E, g_t) = - \frac{1}{\eta_E \gamma_E} + w(p_t^E) \left(\eta_E \gamma_E \eta_R (1+\gamma_R) Z_{t-1} \frac{\partial e(\omega \, g_t, R(r(p_t^E, g_t)))}{\partial R_t} [p_t^E]^{\frac{1}{1-\alpha}} A_{t-1} \, (1-\alpha) \left[\frac{\alpha^2}{\phi_E}\right]^{\frac{\alpha}{1-\alpha}} \right)^{-1}, \tag{19}$$

where we make the dependence of renewable energy R_t on renewable engineers explicit. Substitute $\ell(p_t^E, g_t)$ into equilibrium demand for wage-labor from equation (10):

$$L(p_t^E, g_t) = [\eta_R (1+\gamma_R) \, (1-\alpha)]^{-1} \left[\frac{1-\kappa}{\kappa}\right]^{\sigma} Z_{t-1}^{-1} \left[\frac{\partial e(F_t, R_t)}{\partial R_t}\right]^{-1} e\,(F_t, R_t) \; [w(p_t^E)]^{1-\sigma} [p_t^E]^{\sigma-1}.$$

Also substitute $\ell(p_t^E, g_t)$ into the equilibrium reward to fossil resource exploration from equation (5):

$$\Pi_t^F = \omega \frac{\partial e(F_t, R_t)}{\partial F_t} \left(\eta_R (1+\gamma_R) Z_{t-1} \frac{\partial e(F_t, R_t)}{\partial R} \right)^{-1} w(p_t^E) - \omega \psi(g_t; Q_{t-1}) - \omega \tau.$$

Imposing the labor constraint and indifference between wage-labor and the expected profit from fossil resource exploration, we have the following system of equations in p_t^E and

g_t (where we suppress the dependence of F_t and R_t on these variables for the sake of notation):

$$\begin{aligned}
1 =& g_t + r(p_t^E, g_t) - \frac{1}{\eta_E \gamma_E} \\
&+ w(p_t^E) \left(\eta_E \gamma_E \eta_R (1+\gamma_R) Z_{t-1} \frac{\partial e(F_t, R_t)}{\partial R_t} [p_t^E]^{\frac{1}{1-\alpha}} A_{t-1} (1-\alpha) \left[\frac{\alpha^2}{\phi_E} \right]^{\frac{\alpha}{1-\alpha}} \right)^{-1} \\
&+ [\eta_R (1+\gamma_R)(1-\alpha)]^{-1} \left[\frac{1-\kappa}{\kappa} \right]^{\sigma} Z_{t-1}^{-1} \left[\frac{\partial e(F_t, R_t)}{\partial R_t} \right]^{-1} e(F_t, R_t) \, [w(p_t^E)]^{1-\sigma} [p_t^E]^{\sigma-1} \\
&\triangleq J_1(p_t^E, g_t) \\
1 =& \omega \frac{\partial e(F_t, R_t)}{\partial F_t} \left(\eta_R (1+\gamma_R) Z_{t-1} \frac{\partial e(F_t, R_t)}{\partial R} \right)^{-1} - \omega \left[\psi(g_t; Q_{t-1}) + \tau \right] [w(p_t^E)]^{-1} \\
&\triangleq J_2(p_t^E, g_t).
\end{aligned}$$

We next analyze the partial derivatives of each equation with respect to the endogenous variables.

$$\begin{aligned}
\frac{\partial J_1}{\partial p_t^E} =& - [p_t^E]^{-1} \left[\frac{1}{1-\alpha} + \Theta \right] \Bigg[\frac{e(F_t, R_t)}{\frac{\partial e(F_t, R_t)}{\partial R_t} \eta_R \gamma_R Z_{t-1}} \\
&\qquad + \left(L(p_t^E, g_t) + \ell(p_t^E, g_t) + \frac{1}{\eta_E \gamma_E} \right) \left(1 - e(F_t, R_t) \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2} \left(\frac{\partial e(F_t, R_t)}{\partial R_t} \right)^{-2} \right) \Bigg] \\
&+ L(p_t^E, g_t)(\sigma - 1) [p_t^E]^{-1} [1 + \Theta].
\end{aligned}$$

This is negative if $\sigma < 1$. Let $\hat{\sigma}_{J,1} > 1$ be the smallest root of $\partial J_1 / \partial p_t^E$. Then $\partial J_1 / \partial p_t^E < 0$ if $\sigma < \hat{\sigma}_{J,1}$ and $\partial J_1 / \partial p_t^E > 0$ for σ between $\hat{\sigma}_{J,1}$ and the next-largest root. For ease of exposition, the proposition assumes that the root is unique. In that case, there is a unique $\hat{\sigma}_{J,1} > 1$ such that

$$\frac{\partial J_1}{\partial p_t^E} < 0 \text{ iff } \sigma < \hat{\sigma}_{J,1}.$$

Differentiating J_1 with respect to g_t, we obtain:

$$\begin{aligned}\frac{\partial J_1}{\partial g_t} =&1 + \frac{\partial r(p_t^E, g_t)}{\partial g_t}\\&- \left[\ell_t + \frac{1}{\eta_E \gamma_E}\right] \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \left[\frac{\partial^2 e(F_t, R_t)}{\partial R_t \partial F_t}\,\omega + \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2}\,\eta_R \gamma_R Z_{t-1}\,\frac{\partial r(p_t^E, g_t)}{\partial g_t}\right]\\&- L_t \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \left[\frac{\partial^2 e(F_t, R_t)}{\partial R_t \partial F_t}\,\omega + \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2}\,\eta_R \gamma_R Z_{t-1}\,\frac{\partial r(p_t^E, g_t)}{\partial g_t}\right]\\&+ L_t \left[e(F_t, R_t)\right]^{-1} \left[\frac{\partial e(F_t, R_t)}{\partial F_t}\,\omega + \frac{\partial e(F_t, R_t)}{\partial R_t}\,\eta_R \gamma_R Z_{t-1}\,\frac{\partial r(p_t^E, g_t)}{\partial g_t}\right]\\=&1\\&- \omega \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \left(L_t + \ell_t + \frac{1}{\eta_E \gamma_E}\right) \left(\frac{\partial^2 e(F_t, R_t)}{\partial F_t \partial R_t} - \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2}\,\frac{\partial e(F_t, R_t)}{\partial F_t} \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1}\right)\\&- \omega \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \frac{\partial e(F_t, R_t)}{\partial F_t} \frac{1}{\eta_R \gamma_R Z_{t-1}}.\end{aligned}$$

The top line is positive and the bottom line two lines are negative. The middle line decreases in $\partial^2 e(F_t, R_t)/\partial F_t \partial R_t$ and in $-\partial^2 e(F_t, R_t)/\partial R_t^2$. The bottom line is proportional to $\partial r(p_t^E, g_t)/\partial g_t$. If $\partial e(F_t, R_t)/\partial R_t$ is constant, then the overall expression is positive if and only if Z_{t-1} is sufficiently large.

The partial derivatives of J_2 are unambiguously negative:

$$\begin{aligned}\frac{\partial J_2}{\partial p_t^E} =&\Pi_t^{F,Rev} [w(p_t^E)]^{-1}\,\frac{\partial r(p_t^E, g_t)}{\partial p_t^E} \eta_R \gamma_R Z_{t-1} \left[\left(\frac{\partial e(F_t, R_t)}{\partial F_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial F_t \partial R_t} - \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2}\right]\\&+ \omega \left[\psi(g_t; Q_{t-1}) + \tau\right] \left[w(p_t^E)\right]^{-2} w'(p_t^E)\\<&0,\\\frac{\partial J_2}{\partial g_t} =&\Pi_t^{F,Rev} [w(p_t^E)]^{-1}\,\omega \left[\left(\frac{\partial e(F_t, R_t)}{\partial F_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial F_t^2} - 2 \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial F_t \partial R_t}\right.\\&\qquad\left. + \frac{\partial e(F_t, R_t)}{\partial F_t} \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-2} \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2}\right]\\&- \omega \frac{\partial \psi(g_t; Q_{t-1})}{\partial g_t} \left[w(p_t^E)\right]^{-1}\\<&0.\end{aligned}$$

Define

$$J \triangleq \begin{bmatrix} \frac{\partial J_1}{\partial p_t^E} & \frac{\partial J_1}{\partial g_t} \\ \frac{\partial J_2}{\partial p_t^E} & \frac{\partial J_2}{\partial g_t} \end{bmatrix}.$$

Define $\hat{\sigma}_{J,2}$ as the smallest σ for which $det(J) = 0$, so that $det(J) > 0$ if $\sigma < \hat{\sigma}_{J,2}$. Clearly $\hat{\sigma}_{J,2} > \hat{\sigma}_{J,1}$ if and only if $\partial J_1/\partial g_t > 0$.

If we define a tâtonnement-style adjustment process as in the setting without engineering, then the equilibrium is unstable for all $\sigma < \hat{\sigma}_{J,2}$ and is also unstable for most $\sigma > \hat{\sigma}_{J,2}$, with the only potential exceptions arising for σ sufficiently close to $\hat{\sigma}_{J,1}$ that $\partial J_1/\partial p_t^E$ is small relative to the magnitude of $\partial J_2/\partial g_t$. If $det(J) < 0$, then the determinant of the tâtonnement matrix is positive and, unless $\partial J_1/\partial p_t^E$ is sufficiently small, its trace is also positive (implying two positive eigenvalues). If $det(J) > 0$, then the determinant of the tâtonnement matrix is negative (implying one positive eigenvalue and one negative eigenvalue).

Consider the effect of exogenously increasing the number of geologists. As described in the text, we will restrict the analysis to those equilibria for which introducing geologists from outside the model reduces the incentive to undertake exploration instead of wage-labor. Because the geologists enter from outside the model, this increase affects the labor constraint only through wage-labor and engineering variables, where it acts like an increase in g_t. Its effect on J_2 is the same as increasing g_t. Using the implicit function theorem, the equilibrium restriction requires that

$$-\frac{det\left(\begin{bmatrix} \frac{\partial J_1}{\partial g_t} - 1 & \frac{\partial J_1}{\partial g_t} \\ \frac{\partial J_2}{\partial g_t} & \frac{\partial J_2}{\partial g_t} \end{bmatrix}\right)}{det(J)} < 0,$$

which holds if and only if $det(J) > 0$. In turn, we have seen that $det(J) > 0$ if $\sigma < \hat{\sigma}_{J,2}$. We therefore only consider cases with $\sigma < \hat{\sigma}_{J,2}$.

Now analyze the effect of a renewable policy, which acts like increasing Z_{t-1}. The partial derivatives with respect to this policy variable are:

$$\begin{aligned}
\frac{\partial J_1}{\partial Z_{t-1}} =& \frac{\partial r(p_t^E, g_t)}{\partial Z_{t-1}} \\
&- Z_{t-1}^{-1}\left[\ell_t + \frac{1}{\eta_E \gamma_E}\right] - \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2} \left[\ell_t + \frac{1}{\eta_E \gamma_E}\right] \left[(1 + \eta_R \gamma_R r_t) + \eta_R \gamma_R Z_{t-1} \frac{\partial r(p_t^E, g_t)}{\partial Z_{t-1}}\right] \\
&- Z_{t-1}^{-1} L_t - \left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1} \frac{\partial^2 e(F_t, R_t)}{\partial R_t^2} L_t \left[(1 + \eta_R \gamma_R r_t) + \eta_R \gamma_R Z_{t-1} \frac{\partial r(p_t^E, g_t)}{\partial Z_{t-1}}\right] \\
&+ [e(F_t, R_t)]^{-1} \frac{\partial e(F_t, R_t)}{\partial R_t} L_t \left[(1 + \eta_R \gamma_R r_t) + \eta_R \gamma_R Z_{t-1} \frac{\partial r(p_t^E, g_t)}{\partial Z_{t-1}}\right] \\
=& - Z_{t-1}^{-1}\left[L_t + \ell_t + r_t + \frac{1}{\eta_R \gamma_R} + \frac{1}{\eta_E \gamma_E}\right] \\
<& 0.
\end{aligned}$$

$$\begin{aligned}\frac{\partial J_2}{\partial Z_{t-1}} =&\Pi_t^{F,Rev}\,[w(p_t^E)]^{-1}\Bigg\{-Z_{t-1}^{-1}\\&-\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-1}\frac{\partial^2 e(F_t,R_t)}{\partial R_t^2}\left[(1+\eta_R\,\gamma_R\,r_t)+\eta_R\gamma_R Z_{t-1}\frac{\partial r(p_t^E,g_t)}{\partial Z_{t-1}}\right]\\&+\left(\frac{\partial e(F_t,R_t)}{\partial F_t}\right)^{-1}\frac{\partial^2 e(F_t,R_t)}{\partial F_t\partial R_t}\left[(1+\eta_R\,\gamma_R\,r_t)+\eta_R\gamma_R Z_{t-1}\frac{\partial r(p_t^E,g_t)}{\partial Z_{t-1}}\right]\Bigg\}\\=&-Z_{t-1}^{-1}\,\Pi_t^{F,Rev}\,[w(p_t^E)]^{-1}\\<&0.\end{aligned}$$

Using the implicit function theorem, we have:

$$\frac{\partial g_t}{\partial Z_{t-1}} = -\frac{det\left(\begin{bmatrix}\frac{\partial J_1}{\partial p_t^E} & \frac{\partial J_1}{\partial Z_{t-1}}\\ \frac{\partial J_2}{\partial p_t^E} & \frac{\partial J_2}{\partial Z_{t-1}}\end{bmatrix}\right)}{det(J)}.$$

If $\hat{\sigma}_{J,1} < \hat{\sigma}_{J,2}$ and $\sigma \in (\hat{\sigma}_{J,1}, \hat{\sigma}_{J,2})$, then the comparative static is positive. It is ambiguous otherwise, with negative values occurring when the main diagonal dominates. Therefore, the comparative static is positive for σ sufficiently close to $\hat{\sigma}_{J,1}$ and potentially negative for σ sufficiently small. Analyzing the determinant, we find:

$$\begin{aligned}\frac{\partial g_t}{\partial Z_{t-1}} \propto& -\Pi_t^{F,Rev}\,[p_t^E]^{-1}\left[\frac{1}{1-\alpha}+\Theta\right]\\&\Bigg\{\frac{e(F_t,R_t)}{\frac{\partial e(F_t,R_t)}{\partial R_t}\eta_R\gamma_R\,Z_{t-1}} + L(p_t^E,g_t)+\ell(p_t^E,g_t)+\frac{1}{\eta_E\gamma_E}\\&-\left[L_t+\ell_t+r_t+\frac{1}{\eta_R\gamma_R}+\frac{1}{\eta_E\gamma_E}\right]e(F_t,R_t)\left(\frac{\partial e(F_t,R_t)}{\partial F_t}\right)^{-1}\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-1}\frac{\partial^2 e(F_t,R_t)}{\partial F_t\partial R_t}\\&+\left[r_t+\frac{1}{\eta_R\gamma_R}\right]e(F_t,R_t)\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-2}\frac{\partial^2 e(F_t,R_t)}{\partial R_t^2}\Bigg\}\\&+\left[L_t+\ell_t+r_t+\frac{1}{\eta_R\gamma_R}+\frac{1}{\eta_E\gamma_E}\right]\omega\,[\psi(g_t;Q_{t-1})+\tau]\,[p_t^E]^{-1}\,\Theta\\&+\Pi_t^{F,Rev}\,L(p_t^E,g_t)\,(\sigma-1)\,[p_t^E]^{-1}\,[1+\Theta]\,.\end{aligned}$$

The expression increases in Q_{t-1}, in $\partial^2 e(F_t,R_t)/\partial F_t\partial R_t$, and in $-\partial^2 e(F_t,R_t)/\partial R_t^2$.

The partial derivatives of J_1 and J_2 with respect to A_{t-1} are also negative:

$$\begin{aligned}\frac{\partial J_1}{\partial A_{t-1}} =& - A_{t-1}^{-1}\left\{ \frac{e(F_t,R_t)}{\frac{\partial e(F_t,R_t)}{\partial R_t}\eta_R\gamma_R Z_{t-1}} + \left[L_t+\ell_t+\frac{1}{\eta_E\gamma_E}\right]\left[1-\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-2}\frac{\partial^2 e(F_t,R_t)}{\partial R_t^2}\, e(F_t,R_t)\right]\right\}\\ <&0,\\ \frac{\partial J_2}{\partial A_{t-1}} =& A_{t-1}^{-1}\,\Pi_t^{F,Rev}\,[w(p_t^E)]^{-1}\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-1} e(F_t,R_t)\\ &\left\{\left(\frac{\partial e(F_t,R_t)}{\partial R_t}\right)^{-1}\frac{\partial^2 e(F_t,R_t)}{\partial R_t^2} - \left(\frac{\partial e(F_t,R_t)}{\partial F_t}\right)^{-1}\frac{\partial^2 e(F_t,R_t)}{\partial F_t\partial R_t}\right\}\\ \leq&0,\end{aligned}$$

where $\partial J_2/\partial A_{t-1} = 0$ if and only if $\partial e(F_t,R_t)/\partial R_t$ is constant. Using the implicit function theorem, we have:

$$\frac{\partial g_t}{\partial A_{t-1}} = -\frac{det\left(\begin{bmatrix}\frac{\partial J_1}{\partial p_t^E} & \frac{\partial J_1}{\partial A_{t-1}}\\ \frac{\partial J_2}{\partial p_t^E} & \frac{\partial J_2}{\partial A_{t-1}}\end{bmatrix}\right)}{det(J)}.$$

This comparative static is unambiguously positive if $\partial e(F_t,R_t)/\partial R_t$ is constant. Otherwise, it could be negative for σ sufficiently small. The numerator increases in Q_{t-1} via $\partial J_2/\partial p_t^E$.

Now consider the effect of a supply restriction. The partial derivatives of J_1 and J_2 with respect to Q_{t-1} are:

$$\begin{aligned}\frac{\partial J_1}{\partial Q_{t-1}} =&0,\\ \frac{\partial J_2}{\partial Q_{t-1}} =& -\omega\,\frac{\partial \psi(g_t;Q_{t-1})}{\partial Q_{t-1}}\,[w(p_t^E)]^{-1} < 0.\end{aligned}$$

Using the implicit function theorem, it is easy to see that $\partial p_t^E/\partial Q_{t-1} < 0$ if and only if $\partial^2 e(F_t,R_t)/\partial F_t\partial R_t$ and $-\partial^2 e(F_t,R_t)/\partial R_t^2$ are sufficiently small and Z_{t-1} is sufficiently large. Similarly, it is easy to see via the implicit function theorem that $\partial g_t/\partial Q_{t-1} < 0$ if and only if $\sigma < \hat{\sigma}_{J,1}$, and we know from the above that $\hat{\sigma}_{J,1} < \hat{\sigma}_{J,2}$ if and only if $\partial^2 e(F_t,R_t)/\partial F_t\partial R_t$ and $-\partial^2 e(F_t,R_t)/\partial R_t^2$ are sufficiently small and Z_{t-1} is sufficiently large. Also, the expression for $\partial J_1/\partial p_t^E$ shows that $\hat{\sigma}_{J,1}$ increases in $-\partial^2 e(F_t,R_t)/\partial R_t^2$.

The text mentions how engineering responds to a supply restriction. The equilibrium response of renewable engineers is:

$$\begin{aligned}\frac{\partial r_t}{\partial Q_{t-1}} =& \frac{\partial r(p_t^E, g_t)}{\partial Q_{t-1}} + \frac{\partial r(p_t^E, g_t)}{\partial p_t^E}\frac{\partial p(p_t^E, g_t)}{\partial Q_{t-1}} + \frac{\partial r(p_t^E, g_t)}{\partial g_t}\frac{\partial g_t}{\partial Q_{t-1}} \\ \propto& [p_t^E]^{-1}\left[\frac{1}{1-\alpha} + \Theta\right] \\ & \left\{ e(F_t, R_t) + \omega \left(L_t + \ell_t + \frac{1}{\eta_E \gamma_E}\right)\left(\frac{\partial e(F_t, R_t)}{\partial F_t} - e(F_t, R_t)\left(\frac{\partial e(F_t, R_t)}{\partial R_t}\right)^{-1}\frac{\partial^2 e(F_t, R_t)}{\partial F_t \partial R_t}\right)\right\} \\ & - \omega \frac{\partial e(F_t, R_t)}{\partial F_t} L(p_t^E, g_t)\,(\sigma - 1)\,[p_t^E]^{-1}\,[1 + \Theta]\,.\end{aligned}$$

The larger is the cross-partial $\partial^2 e(F_t, R_t)/\partial F_t \partial R_t$, the broader the conditions under which $\partial r_t/\partial Q_{t-1} < 0$. Also, via the bottom line, greater σ tends to favor $\partial r_t/\partial Q_{t-1} < 0$.

The partial derivatives of J_1 and J_2 with respect to τ are:

$$\begin{aligned}\frac{\partial J_1}{\partial \tau} =& 0, \\ \frac{\partial J_2}{\partial \tau} =& -\omega\,[w(p_t^E)]^{-1} < 0.\end{aligned}$$

It is easy to show that the effects of a resource tax are qualitatively similar to those of a supply restriction.