Imprecision as an Account of Violations of Independence

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ABSTRACT

In an earlier paper we showed that a different type of model based on imprecise preferences could account for different forms of preference reversal. In this paper we show that the same model can also explain the best-known violations of independence and betweenness. Thus it would appear that a simple model of imprecise preferences can account for a broader range of anomalies than any one of the more elaborate alternative theories developed to date.

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During the last three decades, hundreds of studies have been published which report a variety of seemingly systematic violations of expected utility (EU) theory. The weight and breadth of this evidence has inspired more than a dozen alternative decision theories (for a review see Starmer, 2000). However, no single model has been able to accommodate more than a subset of these patterns. For example, different rank-dependent expected utility (RDEU) models can account for violations of betweenness and independence, but cannot explain systematic intransitivity or the preference reversal phenomenon, while other models such as regret theory allow standard preference reversals and certain patterns of cyclical choice but cannot explain most of the violations of betweenness and independence[1].

This is a puzzle. After all, each violation has proved robust to frequent replication with samples that are much like one another: indeed, the different patterns can quite easily be reproduced using the same individuals and in the course of the same experimental session. Yet although the same kinds of people can be relied upon to exhibit the various patterns of behaviour, for some reason these people's behaviour does not seem to conform to any single reasonably parsimonious decision theory. What sense can we make of this?

One approach is to combine a deterministic 'core' preference model with some specification of error in the computation or execution of the underlying preference. In this tradition, the standard error specification takes a Fechnerian structure (Fechner 1860/1966) in which the less preferred option is more likely to be chosen, the closer together the core utilities of the choice objects are. However there are serious problems with this specification, as it predicts frequent violations of transparent dominance if the dominated option is very close in core utility to the dominated option, contrary to the evidence. Loomes (2005) elaborates on this and other problems with the ‘core theory plus errors’ approach in more detail.

Another possibility, suggested by psychology, is to explain the distinct violations in terms of different tasks prompting different heuristics. For example, the systematic disparity between valuation and choice evident in the preference reversal phenomenon may be attributed to people using an approach which anchors on payoffs to generate values, while placing relatively more weight on probabilities when making choices. Meanwhile, for tasks within the realm of pairwise choices, the same people may apply a different heuristic whereby they underweight some probabilities and overweight others in ways which produce the violations of betweenness and independence referred to above.

For the same people to produce the variety of different patterns reported in the literature, this type of explanation would require some sort of two-stage model, where the first stage entails the selection of a particular heuristic for a specific task, while the second stage involves the application of that particular heuristic to generate the non-EU behaviour in question. On this account, what we need is a kind of meta-theory of why/how so many individuals come to share the same ‘toolkit’ of heuristics and why/how they then concur so reliably in the ways they match different heuristics to different decision tasks. However, as far as we are aware, no meta-theory of this kind has yet been formulated.

Yet another possibility is that there ***is***, after all, a single model that predicts all of the well-established patterns, but that this model is not in the mainstream tradition of deterministic decision theories, where people operate as if according to some definitive set of principles or axioms. Instead, we might start from the (behaviourally much more plausible) premise that people’s preferences are often only rather loosely specified, entailing a good deal of ‘noise’ and imprecision, and consider the implications of such a proposition. Even if only minimal detailed assumptions are then made about the nature of noise, clear and refutable predictions can still be made.

In an earlier paper (Butler and Loomes, 2007), we showed how a simple model of imprecise preferences along those lines not only provided a good account of the ‘standard’ preference reversal phenomenon involving a systematic disparity between choice and certainty equivalent valuation, but also predicted the *opposite* asymmetry involving choice and probability equivalents – a novel prediction for data unobserved previously that was confirmed by the experiment we reported in that paper.

We emphasise again that although the model we presented in that earlier paper and in the current paper uses minimal assumptions, this does ***not*** mean it is therefore consistent with all choice patterns; quite the contrary, it makes clear, refutable predictions regarding which patterns of violations it would be consistent with and which would not. In the current paper, we consider whether an extension of our simple model would also predict systematic violations of betweenness and independence, and we report experimental data suggesting that this is indeed a possibility.

I. The Model

We take as our starting point an idea discussed in an unpublished paper by MacCrimmon and Smith (1986) – henceforth, M&S. Their concern at the time was to explain two forms of preference reversal: those involving certainty equivalents, and those involving probability equivalents. For our present purposes, we focus on the second of these.

Consider two binary lotteries: one offers a relatively large chance of a modest sum of money (and a residual chance of zero); the other offers a much smaller chance of a considerably larger prize (and a correspondingly larger chance of zero). Since Lichtenstein and Slovic (1971), these have come to be known respectively as the P-bet and the $-bet.

Examples of each bet can be represented diagrammatically as in Figure 1, where the vertical axis shows the probability of the positive payoff and the horizontal axis shows the size of that payoff. Figure 1 shows the pair of bets used in the preference reversal experiment in Butler and Loomes (2007), where the P-bet offered a 0.70 chance of 24 Australian dollars (A$) while the $-bet offered a 0.25 chance of A$80.

FIGURE 1 HERE

In that experiment, we replicated the usual preference reversal phenomenon by asking respondents to make a straight choice between the two bets and also eliciting from them their 'certainty equivalents' (i.e. the sure sums of money that they would regard as just as good as each bet). In addition, we asked them to identify the probability $\boldsymbol{q}$ of receiving A$160 which would make them indifferent between that prospect and each bet. The values of $\boldsymbol{q}$ identified by an individual are her 'probability equivalents' of the two bets.

In a deterministic model, each individual would have some 'true' value of $\boldsymbol{q}$ for each bet which would be revealed in a reasonably accurate and unbiased manner by a suitable elicitation procedure. According to all transitive models, the ordering of bets by their ***q*** should correspond with their ordering by direct pairwise choice (and, of course, by their certainty equivalents).

However, M&S considered how things might look in a world where respondents do not have such precise preferences. Giving a probability equivalence involves a mapping from each bet to a point on the right hand edge of the rectangle in Figure 1. But M&S suggested that if individuals do not have such precise preferences, it might be better to think of the equivalence as an interval rather than as a point: that is, a range of values of ***q*** for which someone finds it difficult to feel sure that the alternative on the edge is preferred or inferior to the stimulus being evaluated. What properties might such an 'imprecision interval' have?

To begin with, we may expect that transparent dominance will lead to a strong preference for the dominating option[2], so that the upper bound of the interval for the $-bet must be less than 0.25, while for the P-bet it must be less than 0.70. The lower bound in both cases would be zero. Although all individuals are likely to be able to narrow down their intervals further than this, some individuals may have sharper preferences than others, including a few who profess completely precise preferences. But what M&S suggested was that for most individuals, the interval was liable to become wider the further away the bet was from the relevant edge. The intuition is that greater dissimilarity between two prospects is likely to entail a greater degree of uncertainty about how one stands in relation to the other. Thus Figure 1 illustrates the case of an individual who is not exactly sure how a 0.25 chance of A$80 maps to some chance of A$160 in terms of his personal preferences, but feels confident that the probability equivalent for the $-bet lies somewhere between 0.1 and 0.2. At the same time, the greater difference between A$24 and A$160 makes it harder to judge the mapping for the P-bet, so his ‘imprecision interval’ spans a broader range – in this case, from 0.05 to 0.35.

Of course, this is just an example, and there is nothing in the basic model which requires that the P-bet’s broader interval should completely encompass the $-bet’s narrower one[3]. What M&S were primarily concerned to draw attention to was the broader span of the P-bet interval, and in particular the fact that its upper bound was far less constrained than that of the $-bet and could, as in Figure 1, exceed the limit imposed upon the $-bet by respect for dominance. The key point that follows from this is that when respondents are required by the elicitation task to nominate a single value of ***q*** from the interval as equivalent to the other bet, there is considerable scope for the probability equivalent of the P-bet to be higher than the probability equivalent of the $-bet, even for those who may have recorded a preference for the $-bet in the pairwise choice.

So the M&S model entails the possibility that there might be a much greater tendency for respondents to commit a reversal in the form of choosing the $-bet but placing a higher ***q*** on the P-bet than for the opposite reversal to occur whereby respondents choose the P-bet but attach a higher ***q*** to the $-bet. The same model also predicts the usual pattern of reversals for certainty equivalents, where the greater dissimilarity between the $-bet and the certainties on the top edge of the rectangle allows a broader interval for the $-bet *on that edge* and permits the ‘standard’ asymmetry. Thus their model allows that the same people will exhibit one form of asymmetry for probability equivalents and the opposite asymmetry for certainty equivalents. As Butler and Loomes (2007) reported, this is exactly what happened.

But can the basic intuitions underpinning the M&S model also accommodate behaviour which violates independence and betweenness? Although M&S made no claims in this regard, consider the Marschak-Machina triangle diagram in Figure 2 to see how their model might be extended. This diagram enables us to depict the kinds of lotteries most often deployed in tests of independence and betweenness: namely, lotteries involving combinations of up to three payoffs – x_1, x_2 and x_3, where $x_3 > x_2 > x_1$ (and where, in most cases, x_1 is set at 0). In this diagram, the vertical axis shows the probability of x_3 and the horizontal axis shows the probability of x_1, with the probability of x_2 being given by $1\text{-pr}(x_3)\text{-pr}(x_1)$. Hence the point on the hypotenuse labelled M_1 depicts a lottery offering a 0.8 chance of x_3 and a 0.2 chance of x_1, while M_5 represents a lottery involving a 0.2 chance of x_3 and a 0.2 chance of x_1, with a 0.6 chance of the intermediate payoff x_2.

FIGURE 2 HERE

Machina (1982) showed that an EU maximiser's preferences over such lotteries can be represented by indifference loci within the triangle that are linear and parallel, each sloping up from the south-west to the north-east and with the slope reflecting the individual's risk attitude (the steeper the slope, the more risk averse the individual). The most frequently reported violations of betweenness are consistent with convexity rather than linearity of those indifference loci, while the most frequently reported violations of independence can be represented by curves that are not parallel but 'fan out' as if from some point of convergence to the south-west of the right angle of the triangle. Could behaviour taking on this appearance be consistent with the intuition behind the M&S model of imprecise preferences?

Before we investigate this possibility, we should explain that the preference reversal data reported in Butler and Loomes (2007) suggested that the best guide to the width of an imprecision interval was the range of lotteries that separated weak dominance *of* the \$ (or P) bet from weak dominance *by* the \$ (or P) bet. Consequently we might ask whether *this* version of the imprecision model can offer an account for violations of independence and betweenness.[4]

Let us start with the case (which was one we investigated experimentally) where x_3 = A\$40, x_2 = A\$20 and x_3 = 0, and suppose that a respondent is asked to identify an equivalent lottery for each of M_1 to M_5 in turn – those equivalents being denoted respectively by L_1^* to L_5^* - on either the vertical or else the horizontal edge of the triangle. Suppose we begin with M_1, which can be written as (40, 0.8; 0, 0.2). Respect for transparent dominance requires that L_1^* must lie below (40, 0.8; 20, 0.2), labelled L_1^+ in Figure 2, but must lie to the left of (20, 0.8; 0, 0.2), labelled L_1^-.

Within that range, various possibilities may seem plausible to many respondents. Of course, different individuals may have different views about the location and width of the interval that seems most compatible with their own intuitions, but it would not be ridiculous to imagine an individual who is fairly confident that she would prefer L_1 to M_1 if L_1 were higher on the side edge than, say, (40, 0.55; 20, 0.45) and is equally confident that she would prefer M_1 if L_1 were lower down the side edge than, say, (40, 0.25; 20, 0.75) – which includes anywhere on the bottom edge between the corner and L_1^-. However, this leaves an interval on the side edge between 0.55 and 0.25 where the individual is less than sure about her preferences but from within which she is required to identify a point of equivalence. To keep the example simple, suppose that if a sample of people of this kind were each asked to pick some single point from the interval, they would, between them, generate a distribution of points, the median of which (let us say) happens to be the mid-point[5] of the interval, i.e. (40, 0.4; 20, 0.6). For the purposes of the current exposition, let us take this as the 'representative' response and label it L_1* in Figure 2.

Now consider M_4 – (40, 0.2; 0, 0.8) – and the corresponding analysis. Dominance entails L_4^+ = (40, 0.2; 20, 0.8) and L_4^- = (20, 0.2; 0, 0.8). The typical individual is, let us say, sure she would prefer L_4 if it offered anything better than (20, 0.6; 0, 0.4) and equally sure she would prefer M_4 if L_4 were worse than (20, 0.3; 0, 0.7). Suppose once again that the representative response is the mid-point of the interval – in this case, (20, 0.45; 0, 0.55), which we label L_4* in Figure 2.

These are only examples, of course, and their purpose is simply to illustrate how the intuitions behind the M&S model might be extended to the Marschak-Machina framework, and in so doing, to indicate the *potential* for accommodating behaviour within that environment which violates EU in the ways so frequently reported. Focusing just on M_1, L_1*, M_4 and L_4*, Figure 2 shows that the gradient of the slope joining M_1 and L_1* is 2, considerably steeper than the slope of 0.8 joining M_4 and L_4*, and consistent with the 'fanning out' pattern characteristic of many data sets.

For those readers who find the above examples too *ad ho*c and specific, an alternative, more general way of thinking about the model may be helpful. Consider first the lottery (40, 0.6; 20, 0.4) on the side edge which has the same *expected value* as M_1 and so marks the boundary between risk aversion and risk seeking. Four-fifths of the interval between L_1^+ and L_1^- lie below that boundary, indicating the scope for imprecision to favour equivalences for M_1 that show up as risk averse. For M_4, the lottery (20, 0.4; 0, 0.6) on the bottom edge is the one that has the same expected value. In this case, four-fifths of the interval between L_4^+ and L_4^- lie *to the left* of that lottery, so that imprecision would be more likely to pull equivalences for M_4 in the direction of risk *seeking*. The specific examples shown in Figure 2 are just particular cases of the general tendency, consistent with the body of past evidence, for individuals to give responses which look risk averse in the middle and upper part of the triangle but appear to be risk seeking in the bottom right hand area.

The role of imprecision in accounting for violations of betweenness may be explained in conjunction with Figure 3.

FIGURE 3 HERE

Consider the case where the values of x_3, x_2 and x_1 are such that the representative equivalent L_2* for M_2 is the lottery (x_2, 0.9; x_1, 0.1) on the bottom edge. If we suppose, as above, that this lottery is located at the mid-point of a typical interval of imprecision, and if that interval is, as in the earlier example, 0.3 wide, the upper bound of that interval is (x_3, 0.05; x_2, 0.95) while the lower bound is (x_2, 0.75; x_1, 0.25).

A straight line connecting L_2* to M_2 would have a gradient of 2 and pass through M_5, so that if betweenness held, the representative equivalent for M_5, denoted L_5*, would also be (x_2, 0.9; x_1, 0.1). However, in our example the interval of possible equivalences for M_5 cannot be the same as that for M_2: transparent dominance constrains the interval for M_5 to lie somewhere inside the range from L_5^+, which is (x_3, 0.2; x_2, 0.8), to L_5^-, which is (x_2, 0.8; x_1, 0.2). Thus that part of the interval of possible equivalences for M_2 which lies to the right of (x_2, 0.8; x_1, 0.2) is disallowed by dominance for M_5. On the other hand, no part of the upper bound of the interval for M_2 is disallowed for M_5; on the contrary, there is room *above* the upper bound of the interval for M_2 before L_5^+ is reached. Thus there is scope for the interval of possible equivalences for M_5 to be shifted clockwise relative to the interval for M_2, so that if L_5^* were to be in the middle of *this* interval, it would lie to the left of L_2^*. On this basis, it would appear that M_5 is strictly preferred both to M_2 and to (x_2, 0.9; x_1, 0.1), a result which would violate betweenness in the direction of convexity, as has often been reported – see, for example, Camerer (1995).

In a nutshell, the claims behind our version of the M&S model, applied to this context, are as follows:

a) Most individuals will be aware of a significant degree of imprecision in their preferences, leading in our context to an 'imprecision interval' for each M-lottery

b) Individuals will feel no imprecision in their preferences if an M-lottery dominates (L^-), or is dominated by (L^+), an L-lottery, constraining the location of an imprecision interval

c) The average size of these imprecision intervals will be proportional to the 'length' of the line of L-lotteries between L^+ and L^-

d) The L-lottery representing the expected value of an M-lottery may lie much closer to L^+ for some M, and closer to L^- for another M. This asymmetry will exert a 'pull' on the *location* of the imprecision interval, causing choices to appear more (less) risk averse in some evaluations than in others.

e) Choices will be distributed over the range of the imprecision interval, with the median choice close to the centre of the interval.

Point d) above suggests a deeper, unconscious form of imprecision, which allows the *location* of the imprecision interval to be tugged, upwards or downwards, into the 'space' available for imprecision to spill into (anchoring effects, if present, may be another example). A likely cause is that subjects need to construct a preference on each occasion, leaving them vulnerable to framing and other contextual effects of which they are unaware. For the preference reversal investigation in Butler and Loomes (2007), points a) through c) were sufficient to explain the key patterns, whereas for violations of independence we rely also on point d). Of course, the latter held for the preference reversal story also, but there the dramatic contrasts in both the ranges of the non-dominated lotteries and the positions of the dominant and dominated lotteries were sufficient to explain all the phenomena of interest.

To investigate how far the various possibilities outlined above are manifested in actual behaviour, we conducted an experiment. The next section describes the design and implementation of that experiment, and Section III reports the results and concludes with a discussion of the interpretation and possible implications of those results.

II. Design and Implementation of the Experiment

Because we were interested in the role of imprecision in explaining behaviour, we set out not only to identify respondents' stated preferences but also aimed to obtain some measure of the confidence with which those preferences were recorded.

The design was built around two Marschak-Machina triangles. These were the one discussed in the previous section, and another which was the same in every respect except that x_3 was set at A$60 rather than A$40. Respondents were allocated at random to one or other of the two triangles.

Our first objective, in Stage 1 of the experiment, was to get respondents to compare each of the fixed lotteries M_1-M_5 with a series of alternative L lotteries located on the vertical and horizontal edges of the triangle, and to identify the point at which they switched between the M lottery and the L alternative.

To illustrate how we did this, take the case where the fixed lottery was M_2 in the A$60 sub-sample: that is, it offered a 0.60 chance of A$60 and a 0.40 chance of 0. This lottery was presented on a computer screen as option A. The alternative, option B, was a lottery on the vertical or horizontal edge. For half of each sub-sample (again, determined at random), B was initially located on the vertical edge at L_2^+: so in this case, for that half of the sub-sample, B initially offered a 0.60 chance of A$60 and a 0.40 chance of A$20. For the other half of the sub-sample, B was initially located on the horizontal edge at L_2^-, offering a 0.60 chance of A$20 and a 0.40 chance of 0.

Respondents were then asked to respond in one of four ways, which we recorded on a 1-4 scale: if they “definitely preferred” option A, we coded it as 1; if they “probably preferred” A, a 2 was recorded; 3 signified “probably preferring” B; and a definite preference for B was coded as 4. (The instructions, available on request, explained the terms “definitely prefer” and “probably prefer” in more detail.)

To illustrate how this might be expected to work, consider first a respondent initially presented with a choice between A and L_2^+. Since B here dominates A, almost every respondent would signify a definite preference for B, coded as 4. Once the initial response had been recorded, the computer program changed B, making it two points worse: that is, displaying a lottery which offered (A$60, 0.58; A$20, 0.42) instead of the initial (A$60, 0.6; A$20, 0.4). The respondent was then asked again to state their preference and the confidence with which they held it. Thereafter, B was made progressively worse, so that it moved steadily down the vertical edge, reducing the chances of A$60 and increasing the chances of A$20, until it reached the corner (the certainty of A$20), after which point it moved along the horizontal edge until it eventually became L_2^-, where the procedure came to an end.

So for those starting on the vertical edge and initially recording 4's, there might come a point at which they indicated that they still chose B but no longer felt so sure, coded as 3. As B was degraded further, there would come a point at which the respondent switched from B to A: if this was initially a 'probable' preference for A, it was recorded as 2; when it became a definite preference for A, it was recorded as 1.

We refer to the treatment where B initially dominated A, and then was progressively degraded, as 'iterating down'. For the other half of the sub-sample, B was initially set at L_2^- and the program progressively improved it, moving it along the horizontal edge towards the corner, then up the vertical until it reached L_2^+. We refer to this treatment as 'iterating up'. Within a sub-sample, the same direction of iteration was used for all five fixed lotteries, the only difference being that for M_5 the iteration involved decrements or increments of one point at a time, rather than the two-point changes used for each of the four lotteries on the hypotenuse[6].

In this way, in the course of Stage 1 of the experiment, we elicited from each respondent not only their points of indifference between the M and L lotteries (the 2↔3 switch points) but also some indication of the intervals (between 1↔2 and 3↔4) over which they considered themselves to be less than sure about their preference[7]. We do not claim that this represents the same level of confidence (e.g. a 90% confidence interval) for different respondents, but only that whatever a particular respondent regarded as the point of transition between a 'definite' and a 'probable' preference in the case of one pair of lotteries would roughly correspond with that same respondent's judgment of their own confidence for the other pairs.

Of course, the overriding purpose of the experiment was to explore how far the M&S model or our modified version might be able to account for the body of earlier evidence about violations of betweenness and independence. However, the bulk of that body of evidence has taken the form of pairwise choice data, so it was important to see how the patterns yielded in Stage 1 by iterating through a succession of very similar pairwise choices would compare with the usual approach of asking respondents to make a number of separate one-off choices between a variety of predetermined pairs. Note these pairs were the same for all subjects and fixed in advance, so subjects could not affect their future choices by their responses in Stage 1.

To this end, Stage 2 of the experiment involved presenting each sub-sample with a set of 20 pairwise choices: 4 B's for each of M_1-M_5, with each B chosen to produce a particular gradient of the line connecting it to A, as shown in Figure 4 for the A$40 triangle. These gradients, which we shall denote by g_1 ... g_4, were as follows:

	g_1	**g_2**	**g_3**	**g_4**
When x_3 = A$40, the gradients were:	**1,**	**1⅔,**	**2½,**	**5**
When x_3 = A$60, the gradients were:	**½,**	**1,**	**2,**	**4**

FIGURE 4 HERE

To motivate respondents, the random lottery incentive system was used: i.e. respondents were told that at the end of the session one of their choices, selected at random, would be played out for real. We could find no way to make the distinction between 'definitely prefer' and 'maybe prefer' the same option incentive compatible. We doubt that such a mechanism exists. However, the patterns in the data generated by the iterative procedure strongly suggest subjects made a genuine effort to make their choices reflect their feelings. And as we shall see, the incentive-linked choices also displayed patterns consistent with the proposed model, suggesting that the absence of direct financial incentives for the iterative tasks is not a reason to dismiss the data about imprecision they generated. We elaborate on this point in more detail in Butler and Loomes (2007).

A total of 89 individuals from the University of Western Australia took part, mostly undergraduates from a variety of disciplines but also including a few postgraduates and academic staff. Verbal and on-screen explanations plus on-screen practice questions for both stages preceded the experiment proper. 45 participants were allocated at random to the A$40 triangle (of whom 23 iterated down and 22 iterated up in Stage 1) and 44 to the A$60 triangle (with equal numbers iterating in each direction). In both stages, the questions which are the primary focus of this paper were alternated with the preference reversal questions reported in Butler and Loomes (2007). Average earnings across the sample were A$26, ranging from a low of A$0 to a high of A$160, for about an hour of their time.

The main issues we hoped that the data would illuminate were as follows. First, do people typically have non-trivial imprecision intervals (i.e. between 1↔2 and 3↔4)? And if so, what determines the widths of these imprecision intervals? With reference to preference reversals, M&S had speculated that the widths of such intervals might be an increasing function of the distance between the position of the bet in the Figure 1 rectangle and the side of that rectangle upon which the response is recorded. But as noted in Section 1, our evidence had suggested a somewhat different possibility: namely, that the widths of the imprecision intervals were proportionally related to the ranges of un-dominated alternatives. The preference reversal data alone could not discriminate between the different conjectures, but the independence-betweenness design might, as we now explain.

If M&S's conjecture were correct, we might not only find the interval for M_5 markedly smaller than for M_2 or M_3, but might also find some differentiation between the four lotteries on the hypotenuse. As Figure 4 shows, all other things being equal, which distances are greater or smaller depends in part on the degree of risk aversion expressed in responses. In the case of risk neutrality, here represented by a gradient of 1, M_2 and M_3 would be (equal) furthest from their counterparts on the side and bottom edges, with M_1, M_4 and M_5 (though the latter is not shown in Figure 4) being an equal but lesser distance away. Whereas if there were high levels of risk aversion – corresponding, say, with a gradient of 5 – then the distances would be in descending order from M_1 to M_4, with the M_5 distance being equal to that for M_4.

The *caveat* 'all other things being equal' in the previous paragraph refers to the fact that all lotteries in a given triangle involve the same three payoffs (or some subset thereof). But we were also interested to see if there were any differences in interval widths *between* the two triangles. In particular, we speculated that if we held the probability distributions constant, the bigger spread of payoff differences in the case where x_3 = A$60 might entail greater imprecision: that is, trying to balance the utility difference between A$60 and A$20 against the utility difference between A$20 and 0 might involve greater imprecision than the 'tighter' task of balancing the utility difference between A$40 and A$20 against the utility difference between A$20 and 0.

However, our principal interest was to see whether the locations of any imprecision intervals and the means of the 2↔3 switch points were consistent with our account of ways in which 'fanning out' as well as violations of betweenness might be generated by imprecision.

III. Results

We begin with the Stage 2 pairwise choice data. Table 1 reports, for each triangle and each gradient, the numbers of respondents who chose the riskier M lottery. These choices were interspersed with the choices investigating preference reversals, and in a random order (but fixed for all subjects).

TABLE 1 HERE

Reading *down* the rows, things were much as virtually every model would lead us to expect: as the gradient increased, the L lottery became less favourable, and more respondents chose M. The only exception to this was M_1 in the A$40 triangle, where more respondents chose M_1 when the gradient was 1 than when the gradient was either 1⅔ or 2½. This is a case for which we have no explanation except chance aberrations. Further evidence that the top left cell *was* aberrant comes from comparing that whole g_1 row with the g_2 row when x_3 = A$60. Since the gradient was 1 in both cases, the M lotteries should have been chosen by more respondents in the A$60 sub-sample. This was what happened for M_2-M_5, where in each case the number of M choices was about two or three times higher; but not for M_1. Still, with the exception of that top left cell, the data look much as might be expected according to EU and all the main non-EU alternatives.

However, when we read *along* the rows from left to right, we find the kind of patterns consistent with the ‘usual’ departures from EU. There is not much to see in the top row for each triangle, but for the other three gradients in each triangle, there was a clear trend for the numbers of M choices to increase as the lotteries move towards the bottom right hand corner. A within-subject analysis of those choices shows that the numbers choosing L_1 and M_4 outnumbered those choosing M_1 and L_4

to an extent that was significant at the 1% level in all six comparisons (using a McNemar exact binomial test). This is consistent with a significant degree of fanning out. Indeed, 45 of the 89 subjects satisfied *strict* fanning out, while just two satisfied strict fanning in.

Violations of betweenness were also in evidence, directly and indirectly. In the case of the A\$60 triangle, there were two direct tests. When the gradient was ½, the straight line joining M_3 to its L counterpart passed through M_5, and when the gradient was 2, the straight line joining M_2 to *its* L counterpart also passed through M_5. In the first of these cases, there were very few M choices at all, and the fact that 10 chose M_5 over L as opposed to 6 choosing M_3 over the same L is not a statistically significant difference. However, as Table 1 reports, when the gradient was 2, 32 out of 44 respondents chose M_5 as opposed to just 21 who chose M_2, and the asymmetry in the direction consistent with convex indifference curves was significant at the 1% level.

In the A\$40 triangle, the tests were less direct, but the results pointed to the same conclusions. Consider first M_2 and the straight line with gradient 1⅔ linking it to $L_2 = (x_2, 0.96; 0, 0.04)$. This line passes just to the left of M_5 – it goes through $(x_3, 0.2; x_2, 0.64; 0, 0.16)$ – but there is little space between it and the straight line of the same gradient joining M_5 to $L_5 = (x_2, 0.92; 0, 0.08)$. Yet there were 22 respondents choosing M_5 over that L_5 as opposed to just 9 choosing M_2 over the corresponding L_2, with the within-subject asymmetry registering as significant at the 1% level. In case this might be attributed to some very acute fanning out in that thin slice of the triangle, consider M_2 and the gradient 2½ which links it to $(x_2, 0.84; 0, 0.16)$. This straight line passes to the *right* of M_5 through $(x_3, 0.2; x_2, 0.56; 0, 0.24)$ – that is, by the same distance to the right that the previous line passed it to the left – so that any fanning out effect while maintaining linearity might be expected to favour M_2 more than M_5. But once again Table 1 shows that M_5 was chosen much more often – by 31 as opposed to 17 respondents; and once again the asymmetry was significant at the 1% level.

Overall, then, when viewed from the perspective of deterministic models, the patterns of choice in Stage 2 appear entirely consistent with a model of convex indifference curves fanning out as if from some point to the south-west of their respective triangles. But how far do such patterns also show up in the Stage 1 data? And to what extent do they appear explicable by the sorts of propositions about imprecision discussed earlier?

Table 2 shows the data analogous to those in Table 1, but this time drawn from individuals' responses to the Stage 1 iterative procedure. The one additional complication is that with the Stage 1 procedure we may occasionally observe the 2↔3 switching point coincide with the L lottery. In such cases, we have counted this as 0.5 of a choice of each option. Of necessity, the data in Table 2 must be more regular than those in Table 1 when it comes to reading down the columns, with at least as many choices of M at steeper gradients as at shallower ones. However, the important issue is the patterns across the rows. And as far as fanning out is concerned, the picture here is even sharper than it was in Table 1: for all four gradients in both triangles, the differences between M_1 and M_4 patterns of choice are significant at the 1% level.

TABLE 2 HERE

The picture is not quite so sharp with respect to violations of betweenness. Making the same comparisons as in Table 1, all four disparities were in the direction consistent with convex indifference curves. However, once again there were relatively few M choices in the A$60 triangle when the gradient was ½, so that the difference (8.5 of 44 against 7 of 44) was not statistically significant. By contrast, when the gradient was 2, the asymmetry (35.5 of 44 against 20 of 44) was again significant at the 1% level. Meanwhile in the A$40 triangle, the two comparisons between M_2 and M_5 when the lines from M_2 with gradients 1⅔ and 2½ pass either side of M_5 produced one difference that was significant at 10% and another that just failed to be significant at that level.

However, with the iterative procedure we are not confined to looking just at particular choices: with these data we can not only examine the behaviour of the 2↔3 switch-points, but also the widths and locations of the intervals of imprecision around those points. When reporting these results, we shall refer to the mean index values of the various switching points. These index numbers can be best understood with reference to the triangle as follows: the top left point has the value 100 and the numbers fall to 0 at the right angle, and then become progressively more negative as we move along the bottom edge, with the bottom right corner taking the value -100.

Table 3 reports the mean switch-points for all M lotteries in both triangles, as well as (in bold) the implied gradients of straight lines connecting the M lotteries to their respective mean switch-points. The mean intervals between the 3↔4 and the 1↔2 switch-points are also computed. What do these data show?

TABLE 3 HERE

We begin by considering the lotteries on the hypotenuse of both triangles. The gradients from these M's to their 2↔3 switch-points get progressively less steep as we go from M_1 to M_4: in the A$40 triangle, the gradients fall from 3.25 to 0.68, while in the A$60 triangle, the corresponding fall is from 2.89 to 0.56. This pattern of strict fanning-out corresponds very well with the patterns of choice reported in Tables 1 and 2.

However, the data also indicate how this might be explained by our model. First, though, it should be noted that one of the M&S conjectures is *not* borne out. Recall that M&S had speculated that the widths of the imprecision intervals might be an increasing function of the distance between the fixed lottery and the edge upon which the equivalence response is recorded. If we use the length of the straight lines connecting M_1-M_4 to their respective 2↔3 switch points as a rough estimate of that distance[8], then in both triangles M_4 would be closest to the relevant edge, followed by M_3, then M_1, with M_2 furthest away, these last distances being between 75% and 100% greater than those for the respective M_4's. But the widths of the intervals between 1↔2 and 3↔4 did not follow that pattern. Rather, as Table 3 shows, within a given triangle all four interval widths were very similar and there were no significant differences between any two of them. Moreover, in relation to the intervals between L^+ and L^-, the positions of the imprecision intervals were remarkably stable, as Table 4 shows.

TABLE 4 HERE

For M_1-M_4 in the A$40 triangle, the range over which the M lottery is definitely preferred lies between 20.4 and 27.6 points of the dominated L, while the range over which the L lottery is definitely preferred lies between 50.7 and 56.3 points of the L which dominates M. So as we go from M_1 to M_4 and as the positions of L^+ and L^- and the 100-point ranges between them shift, so too do the positions of the intervals between 1↔2 and 3↔4 and the 2↔3 switch-points. The A$60 triangle exhibits similar behaviour, except that, with x_3 being larger, the ranges over which M is definitely preferred are wider and the ranges over which L is definitely preferred are narrower. As we conjectured might be the case, increasing x_3 while keeping x_2 and x_1 constant *did* increase the widths of all of the intervals of imprecision somewhat; but the positions of those intervals within the L^+ to L^- range were as stable for the A$60 triangle as for the A$40 triangle.

The finding that the widths of the imprecision intervals are more a function of the L^+ to L^- range than of the distance from an M to the equivalence edge is given further support by the data relating to M_5. Table 4 reports the actual intervals in the bottom row; but just above, in the row labelled (M_5), these are converted to percentages to make them comparable with the M_1-M_4 data. This shows that, as proportions of the relevant L^+ to L^- range, all of the imprecision intervals within the same triangle are of much the same magnitude: 21-23 percentage points for the A$40 triangle, 25-28 percentage points for the A$60 triangle. It also turns out that these intervals are of a similar magnitude to those found for the preference reversal study: 24 points for the certainty equivalents of the $-bet and P-bet; 28 points for the probability equivalents, when converted to percentages of the non-dominated range.

What the M_5 row also shows is a tendency for the position of the imprecision interval to be shifted somewhat relative to its position for M_1-M_4: with M_5, a relatively larger proportion of the range is associated with a definite preference for M and a correspondingly smaller proportion represents a definite preference for L. This is in line with our conjecture that for M_5 the imprecision interval would be pushed in a clockwise direction, producing an effect that looks like a violation of betweenness consistent with convex indifference loci.

Figures 5 and 6 depict the overall patterns of responses in terms of lines from the M lotteries to their respective 2↔3 switch points. For simplicity, these are shown as straight lines, but the fact that the M_5 line has a shallower slope than might be extrapolated from its position relative to M_2 and M_3 – and indeed, the fact that in the A$60 triangle the M_5 line actually crosses the M_2 line – suggests that if one were to wish to impose an indifference map of the kind typical of deterministic theories, the best fit would be one which involved curves that are convex near the bottom edge and fanning out from the south-west of the triangle: that is, the kind of configuration which some RDEU models are able to generate.

IV. Concluding Remarks

The experiment reported in this paper focused on violations of independence and betweenness among lotteries involving just three payoffs. The model we have proposed has the potential to explain a broader range of phenomena than those examined in our experiment, such as articulating a certainty equivalent for some holiday abroad, or a plasma TV; however we focus on applying it to a simple good here.

Of course, it might be argued that the departures from conventional wisdom discussed in this paper can be explained just as well by one of the class of RDEU models. Against that, there are two counterarguments.

First, while RDEU models may be *technically* capable of accommodating these patterns, they could be regarded as behaviourally implausible: in particular, the process of converting probabilities into the decision weights required to fit the data is one which is quite complex (as anyone who has tried to teach the notion to university students will know). By contrast, the imprecision model is behaviourally very simple

– indeed, for some theorists' tastes has, if anything, too *little* structure. Despite its very modest assumptions, it can make refutable- but not refuted- predictions.

Second, the imprecision model can explain other phenomena – most notably, the two opposite forms of the preference reversal phenomenon described in Butler and Loomes (2007) – which RDEU models simply cannot deal with. While it may be that ultimately no single model can be expected to account for all behaviour in all contexts, it does seem reasonable to expect one model to capture the key phenomena generated by the same subjects performing similar tasks in the course of a single experiment. This consideration alone rules out the 'core theory plus errors' approach as a possible explanation for our results. Thus a relatively simple descriptive model of imprecision is able to accommodate a variety of 'anomalies' that have defied capture by any one of the many alternative deterministic models developed to date.

Figure 1: Illustrating Imprecision – Probability Equivalence

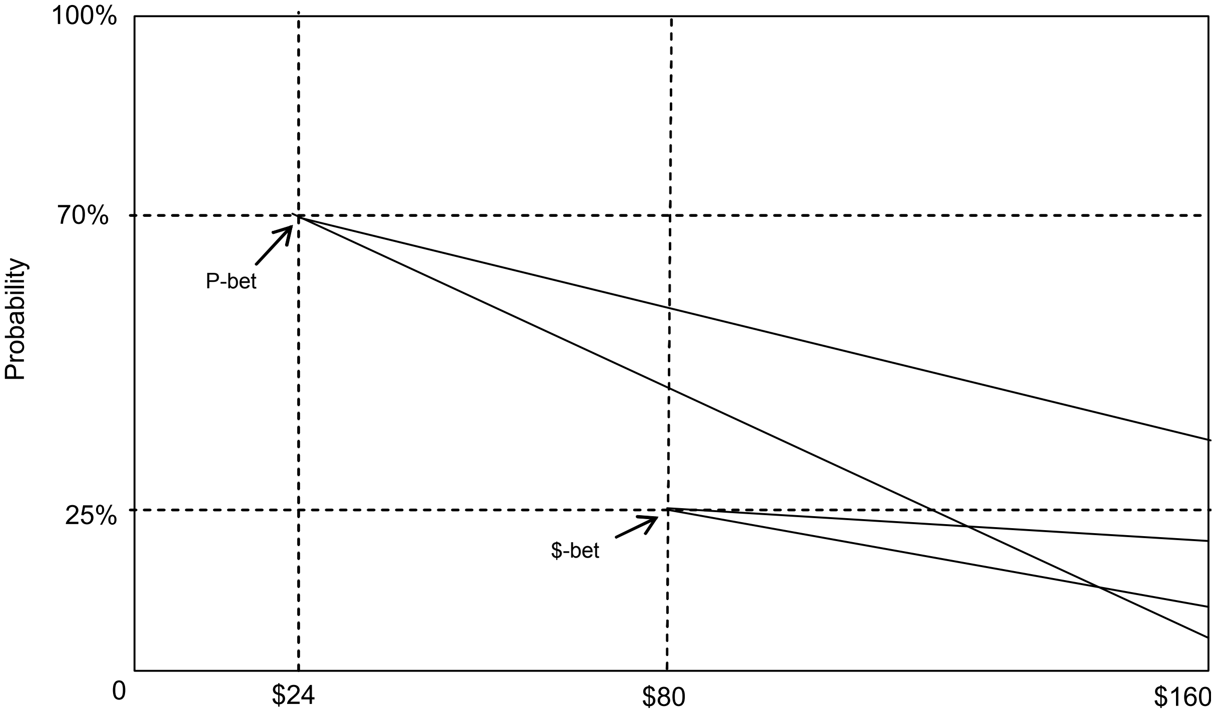

Figure 2: Illustration of Fanning Out Consistent with the Model

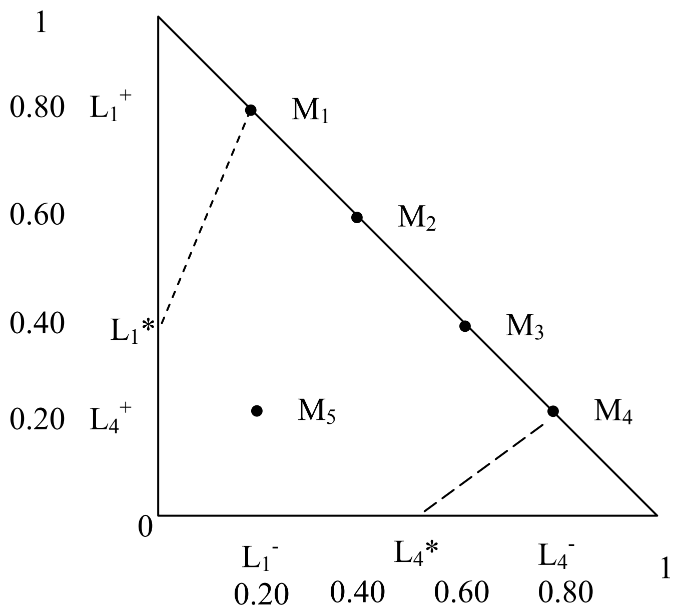

Figure 3: Illustration of Violation of Betweenness

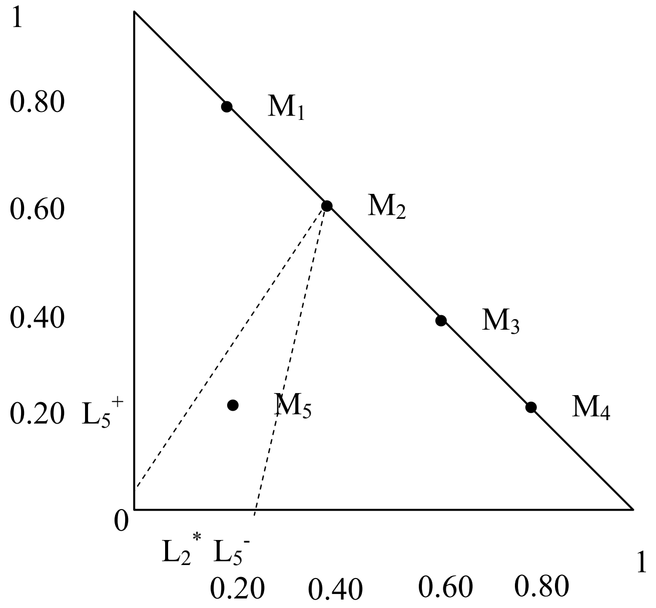

Figure 4: The Gradients Used For Pairwise Choices in A$40 Triangle

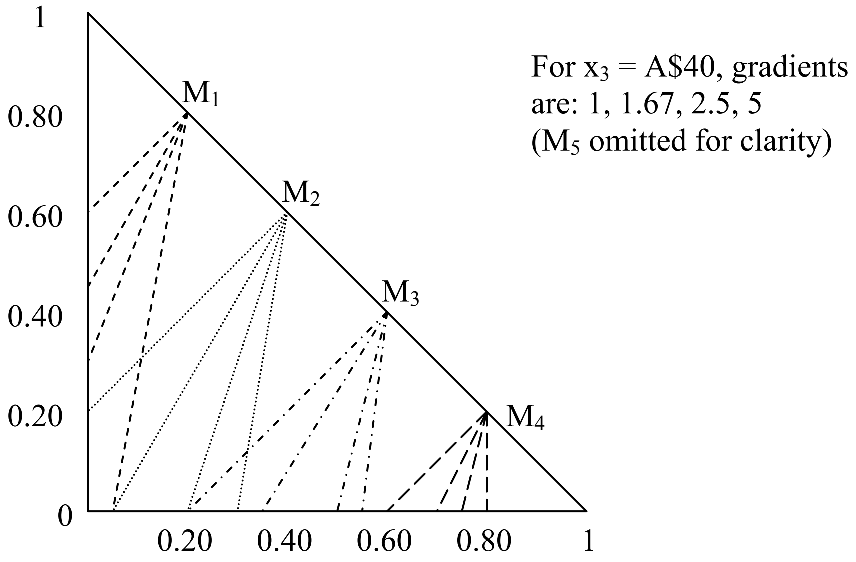

Figure 5: The Fitted Lines from the 2↔3 Switch-points in A$40 Triangle

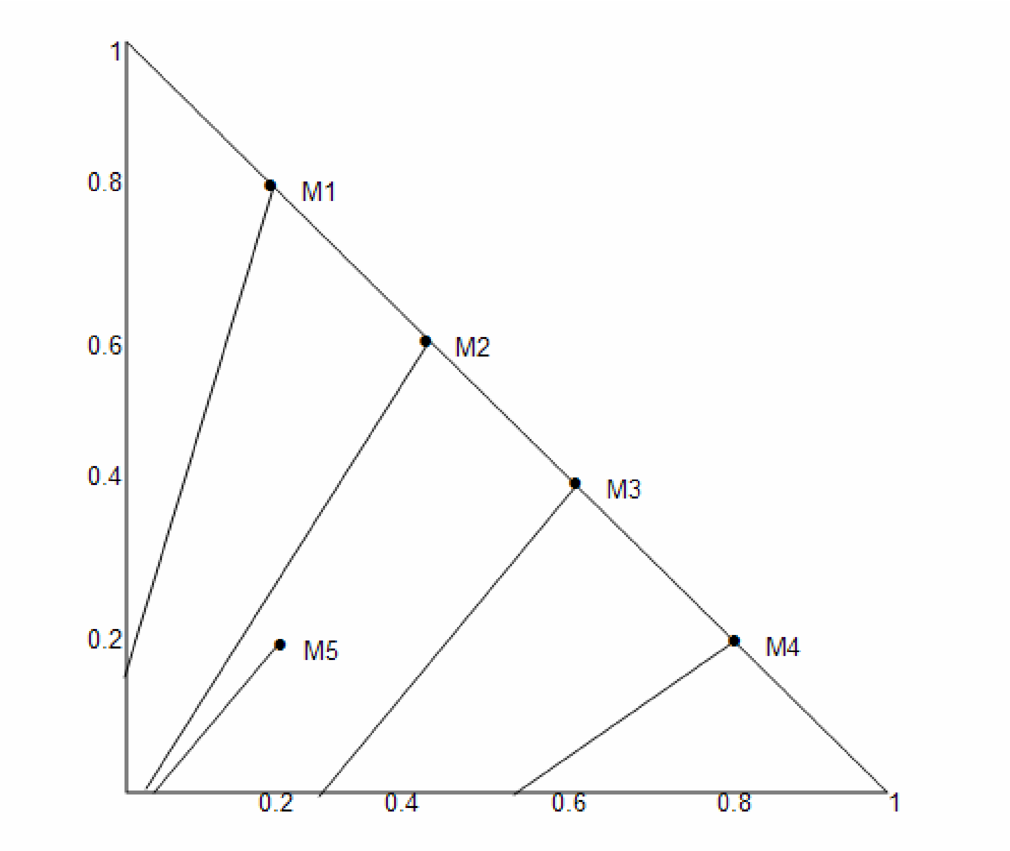

Figure 6: The Fitted Lines from the 2↔3 Switch-points in A$60 Triangle

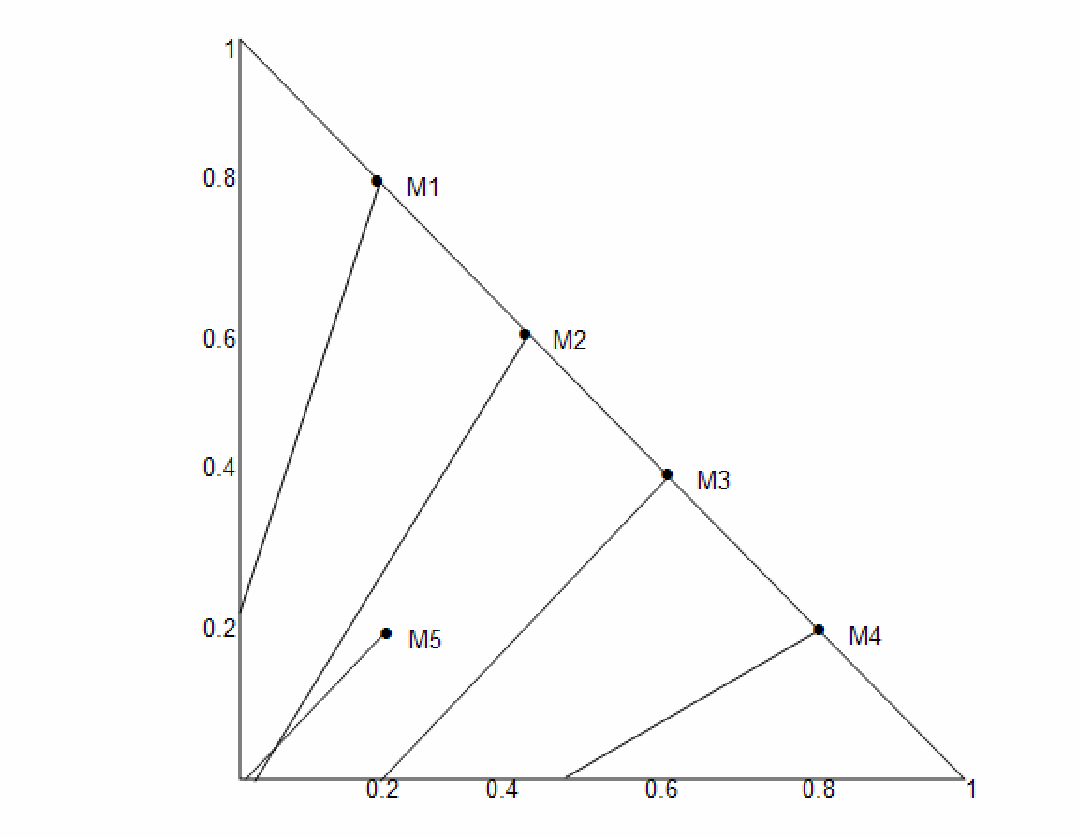

Table 1: Numbers of M Choices in Stage 2

	M_1	M_2	M_3	M_4	M_5
$x_3 = \text{A}\$40$					
g_1 (=1)	12	5	8	8	13
g_2 (=1⅔)	8	9	18	25	22
g_3 (=2½)	11	17	29	36	31
g_4 (=5)	26	34	38	43	36
$x_3 = \text{A}\$60$					
g_1 (=½)	5	2	6	4	10
g_2 (=1)	8	11	18	25	24
g_3 (=2)	17	21	34	36	32
g_4 (=4)	26	35	36	41	33

Table 2: Numbers of M Choices Inferred from Stage 1

	M_1	M_2	M_3	M_4	M_5
$x_3 = \text{A}\$40$					
g_1 (=1)	3.5	12	14	27	15.5
g_2 (=1⅔)	7	18.5	30	38	26
g_3 (=2½)	13	25	35	41	32
g_4 (=5)	30.5	40	43.5	44	36.5
$x_3 = \text{A}\$60$					
g_1 (=½)	1.5	3	7	18	8.5
g_2 (=1)	4.5	13	20.5	34	20
g_3 (=2)	10.5	20	36.5	40	35.5
g_4 (=4)	31.5	36	41	43	41

Table 3: Mean Switch-points, Gradients and Imprecision Intervals

	M_1	**M_2**	**M_3**	**M_4**	**M_5**
$x_3 = \mathrm{A}\$40$					
3↔4 switch-point	27.07	9.33	-13.07	-36.27	1.60
3↔4 **gradient**	**2.65**	**1.27**	**0.85**	**0.46**	**0.92**
2↔3 switch-point	15.02	-3.29	-26.04	-50.58	-3.96
2↔3 **gradient**	**3.25**	**1.63**	**1.18**	**0.68**	**1.25**
1↔2 switch-point	5.91	-12.36	-34.18	-59.60	-7.69
1↔2 **gradient**	**3.70**	**2.17**	**1.55**	**0.98**	**1.62**
1↔2 to 3↔4 interval	21.16	21.69	21.11	23.33	9.29
$x_3 = \mathrm{A}\$60$					
3↔4 switch-point	35.18	15.18	-4.73	-27.32	4.80
3↔4 **gradient**	**2.24**	**1.12**	**0.72**	**0.38**	**0.76**
2↔3 switch-point	22.27	-1.55	-19.50	-44.59	-1.11
2↔3 **gradient**	**2.89**	**1.56**	**0.99**	**0.56**	**1.06**
1↔2 switch-point	9.36	-10.09	-31.95	-54.77	-6.64
1↔2 **gradient**	**3.53**	**2.01**	**1.43**	**0.79**	**1.50**
1↔2 to 3↔4 interval	25.82	25.27	27.23	27.45	11.43

Table 4: Widths of 'Definite' and 'Imprecise' Intervals

	$x_3 = \mathrm{A}\$40$				**$x_3 = \mathrm{A}\$60$**		
	L^- to 1↔2	1↔2 to 3↔4	3↔4 to L^+		L^- to 1↔2	1↔2 to 3↔4	3↔4 to L^+
M_1	25.9	21.2	52.9		29.4	25.8	44.8
M_2	27.6	21.7	50.7		29.9	25.3	44.8
M_3	25.8	21.1	53.1		28.1	27.2	44.7
M_4	20.4	23.3	56.3		25.2	27.5	47.3
(M_5)	(30.8)	(23.2)	(46.0)		(33.4)	(28.6)	(38.0)
M_5	12.3	9.3	18.4		13.4	11.4	15.2

(All figures rounded to one decimal place)

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Footnotes

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[1] Regret theory *can* account for some violations of independence such as the 'common ratio effect', but only by assuming statistical independence between the alternatives (see Loomes and Sugden, 1982). However, many experiments have shown that even when the juxtaposition of payoffs is such that regret effects are controlled for, the common ratio effect continues to be manifested to a degree that regret theory cannot account for.

[2] Tversky and Kahneman (1986) have shown that dominance may be violated by a substantial proportion of a sample if the dominance relation is difficult to detect. However, when dominance is transparent – as in the cases we shall be concerned with – it is rarely violated.

[3] And certainly there is nothing to suggest the intervals are bounded by straight lines from the bets to the edges of the rectangle: curves are behaviourally more plausible, but straight lines simplify the diagram.

[4] Blavatskyy (2007) uses a distinct but related argument to constrain the distribution of *random errors in execution* around a model of *deterministic* expected utility preferences. So a lottery with a utility close to the maximum consequence is then more likely to be undervalued by errors, with the opposite prediction for a lottery with low utility, leading to fanning-out. Despite the apparent similarities, his approach is of the 'core theory plus errors' school which we avoided for reasons given elsewhere in this paper. See also Loomes (2005).

[5] The data in fact show a slight asymmetry in the average size of the imprecision intervals above and below the 2↔3 switch point, of between 0 and 3 percentage points, so preferences are slightly more risk-averse than the mid-point of the interval would imply. However this does not affect the thrust of the argument given here.

[6] This was intended to reduce the difference between the procedure used for M_5 and that used for the other four lotteries, although some disparity remained: i.e. the M_5 procedure involved only 40 changes of B rather than the 50 changes involved in the course of eliciting responses for M_1-M_4.

[7] A respondent who felt no sense of uncertainty could, of course, switch from 4 to 1 (or vice-versa) without ever recording either 2 or 3. A few (male) subjects consistently did just this.

[8] We are not ruling out that the indifference loci might, in fact, be convex; but moderate convexity would not alter the general result.

Acknowledgements

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