Ambiguity aversion solves the conflict between efficiency and incentive compatibility

PRELIMINARY AND INCOMPLETE VERSION

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Abstract

The conflict between Pareto optimality and incentive compatibility, that is, the fact that some Pareto optimal (efficient) allocations are not incentive compatible is a fundamental fact in information economics, mechanism design and general equilibrium with asymmetric information. This important result was obtained assuming that the individuals are expected utility (EU) maximizers. However, a huge literature in decision theory criticize expected utility preferences and propose alternative models. A natural question arises: is the previously mentioned conflict still valid for non-expected utility preferences? In this paper, we departure from the standard setup of EU preferences and show how to model asymmetric information with the maximin expected utility (MEU) of Gilboa and Schmeidler. In the MEU model of an asymmetric information economy, we are able to show that if an allocation is efficient then it is incentive compatible. We also show that the MEU framework provides more efficiency than the standard EU. Based on the MEU, new notions of value, core and equilibrium are introduced, which are always efficient and incentive compatible.

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1 Introduction

One of the fundamental problems in equilibrium theory with asymmetric information is the conflict between efficiency and incentive compatibility. That is, there are allocations that are efficient but not necessarily incentive compatible. This important problem was alluded to in early seminal works by Wilson (1978), Myerson (1979), Holmstrom and Myerson (1983), Myerson and Satterthwaite (1983) and Prescott and Townsend (1984).

Since then, the efforts to partially solve this conflict were central themes in mechanism design and general equilibrium with asymmetric information.[1] In this last field, the key condition which has been used to solve the alluded conflict is the private information measurability of allocations, that is, it is required that allocations are measurable with the respect to the sigma algebra that the private information partition of each agent generates.[2] Such a condition ensures that private information measurable ex-ante efficient allocations are indeed incentive compatible, and in the one good per state it characterizes the set of incentive compatible allocations. It is useful to understand why measurability was used to solve the problem of the conflict between efficiency and incentive compatibility. If an agent accepts to trade a non-measurable contract, this means that the contract makes promises depending on conditions that she cannot verify. Therefore, other agents may have an incentive to cheat her and do not deliver the correct amount in those states. This possibility is exactly the failure of incentive compatibility. To the contrary, if she insists to trade only measurable contracts (allocations), then she cannot be cheated and incentive compatibility is preserved.

However, the requirement of private information measurability has two main concerns. First, it is an exogenous, theoretical requirement, that it may be difficult to justify in real economies. The second, more important point is that the measurability restriction may lead to reduced efficiency and in certain cases even to no-trade situations, as we illustrate in section 3. Therefore, on the one hand, this restriction implies incentive compatibility, but on the other hand, it reduces efficiency. This poses the following question. Is it possible to introduce a new ex-ante efficiency concept which gives higher utility than the private information efficiency, but preserves incentive compatibility?

The answer comes from a reexamination of the problem. When we assume that the information is described by a partition, it is natural to think that the standard

[1] Although our results are also relevant to mechanism design, as we will discuss later, our main focus will be in general equilibrium with asymmetric information.

[2] See Yannelis (1991) and Krasa and Yannelis (1994)). We will use partitions to describe the private information of the agents. This approach was introduced by Radner (1968). However, equivalent analysis go through using the Harsanyi (1967-8)'s type model, as we discuss in section 7.

expected utility preference is defined only for allocations that are measurable with respect to that partition. How could an expected utility decision maker integrate an allocation that is not (private information) measurable? If the integral and, therefore, the expected utility is not defined, then the individual cannot evaluate how good such an allocation is. In other words, the preference of an expected utility individual in an asymmetric information economy should be viewed as *incomplete*, because the individual can compare only privately measurable acts.[3]

This observation is extremely simple, but allows for a new reinterpretation. Indeed, once we recognize that the problem is that the preferences are incomplete, then we have to consider ways in which the individual can complete her preferences. Although there are alternative ways to make this completion, as we discuss in section 2.1, we focus in a specific solution: the maximin criterion. This criterion was discussed by Rawls (1971) and axiomatized by Milnor (1954) and Barbera and Jackson (1988). In its original form, it is viewed as coming from "games against nature" (see Milnor (1954) and Luce and Raiffa (1989)). Later, Gilboa and Schmeidler (1989) provided an axiomatization of a *more permissive* class of preferences, that they called maximin expected utility.[4] The maximin criterion simply says that whenever the individual needs to evaluate a non-measurable allocation, she considers only the worst state that can occur in each element of the partition.

Of course, the maximin criterion can be seen both from a normative and from a descriptive point of view. From a normative perspective, our results show that the maximin criterion provides good outcomes for the individuals that adopt it, thus justifying its use. Moreover, our theory shows that when agents have this kind of preferences, they reach better (more efficient) outcomes because they will be able to trade in situation that agents with standard expected utility preferences will not trade. This result suggests that individuals obeying the maximin criterion will be more successful in the long run.[5] As a descriptive advantage, the maximin criterion rationalizes standard choices as in the Ellsberg paradox (see section 2.1).

Once individuals complete their preferences using the maximin criterion, we show that Pareto optimal outcomes will always be incentive compatible (see our Theorem 4.1). The intuition for this is clear: since they already consider that they will receive the minimum in each situation, no agent can be cheated and all alloca-

[3]Most models do work with complete preferences, by assuming that the probability measure representing the agent's beliefs is defined for all events that are measurable with respect to some finer σ-algebra, not with respect to the σ-algebra that represents the individual's private information. We will discuss this in more details in section 2.1.

[4]See footnote 25 for the difference between our criterion and Gilboa and Schmeidler's maximin expected utility.

[5]This observation can be potentially used to provide an evolutionary justification for these preferences.

tions will be incentive compatible.

The above solution of the conflict between ex-ante efficiency and incentive compatibility, has several interesting and rather surprising implications in equilibrium theory with asymmetric information. First by applying the new maximin expected utility formulation to the core, value and Walrasian equilibrium with asymmetric infromation, we are able to come up with new solution concepts which result in ex-ante efficient allocations which are also incentive compatible.

Second, the free disposal assumption on the feasibility of allocations which created problems for the achievement of incentive compatibility for the standard expected utility setting, is no longer a problem for the maximin formulation. This enables one to avoid complicated arguments needed to bypass the problem that the free disposal creates in the standard expected utility set up as alluded by Radner (1968) (see also Podczeck and Yannelis (2008)).

Finally, the new value, core and Walrasian equilibrium notion yield superior outcomes in terms of efficiency than standard EU and furthermore, the existence of the above notions becomes immediate as we indicate in section 6.

The paper is organized as follows. In section 2.1 we review the famous Ellsberg's paradox and establish a connection between the asymmetric information and the ambiguity aversion literature. Our approach also offers an interpretation of the Ellsberg's paradox that is slightly different from the current one. From this discussion, we are able to explore in more detail our interpretation of incompleteness of the preferences and the connection of this interpretation with the concept of "large worlds" discussed by Savage (1972) and Binmore (2008). Section 2 describes the model, while section 3 discuss various notions of efficiency and incentive compatibility and introduces our notions, based in the MEU preference. In section 4 we present one of the main results, that is, any maximin Pareto optimal allocation is incentive compatible. Section 5 proves that maximin preferences lead to higher efficiency and illustrates this with a comparison between perfect risk sharing and maximin allocations. The value allocation (that comes from a cooperative game) is considered in section 6. The translation of our results to the Harsanyi's type model is presented in section 7. Section 8 includes a review of the literature and discusses other issues. Open questions and concluding remarks are collected in section 9.

2 Economic Model

We consider an economy with differential information, with the partition model.[6] The set of individuals is $I = \{1, \ldots, n\}$. The uncertainty is modeled by a finite set

[6]This model was first introduced by Radner (1968). The modeling of private information by means of partitions seems to be inspired by the work of Radner's advisor, Leonard J. Savage.

of states of nature, Ω. None of our results depend on the fact that Ω is finite, but this assumption is useful to avoid unnecessary technical complications.

The economy is formally described by $\mathcal{E} = \{\Omega, (u_i, \mathcal{F}_i, e_i, \mu_i)_{i\in I}\}$, where the superscript P (for partition) may be omitted and, for each $i \in I$:

1. $u_i : \mathbb{R}^\ell_+ \to \mathbb{R}$ is i's utility function, which is monotonic in all coordinates;[7]

2. $\mathcal{F}_i$ is an algebra generated by a finite measurable partition of Ω, which denotes i's private information;[8]

3. $e_i \in \mathcal{L}_i$, where $\mathcal{L}_i$ is the set of all $\mathcal{F}_i$-measurable functions $f : \Omega \to \mathbb{R}^\ell_+$.[9]

4. $\mu_i : \mathcal{F}_i \to [0, 1]$ is a measure representing i's prior;

Let $\mathcal{F}_i(\omega)$ denotes the element of the partition $\mathcal{F}_i$ that contains ω. We will assume the following:

Assumption 2.1 *For each $i \in I$ and $\omega \in \Omega$, we have $\mu_i(\mathcal{F}_i(\omega)) > 0$.*

The following assumption is trivial, since in the case $\cap_{i\in I}\mathcal{F}_i(\omega)$ is not unitary, we can collapse (treat as equal) all states in it.

Assumption 2.2 *For each $\omega \in \Omega$, $\cap_{i\in I}\mathcal{F}_i(\omega) = \{\omega\}$.*

The following standard concepts will be used below. An *allocation* is a profile $(x_i)_{i\in I}$ such that each x_i is a function $x_i : \Omega \to \mathbb{R}^\ell_+$. An allocation $(x_i)_{i\in I}$ is *feasible* if $\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega)$, for all $\omega \in \Omega$. For each agent $i \in I$, $z_i = x_i - e_i$ denotes the *net trade* of agent i.

2.1 Incompleteness and the Ellsberg's Urn Example

One of the objectives of this paper is to connect two separate literatures, the ambiguity aversion (decision theory) and the asymmetric information literatures. In doing so, we use insights of both literatures that help to illuminate the issues. It is useful to consider the thought experiment proposed by Ellsberg (1961), which

[7] All of our results hold for state-dependent utilities, that is, utility functions of the form $u_i : \Omega \times \mathbb{R}^\ell_+ \to \mathbb{R}$, provided that $\omega \mapsto u_i(\omega, x_i)$ is $\mathcal{F}_i$-measurable for every $x_i \in \mathbb{R}^\ell_+$.

[8] In some parts of the paper, we are going to abuse notation and definitions, and refer to $\mathcal{F}_i$ both as algebra and as partition. Since there is an one-to-one correspondence between the partition and the algebra generated, this will not cause confusion.

[9] A function $f : \Omega \to X$ is $\mathcal{F}_i$-measurable if $f(\omega) = f(\omega')$ for every $\omega \in \mathcal{F}_i(\omega')$, where $\mathcal{F}_i(\omega)$ denotes the element of the partition describing i's private information.

became known as Ellsberg's paradox and served as an inspiration for most of the ambiguity literature.

Consider an urn with three balls, one of which is red, and the other two are either black or yellow, but the exact composition is unknown. We will drawn a ball from this urn and we offer two different pair of bets for an individual to choose. In the first pair, it is offered the choice between the act[10] f_1 that pays $\$1$ if the red ball is drawn and zero otherwise and the act f_2 that pays $\$1$ if the ball is black and zero otherwise. In the second pair, the choice is between an act f_3 that pays $\$1$ if the ball is either red or yellow and zero otherwise and the act f_4 that pays $\$1$ if the ball is either black or yellow and zero otherwise. To summarize, f_i is given, for $i = 1, ..., 4$ as follows:

$$f_1(\omega) = \begin{cases} 1, & \omega = R \\ 0, & \text{otherwise} \end{cases} \qquad f_2(\omega) = \begin{cases} 1, & \omega = B \\ 0, & \text{otherwise} \end{cases}$$
$$f_3(\omega) = \begin{cases} 1, & \omega \in \{R, Y\} \\ 0, & \text{otherwise} \end{cases} \qquad f_4(\omega) = \begin{cases} 1, & \omega \in \{B, Y\} \\ 0, & \text{otherwise} \end{cases}$$

Most individuals will exhibit preferences as: $f_1 \succ f_2$ and $f_4 \succ f_3$.[11] This is called the Ellsberg Paradox because there is no expected utility that can rationalize this choice, since the first preference would imply $\pi(\{R\}) > \pi(\{B\}$, while the second,

$$\pi(\{B, Y\}) = \pi(\{B\}) + \pi(\{Y\}) > \pi(\{R, Y\} = \pi(\{R\}) + \pi(\{Y\}),$$

that is, $\pi(\{B\}) > \pi(\{Y\})$ and these implications contradict each other.

Now, let's formulate this example in the asymmetric information terminology. Let $\Omega = \{R, B, Y\}$, corresponding to the color of a ball (red, black, yellow) to be extracted from an urn. For simplicity, let us assume that the utility index of the individual is $u(x) = x$. The agent's information about the state of the nature is described by the algebra generated by the following partition: $\mathcal{F} = \{\{R\}, \{B, Y\}\}$, and his belief $\mu : \mathcal{F} \to [0, 1]$ is given by $\mu(\{R\}) = \frac{1}{3}$ and $\mu(\{B, Y\}) = \frac{2}{3}$. Therefore, the acts $f_1 = 1_{\{R\}}$ and $f_4 = 1_{\{B,Y\}}$ are measurable, while the acts $f_2 = 1_{\{B\}}$ and $f_3 = 1_{\{R,Y\}}$ are not. Thus, while $U(f_1) = \int u(f_1)\, d\mu = \mu(\{R\}) = \frac{1}{3}$ and $U(f_4) = \int u(f_4)\, d\mu = \mu(\{B, Y\}) = \frac{2}{3}$, the integrals $U(f_2) = \int u(f_2)\, d\mu$ and $U(f_3) = \int u(f_3)\, d\mu$ are not defined! Therefore, in this standard preference, the individual is unable to compare act f_1 with f_2 (and f_4 with f_3). In other words, this

[10] "Acts" is the terminology used by Savage (1972) for bets.

[11] Through the paper we use the standard notation for preferences: given a preference $\succcurlyeq$, we write $x \succ y$ if $x \succcurlyeq y$ but it is not true that $y \succcurlyeq x$. Similarly we write $x \sim y$ if $x \succcurlyeq y$ and $y \succcurlyeq x$.

preference is *incomplete*, because it does not obey the completeness axiom, which requires that either $f_1 \succcurlyeq f_2$ or $f_2 \succcurlyeq f_1$. However, in the above example, we forced the individual to make a choice. This means that the individual has to find a way to complete her preferences.

The need of completing preferences in situations of ignorance was a problem that worried one of the most important proponents of the expected utility theory, Leonard Savage. In fact, Savage (1972, 1972) devotes more than half of his seminal book to discuss his proposed solution to the problem, that is, the minimax regret criterion. Binmore (2008, Chapter 9) discusses the Wald (1950) and the Savage minimax regret and also three other criteria. These criteria are the principle of insufficient reason, the Hurwicz criterion and the one that we adopt here, the maximin criterion. Note that Savage prescribed his expected utility to be used in "small worlds", which are worlds about which the decision maker knows enough to be capable of evaluating the odds. Thus, the need of the extension of the preference arises as long as the decision maker faces a "large world", that is, a world in which she cannot properly evaluate the likelihood of possible outcomes.

Now, of course a modeler could ignore Savage's worries and assume that the decision maker actually attributes probabilities to all events (a position known as "Bayesian doctrine"). However, the choices obtained in the Ellsberg's paradox show that this is not consistent with the way that many people make choices. The impossibility of accommodating both the assumption of expected utility defined for all events and the choices in the Ellsberg's paradox, motivated the ambiguity aversion literature to reject the expected utility framework and consider other forms of preferences.

However, the simple interpretation of incompleteness discussed above easily solves the Ellsberg's paradox. In fact, if the decision maker extends her choices using the maximin criterion, that is, considering the worst state scenario in each case, then the Ellsberg choices are justified.[12] It should be noted also that this solution is consistent with Savage's original intuition of the scope of the applicability of his theory, as we discuss below.

This easy and simple interpretation seems to be missing in the literature. Of course there are *many explanations* of the Ellsberg choices, that is, axiomatizations of preferences that rationalize those choices. Examples of these preferences began with Choquet Expected Utility by Schmeidler (1989) and the Maximin Expected Utility (MEU) by Gilboa and Schmeidler (1989). For more recent developments see Maccheroni, Marinacci, and Rustichini (2006), Cerreia, Maccheroni, Marinacci, and Montrucchio (2008) and the references therein. Since our complete (or, better, completed) preference is a special case of MEU, it is not a novelty that

[12]See example 2.5 for a formal demonstration.

our preferences would rationalize Ellsberg's choices. Thus, our point here is not to offer *another* explanation in this sense. What we claim is that a *minor* adaptation of Savage's expected utility (to see the expected utility as incomplete) together with the use of a classical concept as the maximin criterion to complete the preference is already sufficient to explain Ellsberg's behavior.[13] This allow us to be as close as possible to the classical model of choices, enhance the descriptive power of the theory and, as we will argue below, to make the expected utility framework more reasonable for economic applications.

Since our approach is based in the incompleteness of the preferences and Bewley (1986, 2002) was a precursor in the use of this class of preferences, it is useful to revisit his work.[14] Bewley (1986, 2002) mentions the Ellsberg's thought experiments in his introduction to motivate the shortcomings of the expected utility theory, but he does not offer his model of incompleteness as an explanation for the Ellsberg's paradox. Although this position is consistent with his commitment to describe *only* incomplete preferences, it is interesting to see what he writes about this:

> "One might imagine that Ellsberg's (1961) experiments lend support to the Knightian theory. However, the choices among the alternatives he offered would be indeterminate according to the theory presented here, so that his experiments neither confirm nor contradict the theory."[15]

We do not want to force this explanation too far, though. In an experiment as simple as this, it is hard to think that the world is "large". In fact, it is possible that Savage himself would consider the Ellsberg urn as a "small world". Also, the intuition of a "game against nature" that motivates the maximin criterion can be inappropriate in this case, where nature has absolutely no strategic behavior.[16] Thus, the approach suggested here may be considered disputable as an "explanation" of the Ellsberg paradox by decision theorists (although the connection with asymmetric information can offer some insights). It should be noted that in the situations that we are going to analyze, the individuals face strategic opponents,

[13]The relaxation of completeness does not seem a *minor* change in the original Savage theory. However, Kopylov (2007) has shown that completeness is not essential at all. That is, Savage's expected utility theory can be developed in such a way that the probability is defined only in a restricted class of events, exactly as we do here. Lehrer (2008) also presents an axiomatization of partially defined probabilities.

[14]Our model of incomplete preferences is actually *simpler* than Bewley's. See section 8.2.

[15]Bewley (2002, p. 100).

[16]It is also sometimes argued that the maximin criterion is too pessimistic. See a discussion about this on section 8.4.

that may change the likelihood of perceived events simply by misreporting their information. Therefore, the interpretation of "games" is completely natural. Moreover, our results suggest that maximin expected utility will allow Pareto improving trades where the standard expected utility will force agents to no trade situations as we will see in section 5.1. Thus, the maximin expected utility preferences can be naturally "selected".

It is also useful to remark that we are making subtle changes in some basic concepts *on purpose*. Those are:

- First, we are understanding "large worlds" as worlds where there is missing information, exactly as it occurs in asymmetric information economies. As we mentioned above, this is not necessarily what Savage had in mind.[17]

- Second, we view the preferences as something to be completed by the individual, rather than some exogenously given and fixed characteristic. This view is unusual in decision theory.[18]

- Third, the partition model is taken more seriously than in the standard asymmetric information models, because we constrain the prior of each agent to be defined only on the events in her partition.[19]

Of course these conceptual changes can be criticized on different grounds and we do not pretend they are uncontroversial, although we have justifications for each of them. First, the concept of "large worlds" in Savage is not sufficiently clear (in particular, it does not rule out our interpretation), and there is no much guidance on how to distinguish a "small world" from a "large world". Our definition of "large world" (a world with missing information) is precise and offer grounds for a clear distinction.

Second, our perspective on the construction of preferences is really just a pedagogical device to distinguish the expected utility, based in the private information, and the part of the preference based on the maximin criterion. At the end, each individual will act as having a definitive preference, so that no principle or standard practice in decision theory is violated. However, the exposition in terms of

[17] However, the relation between the small world and the large world that Savage had in mind is exactly the same relation that we have (partitions). Consider the following quote: "It will be noted that the small-world states are in fact events in the grand world, that indeed they constitute a partition of the grand world." Savage (1972, p.84).

[18] See, however, section 8.1 for a discussion on related papers.

[19] In the standard expected utility model with asymmetric information, priors are defined in all possible events and not just in the events of the partition of each agent. Both in our model and in the standard expected utility model, the elements of the partition represent the set of states that the agent cannot distinguish.

completion of the preference, as we presented above, seems more intuitive given the setup considered and allow for a consideration of how the preference should be completed.[20]

For the third point, recall that each agent cannot distinguish the states in the same element of her partition. If an agent follows the Bayesian doctrine, she will attribute probabilities to each of those states, although she is unenable to distinguish them. How can she meaningfully do this? Moreover, since the distinction of states will come from the reports of other agents, any assignment of probabilities will be arbitrary.[21]

Although each of these modifications can be disputed in one or other ground, all of them are subtle but important *conceptual* changes and they may be considered some of the main conceptual contributions of this paper.

2.2 Incomplete Expected Utility Preferences and Maximin Completion

The agents have standard Expected Utility preferences, as usually described in the asymmetric information literature, with a subtle caveat that we will explain now.[22] For each agent $i \in I$, $(\Omega, \mathcal{F}_i, \mu_i)$ is a probability space. Then, the preference $\succcurlyeq_i^\circ$ of the individual i is described as:[23]

$$f \succcurlyeq_i^\circ g \iff \int_\Omega u_i(f(\omega))\mu_i(d\omega) \geqslant \int_\Omega u_i(g(\omega))\mu_i(d\omega), \forall f, g \in \mathcal{L}_i. \quad (1)$$

Since all integrations in this paper will be in Ω, we will omit it in next integrals.[24]

[20] If the reader is too uncomfortable with this view, then it will do no harm to see our paper as just assuming that individuals' preferences $\succcurlyeq_i$ are given by (3) below. All definitions and results can be reinterpreted to accommodate this view, with some conceptual losses though.

[21] Observe also that the first and third points correspond to the abandonment of the Bayesian doctrine, which assumes that each individual should attach probability to everything that she does not know. In fact, one can interpret Ellsberg's paradox as a rejection of the Bayesian doctrine, not of the expected utility theory. As our theory shows, it is possible and reasonable to maintain the expected utility preferences whenever the decision maker is ready for attributing probabilities, and allow different completions when this is not the case. Some reasonable completions (as the one following the maximin criterion) will accommodate Ellsberg's choices. In other words, it is the Bayesian doctrine that should be reformulated, not necessarily the expected utility theory. Given the practical purposes of this paper, we must limit the discussion of this important issue to these brief comments, but see also section 8.3.

[22] In this subsection, we restrict our discussion to the partition model. The translation to the types model is immediate.

[23] We use the notation $\succcurlyeq_i^\circ$ for this incomplete expected utility preference and reserve the more standard $\succcurlyeq_i$ for the complete preferences given in (3) below . Note that μ_i is just a partial probability, that is, a probability restricted only to some events (those in $\mathcal{F}_i$).

[24] Although Ω is finite, the integral $\int f \, d\mu_i$ of some function $f : \Omega \to \mathbb{R}$ *is not* equal to $\sum_{\omega \in \Omega} f(\omega)\mu_i(\{\omega\})$. The reason is that $\mu_i(\{\omega\})$ is defined only if $\{\omega\} \in \mathcal{F}_i$. The correct def-

We insist that the preferences above described are completely standard: they are just expected utility preferences which take in account the private information of each individual. However, the following trivial observation seems to have received little attention in the literature: these preferences are incomplete! To see this, it is sufficient to observe that the preference is capable of comparing only $\mathcal{F}_i$-measurable acts. If h is not $\mathcal{F}_i$-measurable, its integral $\int h \, d\mu_i$ is not defined and, therefore, it is not possible for individual i to compare h with any other act. In other words: neither $f \succcurlyeq_i^\circ h$ nor $h \succcurlyeq_i^\circ f$ hold for any act f, which is the same as saying that the preference is *incomplete*. This was exemplified in the discussion of the Ellsberg paradox in section 2.1. We solve the problem of incompleteness by adopting the maximin expected utility, as we will show below.

Most problems that arise in the asymmetric information literature come from the difficulty in dealing with the incompleteness of these preferences. However, it seems that we are the first to see the problem in this way. What most papers in this literature do is to assume that μ_i is in fact defined for all sets in $\mathcal{F}$, where $\mathcal{F}$ is the power set of Ω in the case of finite Ω, or a σ-algebra in the general case. This extends the preference to all measurable acts, but ignores the information constraint that individual i has.

Our simple "incompleteness view" of these preferences will lead us to an interesting new way to look at the problem of modeling asymmetric information and allow new solutions for classic problems, as we will show.

Although this view was never undertaken or at least well explored in the asymmetric information literature, it reflects a basic problem in the Bayesian decision making. In fact, it was recognized by Savage (1972) himself, who devoted almost half of his important book to it. Savage's approach (minimax regret) *is not* the approach that we are going to undertaken here, although there are some similarities. Essentially, we will assume that the individuals will complete their preferences following the maximin criterion, which is a classical normative criterion (see Milnor (1954) and Luce and Raiffa (1989)). The next sections will make clear why this specific way of completing preferences is natural in our context. But before that, let us formally introduce the preferences that we will consider.

Let Δ denote the set of measures $\pi : \mathcal{F} \to [0,1]$. Define, for each i, the following set:

$$\mathcal{P}_i \equiv \{\pi \in \Delta : \pi(A) = \mu_i(A), \forall A \in \mathcal{F}_i\}. \tag{2}$$

Thus, $\mathcal{P}_i$ is the set of all extensions of μ_i to from $\mathcal{F}_i$ to $\mathcal{F}$, that is, the set of all

inition of the integral would be as follows. Let agent i's partition be $\mathcal{F}_i \equiv \{A_1, ..., A_n\}$ and fix any $\omega_k \in A_k$. If $f : \Omega \to \mathbb{R}$ is $\mathcal{F}_i$-measurable, then $f(\omega) = f(\omega_k)$ for any $\omega \in A_k$. Then, $\int f \, d\mu_i = \sum_{k=1}^n f(\omega_k)\mu_i(A_k)$. From this, we see that the integral notation is simpler than the sum notation, and this is the reason why we write integrals.

probability measures defined in $\mathcal{F}$ that agree with μ_i in the events that individual i is informed about. Let $\mathcal{L}$ denote the set of all acts $f : \Omega \to \mathbb{R}^{\ell}_{+}$. Then, we consider the preference $\succcurlyeq_i$ which extends $\succcurlyeq_i^{\circ}$ from $\mathcal{L}_i$ to the set of all acts, $\mathcal{L}$:[25]

$$f \succcurlyeq_i g \iff \min_{\pi \in \mathcal{P}_i} \int_{\Omega} u_i(f(\omega))\pi(d\omega) \geqslant \min_{\pi \in \mathcal{P}_i} \int_{\Omega} u_i(g(\omega))\pi(d\omega), \forall f, g \in \mathcal{L}. \tag{3}$$

In fact, we can describe the above preferences in a simpler way, following an idea of Gilboa and Schmeidler (1994). For this, let us define, for individual i with partition $\mathcal{F}_i$ and for each function $h : \Omega \to \mathbb{R}$, the underline h, denoted $\underline{h}$ and given by:[26]

$$\underline{h}(\omega) \equiv \min_{\omega' \in \mathcal{F}_i(\omega)} h(\omega'). \tag{4}$$

Note that for any function $h : \Omega \to \mathbb{R}$, the function $\underline{h}$ is $\mathcal{F}_i$-measurable and, therefore, integrable. Of course, we are interested in applying this concept to the functions like $u_i \circ f$, for $f : \Omega \to \mathbb{R}^{\ell}_{+}$ and $u_i : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$. In this case, we will slightly abuse notation by writing $\underline{u}_i(f(\cdot))$ instead of $\underline{u_i(f(\cdot))}$. This allows us to have the following:

Lemma 2.3 *The preference $\succcurlyeq_i$ given by (3) is equivalently characterized by:*

$$f \succcurlyeq_i g \iff \int_{\Omega} \underline{u}_i(f(\omega))\mu_i(d\omega) \geqslant \int_{\Omega} \underline{u}_i(g(\omega))\mu_i(d\omega), \forall f, g \in \mathcal{L}. \tag{5}$$

Proof: It is sufficient to prove that $\int \underline{u}_i(f(\cdot))\, d\mu_i = \min_{\pi \in \mathcal{P}_i} \int \underline{u}_i(f(\cdot))\, d\pi$ for every $f \in \mathcal{L}$. Let agent i's partition be $\mathcal{F}_i \equiv \{A^1, ..., A^n\}$ and let $\mathcal{P}_i^k$ denote the set $\{\pi \in \Delta : \pi(A^k) = \mu_i(A^k)\}$. Observe that $\mathcal{P}_i = \cap_{k=1}^{n} \mathcal{P}_i^k$ and the minimization in different A^k is independent, that is, if $\pi^k \in \arg\min_{\pi \in \mathcal{P}_i^k} \int_{A^k} u_i(f(\cdot))\, d\pi$ we can define π° as $\pi^{\circ}(A) = \pi^k(A)$ if $A \subset A^k$. For this π°, we have:

$$\min_{\pi \in \mathcal{P}_i} \int_{\Omega} u_i(f(\cdot))\, d\pi = \sum_{k=1}^{n} \int_{A^k} u_i(f(\cdot))\, d\pi^k = \int u_i(f(\cdot))\, d\pi^{\circ}. \tag{6}$$

From the definition of $\mathcal{P}_i^k$, it is not difficult to see that:

$$\int_{A^k} u_i(f(\cdot))\, d\pi^{\circ} = \int_{A^k} \min_{\omega \in A^k} u_i(f(\omega))\, d\mu_i. \tag{7}$$

[25] Note that this is a *particular case* of the maximin expected utility axiomatized by Gilboa and Schmeidler (1989). The difference is that while we require $\mathcal{P}_i$ to have the format given by (2), the set $\mathcal{P}_i$ in Gilboa and Schmeidler (1989) has to be only compact and convex. See a related discussion about Bewley's preferences in section 8.2.

[26] We denote by $\mathcal{F}_i(\omega)$ the element of the partition $\mathcal{F}_i$ that contains ω.

Therefore, (6) and (7) give:

$$\begin{aligned}\min_{\pi\in\mathcal{P}_i}\int_\Omega u_i(f(\cdot))\,d\pi &= \sum_{k=1}^{n}\min_{\pi\in\mathcal{P}_i^k}\int_{A^k}u_i(f(\cdot))\,d\pi\\ &= \sum_{k=1}^{n}\int_{A^k}\min_{\omega\in A^k}u_i(f(\omega))\mu_i(d\omega)=\int \underline{u}_i(f(\cdot))\,d\mu_i,\end{aligned}$$

as we wanted to show. ■

The following easy observation is also useful:

Lemma 2.4 *If $f:\Omega\to\mathbb{R}^\ell_+$ and $g:\Omega\to\mathbb{R}^\ell_+$ are $\mathcal{F}_i$-measurable, then*

$$f\succcurlyeq_i^\circ g \iff f\succcurlyeq_i g.$$

Proof: If f is $\mathcal{F}_i$-measurable, it is constant on any event in $\mathcal{F}_i$ and therefore, for every $\tilde{\omega}\in\mathcal{F}_i(\omega)\in\mathcal{F}_i$, then

$$\underline{u}_i(f(\tilde{\omega}))=\min_{\omega'\in\mathcal{F}_i(\omega)}u_i(f(\omega'))=u_i(f(\tilde{\omega})).$$

Consequently,

$$\int_\Omega \underline{u}_i(f(\omega))\mu_i(d\omega)=\int_\Omega u_i(f(\omega))\mu_i(d\omega).$$

Since a similar equality holds for g, the result follows from (1) and (5). ■

Now, we can return to the Ellsberg's urn example and show how the choices described in section 2.1 are represented by the maximin expected utilities. The following example clarifies the issue:[27]

Example 2.5 (Ellsberg's Experiment) *See section 2.1 for a description of the Ellsberg's thought experiment. Given the partition: $\mathcal{F}=\{\{R\},\{B,Y\}\}$, the set of probabilities defined by (2) is:*

$$\mathcal{P}_i\equiv\{\pi\in\Delta:\pi(\{R\})=\frac{1}{3};\pi(\{B,Y\})=\frac{2}{3}\}.$$

[27] As discussed in section 2.1, it is well known that this class of preferences can rationalize the Ellsberg choices.

Let us assume that $0 = u(0) < u(1) = 1$. *Thus,*

$$\begin{aligned}
\underline{U}(f_1) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{R\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{R\}) = \frac{1}{3};\\
\underline{U}(f_2) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{B\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{B\}) = 0;\\
\underline{U}(f_3) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{R,Y\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{R,Y\}) = \frac{1}{3};\\
\underline{U}(f_4) &= \min_{\pi \in \mathcal{P}_i} \int_\Omega 1_{\{B,Y\}}\, d\pi = \min_{\pi \in \mathcal{P}_i} \pi(\{B,Y\}) = \frac{2}{3}.
\end{aligned}$$

This implies $f_1 \succ f_2$ *and* $f_4 \succ f_3$*, exactly as in the Ellsberg's thought experiment.*[28] *As we explained in section 2.1, these choices cannot be represented by an expected utility. For, if* π *is the probability of an expected utility, then:*

$$\begin{aligned}
U(f_1) &= \int u(f_1)\, d\pi = \pi(\{R\});\\
U(f_2) &= \int u(f_2)\, d\pi = \pi(\{B\});\\
U(f_3) &= \int u(f_3)\, d\pi = \pi(\{R,Y\});\\
U(f_4) &= \int u(f_4)\, d\pi = \pi(\{B,Y\}).
\end{aligned}$$

In this case, $U(f_1) > U(f_2)$ *and* $U(f_3) < U(f_4)$ *would require the contradictory inequalities* $\pi(\{R\}) > \pi(\{B\})$ *and* $\pi(\{R,Y\}) < \pi(\{B,Y\}) \iff \pi(\{R\}) < \pi(\{B\})$.

We will sometimes assume that there is a probability defined for all events, because this makes the definition of preferences easier. Another occasional reason is to compare maximin expected utilities with those obtained by expected utility completions (following the Bayesian doctrine).

The following lemma will be useful:

Lemma 2.6 *If* u_i *is concave, then* $\succcurlyeq_i$ *is a convex preference, that is, the set* $\{f : \Omega \to \mathbb{R}^\ell_+ : f \succcurlyeq_i h\}$ *is convex for all* $h : \Omega \to \mathbb{R}^\ell_+$.

[28] Note that f_2 and f_3 are not $\mathcal{F}$- measurable and therefore, could not be compared using the expected utility preference $\succcurlyeq_i^\circ$. Now, the preference $\succcurlyeq_i$ is complete and we can compare every acts, including the non-measurable ones. In that sense, the maximin criterion completes the preference relation. See equation (5) and footnote 11 for the meaning of $x \succ y$.

Proof: Let us consider functions f, g such that $f \succcurlyeq_i g$ and $g \succcurlyeq_i h$ for a fixed $h : \Omega \to \mathbb{R}^{\ell}_{+}$. We want to show that $\alpha f + (1-\alpha)g \succcurlyeq_i h$. Since u_i is concave, $u_i(\alpha f(\omega) + (1-\alpha)g(\omega)) \geqslant \alpha u_i(f(\omega)) + (1-\alpha)u_i(g(\omega))$. Therefore,

$$
\begin{aligned}
\underline{u}_i\left(\alpha f(\omega') + (1-\alpha)g(\omega')\right) &= \min_{\omega \in \mathcal{F}_i(\omega')} \left[u_i\left(\alpha f(\omega) + (1-\alpha)g(\omega)\right)\right] \\
&\geqslant \min_{\omega \in \mathcal{F}_i(\omega')} \left[\alpha u_i\left(f(\omega)\right) + (1-\alpha)u_i\left(g(\omega)\right)\right] \\
&\geqslant \alpha \min_{\omega \in \mathcal{F}_i(\omega')} u_i\left(f(\omega)\right) + (1-\alpha) \min_{\omega \in \mathcal{F}_i(\omega')} u_i\left(g(\omega)\right) \\
&= \alpha \underline{u}_i\left(f(\omega')\right) + (1-\alpha)\underline{u}_i\left(g(\omega')\right).
\end{aligned}
$$

In other words, $\underline{u}_i$ is also concave. From this we obtain

$$
\begin{aligned}
\int \underline{u}_i\left(\alpha f(\cdot) + (1-\alpha)g(\cdot)\right)\, d\mu_i &\geqslant \int \left[\alpha \underline{u}_i\left(f(\cdot)\right) + (1-\alpha)\underline{u}_i\left(g(\cdot)\right)\right]\, d\mu_i \\
&\geqslant \alpha \int \underline{u}_i\left(h(\cdot)\right)\, d\mu_i + (1-\alpha)\int \underline{u}_i\left(h(\cdot)\right)\, d\mu_i \\
&= \int \underline{u}_i\left(h(\cdot)\right)\, d\mu_i.
\end{aligned}
$$

Thus, $\alpha f + (1-\alpha)g \succcurlyeq_i h$, as we wanted to show. ■

3 Efficiency and Incentive Compatibility: a primer

3.1 Incentive Compatibility

What does incentive compatibility mean in an asymmetric information economy? What is the appropriate notion of incentive compatibility?

In an economy with asymmetric information agents are characterized by random utility functions, random initial endowments, a private information set and a prior. The random initial endowments necessitates new definitions of incentive compatibility which go beyond the ones introduced in the theory of games and mechanism design. To see this consider a two person one good economy as follows:[29]

Example 3.1 *The state space is $\Omega = \{a, b, c\}$ and the partition information of the two individuals are given by:*

$$\mathcal{F}_1 = \{\{a, b\}, \{c\}\}; \quad \mathcal{F}_2 = \{\{a, c\}, \{b\}\}.$$

[29] Example 3.1 will be a leading example in this paper. We will use it to illustrate a number of points.

Agents' beliefs are defined as follows:[30]

$$\mu_1(\{a,b\}) = \frac{2}{3}; \quad \mu_1(\{c\}) = \frac{1}{3};$$
$$\mu_2(\{a,c\}) = \frac{2}{3}; \quad \mu_2(\{b\}) = \frac{1}{3}.$$

Agent i's utility function is $u_i : \mathbb{R}_+ \to \mathbb{R}_+$, given by $u_i(x_i) = \sqrt{x_i}$. The random initial endowments are:

$$(e_1(a), e_1(b), e_1(c)) = (5, 5, 0);$$
$$(e_2(a), e_2(b), e_2(c)) = (5, 0, 5).$$

The following distribution of the initial endowments

$$(x_1(a), x_1(b), x_1(c)) = (5, 4, 1);$$
$$(x_2(a), x_2(b), x_2(c)) = (5, 1, 4)$$

is not incentive compatible. Let us see why. Suppose that the realized state of nature is state a. Notice that agent 1 does not know if it is state a or b and agent 2 does not know if it is state a or c. Agent 1 is not sure that the state is a (it could be b), but he has an incentive to claim that state c has occurred.[31] Indeed, if agent 2 (who knows that agent 1 knows whether state c has occurred or not) believes the claim of agent 1 that indeed state c has occurred, she gives one unit of the good in this state and therefore agent 1 ends up with his initial 5 units (since $e_1(a) = 5$) and also gets the one unit that she is supposed to receive from agent 2 in state c, that is, $x_1(c) = 1$. Thus, the utility of agent 1 when she misreports will be $u_1(e_1(a) + x_1(c) - e_1(c)) = \sqrt{6}$, which is greater than $u_1(x_1(a)) = \sqrt{5}$, which is the utility when she does not misreport. Consequently, agent 1 has an incentive to misreport the realized state of nature and become better off. Therefore the allocation given above is not incentive compatible.

This example can also be given in the type model. See section 7.

But what is the appropriate definition of incentive compatibility if we have more than two agents? To address this question, let us consider another example with three agents:

[30] Note that these beliefs are given by the restriction of the uniform probability (i.e., $\mu(\{\omega\}) = \frac{1}{3}$, for all $\omega \in \Omega$) to the individual's partition.

[31] If the true state were b, this lie would be detected by agent 2. We will assume that nothing happens in this case, that is, there is no punishment for the lie and the allocation $(x_1(b), x_2(b))$ is implemented. See appendix A for a detailed discussion.

Example 3.2 *There are three agents and two goods. As before, $\Omega = \{a, b, c\}$ and each state occurs with the same probability, i.e., $\mu(\{\omega\}) = \frac{1}{3}$, for all $\omega \in \Omega$.*

$$u_i(x_i, y_i) = \sqrt{x_i y_i};\ \textit{for } i = 1, 2, 3;$$
$$e_1(a, b, c) = ((15, 0), (15, 0), (15, 0));\ \ \mathcal{F}_1 = \{\{a, b, c\}\};$$
$$e_2(a, b, c) = ((0, 15), (0, 15), (0, 12));\ \ \mathcal{F}_2 = \{\{a, b\}, \{c\}\};$$
$$e_3(a, b, c) = ((15, 0), (15, 0), (15, 0));\ \ \mathcal{F}_3 = \{\{a\}, \{b\}, \{c\}\}.$$

The redistribution below is individual incentive compatible because no individual has an incentive to misreport the realized state of nature.

$$x_1(a, b, c) = ((8, 1), (8, 1), (10, 2));$$
$$x_2(a, b, c) = ((7, 4), (7, 4), (2, 2));$$
$$x_3(a, b, c) = ((15, 10), (15, 10), (18, 8)).$$

Notice that the only agent who can misreport either state a or b to agents 1 and 2 is agent 3. Observe that if agent 3 misreports she cannot be caught by either agent, since agent 1 cannot distinguish any state and agent 2 cannot distinguish state a from state b. However, agent 3 has no incentive to misreport because in states a and b she has the same consumption. Hence, the above allocation is individual incentive compatible, although is not coalitional incentive compatible. Observe that the coalition of agents 2 and 3 can cheat agent 1. For example, if the realized state of nature is c, then agents 2 and 3 report to agent 1that it is state b. In this way, agent 1 will get in state b the allocation $x_1(b) = (8, 1)$, and not $x_1(c) = (10, 2)$. Now, agents 2 and 3 will split the difference between $x_1(c) - x_1(b) = (2, 1)$ and become better off.[32] The conclusion to be drawn is that a redistribution or a contract which is individual incentive compatible may not be coalitional incentive compatible and therefore the contract may not be viable or stable. For this reason, we will focus on a coalitional notion of incentive compatibility and an appropriate notion will be introduced in section 3.3, after a motivating discussion in appendix A.

3.2 Efficiency

In the absence of asymmetric information, it is well-known from the state-contigent model of Arrow and Debreu (see Debreu (1959, Chapter 7)) that agents can write

[32]Notice that agent 2 cannot distinguish a and b, but the coalition of 2 and 3 agreed to state one of the two states. It is not necessary for both agents to be able to distinguish the reported state, as long as the coalition benefit and can be sustained. The fact that we do not require all members of the coalition to be able to distinguish the state corresponds to make it easier for coalitions to form. Not that the weaker the requirements, the stronger the incentive compatibility definition. See a longer discussion on this issue after definition 3.6.

ex-ante state-contigent contracts which are ex-post Pareto optimal. Indeed, as it was noted in Debreu (1959) with state dependent preferences and initial endowments all the classical results of existence and optimality of the Walrasian equilibrium continue to hold. Of course in this model one needs to assume that there is an exogenous enforcer (court or government) who makes sure that the state-dependent contingent contracts made ex-ante (period one) are executed ex-post (period two). It should be noted that the issue of incentive compatibility does not arise in the Arrow-Debreu state-contingent model since there is no asymmetric information.

However, in the presence of asymmetric information the situation is quite different as the example below illustrates.

Example 3.3 *Consider again the same basic structure given in example 3.1, but with the following endowments:*

$$(e_1(a), e_1(b), e_1(c)) = (20, 20, 0)\,;$$
$$(e_2(a), e_2(b), e_2(c)) = (20, 0, 20)\,.$$

The perfect risk sharing allocation is:[33]

$$x_1\,(a, b, c) = (20, 10, 10)$$
$$x_2\,(a, b, c) = (20, 10, 10)\,.$$

This allocation is Pareto optimal (it cannot be dominated by any redistribution of the random initial endowment and make agents better off in terms of ex-ante expected utility).[34] However, the above allocation is not incentive compatible, because if a is the realized state of nature, agent 1 has an incentive to report that it is state c (notice that agent 2 cannot distinguish state a from state c) and become better off. In particular, agent 1 will keep her initial endowment in the event $\{a, b\}$ which is 20 units and receive another 10 units from agent 2 in state c (i.e., $u_1\,(e_1\,(a) + x_1\,(c) - e_1\,(c)) = u_1\,(30)$. Notice that $u_1(30)$ is greater than $u_1\,(x_1\,(a)) = u_1\,(20)$, which is the utility of agent 1 when she does not misreport. Obviously, agent 2 is worse off. Similarly, agent 2 has an incentive to report b when he observes $\{a, c\}$.

The above example indicates that *it is not possible to have contracts or allocations which are simultaneously efficient and incentive compatible*. This poses the following question: is it possible to move into a *second best efficiency* concept which is not in conflict with incentive compability? The example below will answer this question in an affirmative way.

---

[33] See section 5.1 for a discussion of perfect risk sharing allocations.

[34] By ex-ante expected utility here, we mean $\sum_{\omega\in\Omega} u_i(x_i(\omega)\mu(\{\omega\})$, where $\mu(\{\omega\}) = \frac{1}{3}$ for $\omega = a, b, c$. See also section 5.1.

Example 3.4 *There are two agents, 1 and 2; two goods, denoted by x and y; and two equally probable states of nature, denoted by a and b. The utilities are given by $u_i(x_i, y_i) = \sqrt{x_i y_i}$, for $i = 1, 2$. The endowments and information are:*

$$e_1(a, b) = ((20, 0), (20, 0)); \quad \mathcal{F}_1 = \{\{a, b\}\};$$
$$e_2(a, b) = ((20, 20), (0, 20)); \quad \mathcal{F}_2 = \{\{a\}, \{b\}\}.$$

It is not difficult to show that the allocation below is (ex post) Pareto optimal:

$$((x_1(a), y_1(a)), (x_1(b), y_1(b))) = ((10, 8), (10, 10));$$
$$((x_2(a), y_2(a)), (x_2(b), y_2(b))) = ((30, 12), (10, 10)).$$

However, this allocation is not incentive compatible because if b is the realized state of nature, agent 2 has an incentive to report state a (observe that agent 1 cannot distinguish state a from state b) and become better off, i.e.,

$$u_2(e_2(b) + (x_2(a), y_2(a)) - e_2(a)) = u_2((0, 20) + (30, 12) - (20, 20))$$
$$= u_2(10, 12) > u_2(10, 10) = u_2(x_2(b), y_2(b)).$$

Notice that agent 1's consumption bundle $((x_1(a), y_1(a)), (x_1(b), y_1(b))) = ((10, 8), (10, 10))$ is not $\mathcal{F}_i$-measurable. If agent 1 had made an $\mathcal{F}_i$-measurable contract, for example had chosen the allocation

$$\begin{aligned} ((x_1(a), y_1(a)), (x_1(b), y_1(b))) &= ((10, 10), (10, 10)); \\ ((x_2(a), y_2(a)), (x_2(b), y_2(b))) &= ((30, 10), (10, 10)), \end{aligned} \tag{8}$$

then indeed (8) is incentive compatible and $\mathcal{F}_i$-measurable Pareto optimal. In another words, it is not possible for the grand coalition to make $\mathcal{F}_i$-measurable redistributions of the random initial endowments and make all agents better off in terms of ex-ante expected utility (see Yannelis (1991)). Indeed, the $\mathcal{F}_i$-measurability is a necessary and sufficient condition for the incentive compatibility of Pareto optimal allocations in the one good per state case (see footnote 38). For more than one good, the $\mathcal{F}_i$-measurability is only a sufficient condition (see for example, Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994)).

The example above provides an affirmative answer to the question as to whether or not agents can write contracts that are incentive compatible and second best Pareto optimal.

It should be noted that in Example 3.3, the $\mathcal{F}_i$-measurability Pareto optimal allocation (which is obviously incentive compatible) is the no trade allocation. It is also clear that there is a tremendous loss of efficiency. Indeed, this example suggests that insisting on $\mathcal{F}_i$-measurable allocations to ensure incentive compatibility may be a bit strong. At the same time the lack of $\mathcal{F}_i$-measuribility leads to allocations which need not be incentive compatible as examples 3.1 and 3.4 demonstrated. Thus, on the one hand $\mathcal{F}_i$-measurability guarantes incentive compatibility and on the other, it reduces efficiency.

Thus, the new question which arises is: *can we dispense with the $\mathcal{F}_i$-measurability condition and have efficient incentive compatible allocations?* This is exactly the question that this paper addresses. We will show that indeed one can drop the $\mathcal{F}_i$-measurability assumption and therefore gain in terms of efficiency, and at the same time maintain incentive compatibility. We will achieve this by making use of the expected utility formulation introduced in section 2.2. In particular, we will define a new notion of efficiency which replaces the standard EU with the MEU.

We will now formally define the notion maximin Pareto optimality.

Definition 3.5 (Maximin Pareto Optimality) *Maximin Pareto optimal allocation is a feasible allocation $x = (x_i)_{i \in I}$ for which there is no allocation $y = (y_i)_{i \in I}$ satisfying $\sum_{i \in C} y_i(\omega) = \sum_{i \in C} e_i(\omega)$, $\forall \omega \in \Omega$, $y_i \succcurlyeq_i x_i$ for all $i \in C$ and $y_i \succ_i x_i$ for at least one $i \in I$.*[35]

Note that this definitions is standard, with preferences $\succcurlyeq_i$, for each $i \in I$. The only difference is that we are not considering the incomplete preference $\succcurlyeq_i^\circ$, and we use the name maximin just to emphasize this.

3.3 Maximin Incentive Compatibility (IC)

In this section we formalize different notions of incentive compatibility. Once we assume that individuals have maximin preferences, as described above, then our definitions are completely standard. We choose to use new names to it just to emphasize the kind of preferences that are considered. Before presenting the definitions and discuss their relations, appendix A motivates the different requirements that can form the incentive compatibility definitions.

Definition 3.6 (Maximin incentive compatibility) *A feasible allocation $(x_i)_{i \in I}$ is maximin coalitional incentive compatible (MCIC) if there is no coalition $C \subset I$, states ω, $\omega' \in \Omega$ and transfers $(\tau_i)_{i \in C} \in \left(\mathbb{R}^\ell_+\right)^{|S|}$ satisfying the following:*

1. *$\mathcal{F}_j(\omega') = \mathcal{F}_j(\omega)$ for all $j \notin C$;*
2. *$\sum_{i \in C} \tau_i = 0$;*
3. *$\underline{u}_i(e_i(\omega) + x_i(\omega') - e_i(\omega') + \tau_i) > \underline{u}_i(x_i(\omega))$ for all $i \in C$.*[36]

[35] Recall $f \succ_i g$ is equivalent to $\int \underline{u}_i(f(\cdot))\, d\mu_i > \int \underline{u}_i(g(\cdot))\, d\mu_i$.

[36] Recall that $\underline{u}_i(x_i(\omega)) = \min_{\tilde{\omega} \in \mathcal{F}_i(\omega)} u_i(x_i(\tilde{\omega}))$.

- *If condition 2 above is changed to $\tau_i = 0$, for all i, that is, transfers are not allowed, then we say that $(x_i)_{i \in I}$ is* weakly maximin coalitional incentive compatible*.*

- *If there is no unitary coalitions C for which 1-3 hold, then we say that $(x_i)_{i \in I}$ is* maximin individual incentive compatibility.

Condition 1 above means that the individuals out of the coalition will not be able to detect the cheating by the coalition. Condition 2 means that the transfer are only among members of the coalition. Condition 3 is the essential condition, that requires that the transfers make all members of the coalition strictly better off in the state where cheating occurs.

As we discussed in appendix A, we could have imposed extra requirements in the event $\mathcal{F}_i(\omega)$, such as that it should be included in the common knowledge partition $\wedge_{i \in C} \mathcal{F}_i$. As we explained above, the imposition of such stronger conditions makes the definition of incentive compatibility weaker, because there will be less opportunities for coalitions to share their information and cheat the complimentary coalition.[37]

Krasa and Yannelis (1994) introduced the following:

Definition 3.7 (coalitional IC) *A feasible allocation $(x_i)_{i \in I}$ is coalitional incentive compatible (CIC, for short) if it satisfies the conditions of the previous definition, with 3 changed to*

$$u_i \left(e_i \left(\omega \right) + x_i \left(\omega' \right) - e_i \left(\omega' \right) + \tau_i \right) > u_i \left(x_i \left(\omega \right) \right), \text{ for all } i \in C.$$

This makes clear that our only modification in the Krasa and Yannelis definition corresponds to considering the utility function that actually represents the preference $\succcurlyeq_i$, instead of the utility function that represents $\succcurlyeq_i^\circ$, as in Krasa and Yannelis (1994) definition. The following result establishes the relationship between the two definitions.

Proposition 3.8 *If a feasible allocation $(x_i)_{i \in I}$ is CIC, then it is maximin-IC, but the converse does not necessarily holds.*

Proof: Assume that x is not maximin-IC. Then there exists a coalition $C \subset I$, states $\omega', \omega'' \in \Omega$ and transfers $(\tau_i)_{i \in C}$ satisfying:

[37] Exactly this condition has been imposed by Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994). In those papers such a condition played an important role in their proofs to maintain the private information measurability of allocations. Since private information measurability does not play a role in our paper, we can use a more general definition of incentive compatibility.

1. $\mathcal{F}_j(\omega') = \mathcal{F}_j(\omega'')$ for all $j \notin C$;
2. $\sum_{i \in C} \tau_i = 0$;
3. $\underline{u}_i(e_i(\omega') + x_i(\omega'') - e_i(\omega'') + \tau_i) > \underline{u}_i(x_i(\omega'))$, for all $i \in C$.

Observe that the last item implies: $u_i(e_i(\omega') + x_i(\omega'') - e_i(\omega'') + t_i) > u_i(x_i(\omega'))$ for all $i \in C$. Therefore, this is also a cheating coalition in the Krasa-Yannelis's sense, establishing that x could not be CIC. For the converse, consider again the example 3.3: there are two agents, 1 and 2; three equally probable states of nature, denoted by a, b and c and one good per state denoted by x. The utility functions are $u_i(x_i) = \sqrt{x_i}$, for $i = 1, 2$. The initial endowments and private information sets are given as follows:

$$e_1(a,b,c) = (20, 20, 0); \mathcal{F}_1 = \{\{a,b\},\{c\}\};$$
$$e_2(a,b,c) = (20, 0, 20); \mathcal{F}_2 = \{\{a,c\},\{b\}\}.$$

It can be easily calculated that the maximin core for this example is:

$$\begin{aligned} x_1(a,b,c) &= (20, 16, 4) \\ x_2(a,b,c) &= (20, 4, 16). \end{aligned} \tag{9}$$

This allocation is maximin incentive compatible since neither agent has an incentive to misreport the state of nature and become better off. For example, suppose that the realized state of nature is a, agent 1 is in the event $\{a, b\}$. If agent 1 reports the false event $\{c\}$ (notice that agent 2 cannot distinguish state a from state c), then her maximin expected utility does not increase since

$$\begin{aligned} \underline{u}_1(e_1(a) + x_1(c) - e_1(c)) &= \min\{u_1(e_1(a) + x_1(c) - e_1(c)), u_1(x_1(b))\} \\ &= \min\{\sqrt{24}, \sqrt{16}\} = 4 \end{aligned}$$

is not bigger than $\underline{u}_1(x_1(a)) = \min\{\sqrt{20}, \sqrt{16}\} = 4$. Similarly, agent 2 does not have an incentive to misreport the realized state of nature and, consequently, the maximin core allocation above is maximin-IC. However, (9) is not CIC. For if a is the realized state of nature, agent 1 has an incentive to report state c and become better off, that is, $u_1(e_1(a) + x_1(c) - e_1(c)) = \sqrt{24}$, which is bigger than $u_1(x_1(a)) = \sqrt{16} = 4$. ■

In the one good per state case, the private information measurability of an allocation is a necessary and sufficient condition for the CIC.[38] Indeed, observe that

[38] See Krasa and Yannelis (1994, Lemma 1), or Glycopantis, Muir, and Yannelis (2003, Proposition 4.1).

the allocation (9) in the proof of Proposition 3.8 is maximin incentive compatible but it is not private information measurable. However, the maximin coalitional incentive compatibility does not imply measurability, even in the one good case, as was indicated in Proposition 3.8. However, the converse holds, that is, private measurability of a feasible allocation implies maximin incentive compatibility as the Proposition below shows.

Proposition 3.9 *Assume that $x = (x_i)_{i\in I}$ is feasible and measurable, that is, x_i is $\mathcal{F}_i$-measurable for each $i \in I$. Moreover assume that there is only one good ($\ell = 1$) and all utilities are monotonic. Then x is maximin coalitional incentive compatible.*

Proof: Assume that $x = (x_i)_{i\in I}$ is feasible and measurable, but it not is maximin coalitional incentive compatible. Then, there exists a coalition $C \subset I$, states ω', $\omega'' \in \Omega$ and transfers $(\tau_i)_{i\in C}$ satisfying:

1. $\mathcal{F}_j(\omega') = \mathcal{F}_j(\omega'')$ for all $j \notin C$;
2. $\sum_{i\in C} \tau_i = 0$;
3. $\underline{u}_i\left(e_i(\omega') + x_i(\omega'') - e_i(\omega'') + \tau_i\right) > \underline{u}_i\left(x_i(\omega')\right)$, for all $i \in C$.

From 1 above, we have that for all $i \notin C$, $x_i(\omega') - e_i(\omega') = x_i(\omega'') - e_i(\omega'')$, because x_i and e_i are $\mathcal{F}_i$-measurable. Therefore,

$$\sum_{i\notin C}\left(x_i(\omega') - e_i(\omega')\right) = \sum_{i\notin C}\left(x_i(\omega'') - e_i(\omega'')\right). \tag{10}$$

On the other hand, conditon 3 above implies that:

$$\sum_{i\in C}\left(e_i(\omega') + x_i(\omega'') - e_i(\omega'') + \tau_i\right) > \sum_{i\in C} x_i(\omega').$$

Since $\sum_{i\in C} \tau_i = 0$, this implies:

$$\sum_{i\in C}\left(x_i(\omega') - e_i(\omega')\right) < \sum_{i\in C}\left(x_i(\omega'') - e_i(\omega'')\right). \tag{11}$$

Summing (10) and (11), we obtain:

$$\sum_{i\in I}\left(x_i(\omega') - e_i(\omega')\right) < \sum_{i\in I}\left(x_i(\omega'') - e_i(\omega'')\right).$$

But feasibility requires that both sides in the above inequality are zero, and this is a contradiction. ■

4 Consistency of Incentive Compatibility and Efficiency

We now show any maximin Pareto optimal allocation is maximin incentive compatible.

Theorem 4.1 *If x is a Maximin Pareto Optimal allocation then x is maximin coalitional incentive compatible.*

Proof: Suppose that x is not maximin coalitional incentive compatible. This means that there exists a coalition $C \subset I$, states $\omega', \omega'' \in \Omega$ and transfers $(\tau_i)_{i \in C}$ satisfying:

1. $\mathcal{F}_j(\omega') = \mathcal{F}_j(\omega'')$ for all $j \notin C$;
2. $\sum_{i \in C} \tau_i = 0$;
3. $\underline{u}_i(e_i(\omega') + x_i(\omega'') - e_i(\omega'') + \tau_i) > \underline{u}_i(x_i(\omega'))$, for all $i \in C$.

We will prove that x cannot be maximin Pareto optimal, by constructing another feasible allocation $y = (y_i)_{i \in I}$ that Pareto improves upon x. For this, define $\tau_j = 0$ if $j \notin C$ and, for all $i \in I$:

$$y_i(\omega) = \begin{cases} x_i(\omega), & \text{if } \omega \neq \omega' \\ e_i(\omega') + x_i(\omega'') - e_i(\omega'') + \tau_i, & \text{otherwise} \end{cases} \tag{12}$$

Observe that $(y_i)_{i \in I}$ is feasible. Indeed, for $\omega \neq \omega'$ this comes from the feasibility of $(x_i)_{i \in I}$ and for $\omega = \omega'$ we have:

$$\sum_{i \in I} y_i(\omega') = \sum_{i \in I} e_i(\omega') + \sum_{i \in I} x_i(\omega'') - \sum_{i \in I} e_i(\omega'') + \sum_{i \in I} \tau_i = \sum_{i \in I} e_i(\omega'),$$

because $\sum_{i \in I} \tau_i = \sum_{i \in C} \tau_i = 0$ and $\sum_{i \in I} x_i(\omega'') = \sum_{i \in I} e_i(\omega'')$ (from the feasibility of x_i at ω'').

From 3 above, we have: $\underline{u}_i(y_i(\omega')) > \underline{u}_i(x_i(\omega'))$, for all $i \in C$. Now, using Assumption 2.1, we obtain:

$$\int \underline{u}_i(y_i(\cdot))\, d\mu_i > \int \underline{u}_i(x_i(\cdot))\, d\mu_i,$$

that is, $y_i \succ_i x_i$. In other words, $(y_i)_{i \in I}$ makes all $i \in C$ strictly better off. It remains to prove that $y_j \succcurlyeq_j x_j$ for $j \notin C$.

Fix $j \notin C$. The measurability of e_j and the fact that $\mathcal{F}_j(\omega') = \mathcal{F}_j(\omega'')$ imply that $e_j(\omega) = e_j(\omega'')$ for all $\omega \in \mathcal{F}_i(\omega')$ (from condition 1 above). Since $\tau_j = 0$,

$$y_j(\omega') = e_j(\omega) + x_j(\omega'') - e_j(\omega'') + \tau_j = x_j(\omega''). \tag{13}$$

Let X_j denote the set $\{x_j(\omega) : \omega \in \mathcal{F}_j(\omega')\}$ and $Y_j \equiv \{y_j(\omega) : \omega \in \mathcal{F}_j(\omega')\}$. Let $\omega \in \mathcal{F}_j(\omega')$. If $\omega \neq \omega'$, the definition (12) of y_j implies that $y_j(\omega) = x_j(\omega) \in X_j$. For $\omega = \omega'$, (13) gives $y_j(\omega') = x_j(\omega'') \in X_j$. Thus, $Y_j \subset X_j$. Now, we can use the definition of $\underline{u}_j$ to obtain:

$$\underline{u}_j\left(y_j\left(\omega'\right)\right) = \min_{x \in Y_j} u_j(x) \geqslant \min_{x \in X_j} u_j(x) = \underline{u}_j\left(x_j\left(\omega'\right)\right). \tag{14}$$

Again by the definition (12) of y_j, $y_j(\omega) = x_j(\omega)$ for $\omega \notin \mathcal{F}_j(\omega')$, this implies that $\underline{u}_j\left(y_j\left(\omega\right)\right) \geqslant \underline{u}_j\left(x_j\left(\omega\right)\right)$, for all $j \notin C$ and $\omega \in \Omega$, that is, $y_j \succcurlyeq_i x_j$ for all $j \notin C$. Thus, y is a Maximin Pareto improving upon x, that is, x is not Maximin Pareto optimal. ■

The reader can observe that the only place where we used the specific definition of $\underline{u}_j$ as the minimum was to conclude (14). Indeed if we were to use other preferences, this step would not go through.

Theorem 4.1 solves the long standing problem of the conflict between efficiency and incentive compatibility. As we will see below and in the next sections, Theorem 4.1 has important implications. Specifically, we will show that several new maximin Pareto optimal equilibrium notions will turn out to be incentive compatible.

5 Maximin preferences lead to higher efficiency

In this section we will show that maximin preferences lead to higher efficiency than the Bayesian setup.[39] By this, we mean that if an allocation is efficient (Pareto optimal) and incentive compatible for Bayesian preferences, then it is also Pareto optimal (and therefore incentive compatible) for maximin preferences, although the converse is not true. Formally, we have the following:

Theorem 5.1 *Consider an economy $\mathcal{E} = \{\Omega, (u_i,\ \mathcal{F}_i,\ e_i, \mu_i)_{i \in I}\}$ with just one good ($\ell = 1$) and consider two variants of it:*

(B) the agents have Bayesian preferences, that is, for each $i \in I$ the belief μ_i is extended to all subsets of Ω. This variant will be denoted $\mathcal{E}^B$;

(M) the agents have maximin preferences. This variant will be denoted $\mathcal{E}^M$.

[39]By Bayesian setup we mean an economy where the agents have complete expected utility preferences, that is, with probabilities defined for all events and not only the privately measurable ones.

If x is a Pareto optimal allocation in $\mathcal{E}^B$ which is also incentive compatible, it is a Pareto optimal allocation in $\mathcal{E}^M$, i.e., x is maximin Pareto optimal, but the reverse is not true.

Proof: Let $\succcurlyeq_i^M$ and $\succcurlyeq_i^B$ denote respectively the Maximin and the Bayeasian completions of $\succcurlyeq_i^\circ$. Assume that x is Bayesian Pareto optimal and incentive compatible. We will prove that x_i is $\mathcal{F}_i$-measurable for each $i \in I$.

For a contradiction, suppose x is coalitional incentive compatible but x_j is not measurable for some $j \in I$, that is, suppose that there exist states $a, b \in \Omega$ such that $x_j(a) \neq x_j(b)$. Without loss of generality, we may assume that $x_j(a) > x_j(b)$. Since e_j is $\mathcal{F}_j$-measurable, $e_j(b) = e_j(a)$ and therefore

$$x_j(a) - e_j(a) > x_j(b) - e_j(b). \tag{15}$$

Let $C \equiv I \setminus \{j\}$. From feasibility of x and (15), we have:

$$\begin{aligned}\sum_{i \in C} [x_i(a) - e_i(a)] &= -[x_j(a) - e_j(a)] \\ &< -[x_j(b) - e_j(b)] \\ &= \sum_{i \in C} [x_i(b) - e_i(b)].\end{aligned}$$

Thus,

$$\delta \equiv \sum_{i \in C} [x_i(b) - e_i(b) - x_j(a) + e_j(a)] > 0.$$

For each $i \in C$, let

$$\tau_i \equiv -x_i(b) + e_i(b) + x_i(a) - e_i(a) + \frac{\delta}{n-1},$$

so that $\sum_{i \in C} \tau_i = 0$ and

$$e_i(a) + x_i(b) - e_i(b) + \tau_i > x_i(a).$$

By the monotonicity of u_i, we can conclude that for all $i \in C$,

$$u_i\left(e_i(a) + x_i(b) - e_i(b) + \tau_i\right) > u_i\left(x_i(a)\right),$$

which contradicts the assumption that x is coalitional incentive compatible.

Now, assume that x is not maximin Pareto optimal. This means that there exists a feasible allocation y such that $y_j \succcurlyeq_i^M x_j$ for all $j \in I$ and there is $i \in I$ such that $y_i \succ_i^M x_i$, that is, $\underline{u}_i(y_i(\omega')) > \underline{u}_i(x_i(\omega'))$ for some $\omega' \in \Omega$. Since x_i is $\mathcal{F}_i$-measurable, this implies that $u_i(y_i(\omega')) > u_i(x_i(\omega'))$ and the monotonicity of u_i now gives $y_i(\omega') > x_i(\omega')$. Similarly, $y_j \succcurlyeq_i^M x_j$ and the fact that x_j is $\mathcal{F}_j$-measurable imply that $y_j(\omega') \geqslant x_j(\omega')$ for all $j \neq i$. But then, $\sum_{i \in I} y_i(\omega') > \sum_{i \in I} x_i(\omega')$ and y is not feasible, which is a contradiction.

The counterexample for the reverse is provided below (subsection 5.1). ■

We illustrate Theorem 5.1 by comparing the maximin Pareto optimal allocation with the perfect risk sharing allocation below.

5.1 Perfect risk sharing efficiency versus maximin efficiency

We will illustrate the previous result with a comparison between perfect risk sharing outcomes and maximin Pareto optimal ones. Consider again example 3.1, where the state space is $\Omega = \{a, b, c\}$. The utility function of each individual is $u_i(x_i) = \sqrt{x_i}$ and endowments and partitions are:

$$\begin{aligned} e_1(a,b,c) &= (5,5,0); \quad \mathcal{F}_1 = \{\{a,b\},\{c\}\}; \\ e_2(a,b,c) &= (5,0,5); \quad \mathcal{F}_2 = \{\{a,c\},\{b\}\}; \\ \mu_1(\{a,b\}) &= \tfrac{2}{3}; \qquad \mu_1(\{c\}) = \frac{1}{3} \\ \mu_2(\{a,c\}) &= \tfrac{2}{3}; \qquad \mu_2(\{b\}) = \frac{1}{3}. \end{aligned}$$

The perfect risk-sharing trade leads to:

$$\begin{aligned} x_1(a,b,c) &= (5, 2.5, 2.5); \\ x_2(a,b,c) &= (5, 2.5, 2.5), \end{aligned}$$

while the maximin Pareto optimal outcome is:

$$\begin{aligned} y_1(a,b,c) &= (5,4,1); \\ y_2(a,b,c) &= (5,1,4), \end{aligned}$$

Let us compare these two equilibrium allocations. Notice that both agents are better off with the maximin Pareto optimal (MPO) allocation y than with the perfect risk-sharing (PRS) allocation x, since

$$\int \underline{u}_i(y_i)\, d\mu_i = \frac{2}{3}\sqrt{\min\{5,4\}} + \frac{1}{3}\sqrt{1} > \frac{2}{3}\sqrt{\min\{5,2.5\}} + \frac{1}{3}\sqrt{2.5} = \int \underline{u}_i(x_i)\, d\mu_i.$$

Clearly, both agents should choose the MPO allocation since it gives higher utility.[40] Moreover, the MPO allocation is incentive compatible (recall Theorem 4.1). To the contrary, if agents have complete von Neumann-Morgensten expected utility, the PRS allocation is efficient, but it is not CIC. To see this, notice that if the realized state of nature is a, agent 1 has an incentive to report state c, because he keeps the initial endowment in state a and adds 2.5 units in state c, that is, $u_1(e_1(a) + x_1(c) - e_1(c)) = u_1(5 + 2.5) > u_1(x_1(a)) = u_1(5)$.[41]

It should be noted that the PRS allocation is not consistent with Bayesian rationality as it is not implementable as a perfect Bayesian equilibrium (see Glycopantis, Muir, and Yannelis (2001, Section 5)).

The above example together with Theorem 4.1 enable us to conclude that the MPO is a reasonable outcome with no conflict between efficiency and incentive compatibility. To the contrary the conflict between efficiency and incentive compatibility seems to be inherent in the perfect risk sharing efficiency as the example above shows.

6 Applications in Asymmetric Information Equilibrium Theory

6.1 Maximin Core

We will now define the notions of maximin core and maximin Pareto optimality.

Definition 6.1 (Maximin Core) *Maximin Core allocation is a feasible allocation $x = (x_i)_{i \in I}$ for which there is no coalition $C \subseteq I$, $C \neq \emptyset$, and allocation $y = (y_i)_{i \in I}$ satisfying $\sum_{i \in C} y_i(\omega) = \sum_{i \in C} e_i(\omega)$, $\forall \omega \in \Omega$, $y_i \succcurlyeq_i x_i$ for all $i \in C$ and $y_i \succ_i x_i$ for at least one $i \in C$.*[42]

The coalition C in the above definition is called a blocking coalition. More precisely:

Definition 6.2 (Blocking coalition) *A blocking coalition C for the allocation x is a nonempty subset of I such that there exists allocation $y = (y_i)_{i \in I}$ satisfying*

[40] In fact, if the extension of μ_1 and μ_2 to 2^Ω is given by $\mu_1(\{a\}) = \frac{1}{9}$; $\mu_1(\{b\}) = \frac{5}{9}$; $\mu_2(\{a\}) = \frac{1}{9}$; $\mu_2(\{c\}) = \frac{5}{9}$; then the perfect risk sharing allocation is also worse than the MPO in the terms of the expected utility: $\frac{1}{9}\sqrt{5} + \frac{5}{9}\sqrt{2.5} + \frac{1}{3}\sqrt{2.5} < \frac{1}{9}\sqrt{5} + \frac{5}{9}\sqrt{4} + \frac{1}{3}\sqrt{1}$.

[41] The no trade is the unique incentive compatible allocation in this example with the standard expected utility formulation. As we remarked earlier, in the one good case, private information measurability characterizes the incentive compatibility.

[42] Recall $f \succ_i g$ is equivalent to $\int \underline{u}_i(f(\cdot))\, d\mu_i > \int \underline{u}_i(g(\cdot))\, d\mu_i$.

$\sum_{i\in C} y_i(\omega) = \sum_{i\in C} e_i(\omega)$, $\forall\omega \in \Omega$, $y_i \succcurlyeq_i x_i$ for all $i \in C$ and $y_i \succ_i x_i$ for at least one $i \in C$.

It is useful to compare the maximin core with the private core (Yannelis (1991)):

Definition 6.3 (Private Core) *A private Core allocation is a feasible and measurable*[43] *allocation $x = (x_i)_{i\in I}$ such that there do not exist coalition $C \subseteq I$, $C \neq \emptyset$ and measurable allocation $y = (y_i)_{i\in I}$, such that: $\sum_{i\in C} y_i(\omega) = \sum_{i\in C} e_i(\omega)$, $\forall\omega \in \Omega$, and, $y_i \succcurlyeq_i x_i$ for all $i \in C$ and $y_i \succ_i x_i$ for at least one $i \in C$.*[44]

The following observation is useful.

Proposition 6.4 *If $x = (x_i)_{i\in I}$ is a maximin core allocation and x_i is $\mathcal{F}_i$-measurable for all $i \in I$, then x is a private core allocation, but the converse is not true.*

Proof: Let $x = (x_i)_{i\in I}$ be a measurable maxmin core allocation. Suppose by way of contradiction that x not a private core allocation. Then, there is coalition $C \subset I$ and allocation $y = (y_i)_{i\in I}$ such that: $\sum_{i\in C} y_i = \sum_{i\in C} e_i$ and, for each $i \in C$, y_i is $\mathcal{F}_i$-measurable and $y_i \succ_i^\circ x_i$. Since both x_i and y_i are $\mathcal{F}_i$-measurable, Lemma 2.4 implies that $y_i \succ_i x_i$. Thus, C is a blocking coliation in the sense of definition 6.2, that is, x is not a maximin core allocation, a contradiction.

To see that the converse is not true, consider an economy with $\Omega = \{a, b, c\}$, $\mu_1(\{\omega\}) = \mu_2(\{\omega\}) = \frac{1}{3}$ for all $\omega \in \Omega$, one good and two consumers with the same utility $\sqrt{x}$. The endowments are $e_1 = (e_1(a), e_1(b), e_1(c)) = (5, 5, 0)$ and $e_2 = (e_2(a), e_2(b), e_2(c)) = (5, 0, 5)$. The partitions are: $\mathcal{F}_1 = \{\{a, b\}, \{c\}\}$ and $\mathcal{F}_2 = \{\{a, c\}, \{b\}\}$. The following allocation is a maximin core which is not measurable: $x = (x_1, x_2) = ((5, 4, 1), (5, 1, 4))$. To see this, observe that a coalition of just one agent would consume only his endowment, which is worse than $x : \frac{2}{3}\sqrt{4} + \frac{1}{3}\sqrt{1} = \frac{5}{3} > \frac{2}{3}\sqrt{5} + \frac{1}{3}\sqrt{0}$. In fact, $\frac{2}{3}\sqrt{4} + \frac{1}{3}\sqrt{1} = \frac{5}{3} = \max_{\varepsilon\in[0,5]} \frac{2}{3}\sqrt{5-\varepsilon} + \frac{1}{3}\sqrt{\varepsilon}$. Thus, the grand coalition cannot improve upon x either. ■

Hence the maximin core and the private core do not need to be a subset of each other and may even have an empty intersection. To see this, recall example 3.1,

[43] We say that an allocation $x = (x_i)_{i\in I}$ is measurable if x_i is $\mathcal{F}_i$-measurable for all $i \in I$.

[44] Recall that

$$y_i \succ_i^\circ x_i \iff \int_\Omega u_i(y_i(\omega))\mu_i(d\omega) > \int_\Omega u_i(x_i(\omega)\mu_i(d\omega).$$

Note that we required that x and y is measurable, that is, x_i and y_i are $\mathcal{F}_i$-measurable for each $i \in I$. Therefore, the integrals in the right are defined for those acts.

where the unique private core is the initial endowment, but the maximin private core is

$$x_1(a,b,c) = (5,4,1);$$
$$x_2(a,b,c) = (5,1,4).$$

However, we have the following:

Corollary 6.5 *Any private core allocation is maximin incentive compatible, but the converse is not true.*

Proof: By Theorem 1 of Koutsougeras and Yannelis (1993), the private core is CIC. By Proposition 3.8, it is also maximin-IC. The example above shows that the converse is not true. ■

As a consequence of Theorem 4.1, we obtain the following immediate corollary:

Corollary 6.6 *Any maximin core allocation is maximin incentive compatible.*

It should be noted that it is straightforward to show the existence of maximin core allocations. The existence follows from the standard balancedness condition (e.g. Scarf (1967)), provided that the utility functions u_i are concave and continuous. Indeed, if u_i is concave, then $\underline{u}_i$ is also concave as Lemma 2.6 establishes. Therefore, the existence of maximin core follows directly from Scarf's theorem.

6.2 Maximin value allocation

In this section, we will introduce the concept of maximin value allocation. We begin with the definition of a game with side payments (or transferable utility game):

Definition 6.7 *A game with side payments $\Gamma = (I, V)$ consist of a finite set of agents $I = \{1, ..., n\}$ and a superadditive, real valued function V defined on 2^I such that $V(\emptyset) = 0$. Each $C \subseteq I$ is called a coalition and $V(S)$ is the 'worth' of the coalition C.*

The Shapley value of the game Γ (Shapley (1953)) assigns to each Agent i a payoff, $Sh_i(V)$, given by the formula below. It measures the sum of the expected marginal contributions an agent can make to all the coalitions of which he/she can be a member.

$$Sh_i(V) = \sum_{\substack{S \subseteq I \\ S \supseteq \{i\}}} \frac{(\mid S \mid -1)!(\mid I \mid - \mid S \mid)!}{\mid I \mid!}[V(S) - V(S \backslash \{i\})]. \tag{16}$$

The Shapley value has the property that $\sum_{i \in I} Sh_i(V) = V(I)$, i.e. the implied allocation of payoffs is Pareto efficient.

We now define for each differential information economy $\mathcal{E}$, with common prior μ, which is assumed for simplicity, and for each set of weights, $\lambda = \{\lambda_i \geq 0 : i = 1, \ldots, n\}$, the associated game with side payments (I, V_λ). We also refer to this as a *transferable utility* (TU) game.

Definition 6.8 (Maximin TU Game) *Given $\{\mathcal{E}, \lambda\}$ an associated game $\Gamma_\lambda = (I, V_\lambda)$ is defined as follows: For every coalition $C \subset I$ let*

$$\begin{aligned} V_\lambda(S) = \max_x \quad & \textstyle\sum_{i \in C} \lambda_i \min_{\pi \in \mathcal{P}_i} \int_\Omega u_i(x_i(\omega))\pi(d\omega) \\ \textit{s.t.} \quad & \textstyle\sum_{i \in C} x_i(\omega) = \sum_{i \in C} e_i(\omega), \forall \omega \in \Omega. \end{aligned} \tag{17}$$

We are now ready to introduce the notion of maximin value allocation.

Definition 6.9 (Maximin Value Allocation) *An allocation x is said to be a* maximin value allocation *(MVA) of the differential information economy $\mathcal{E}$, if the following conditions hold:*

(i) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$ and

(ii) There exist $\lambda_i \geq 0$, for every $i = 1, ..., n$, which are not all equal to zero, such that for all i,

$$\min_{\pi \in \mathcal{P}_i} \int_\Omega \lambda_i u_i(x_i(\omega))\pi(d\omega) = Sh_i(V_\lambda),$$

where $Sh_i(V_\lambda)$ is the Shapley value of Agent i derived from the game (I, V_λ), defined in (17) above.

Conditions (i) is feasibility and (ii) says that the maximin expected utility of each agent multiplied by his/her weight, λ_i, must be equal to his/her Shapley value derived from the TU game (I, V_λ). For the actual utility that the agent will obtain the weight must not be taken into account. An agent could obtain the utility of a positive allocation even if λ_i were zero.

An immediate consequence of Definition 6.8 is that

$$Sh_i(V_\lambda) \geq \min_{\pi \in \mathcal{P}_i} \int_\Omega \lambda_i u_i(e_i(\omega))\pi(d\omega)$$

for every i, i.e. the value allocation is individually rational. Similarly, efficiency of the Shapley value implies that the maximin value allocation is maximin Pareto optimal. Consequently, it follows from Theorem 4.1 that:

Corollary 6.10 *Any maximin value allocation is MCIC.*

We now compare the maximin value allocation with the private value allocation of Krasa and Yannelis (1994). To this end we will need the following definitions:

Definition 6.11 (ex-ante TU Game) *Given $\{\mathcal{E}, \lambda\}$ an associated game $\Gamma_\lambda = (I, V_\lambda)$ is defined as follows: For every coalition $C \subset I$ let*

$$V_\lambda(S) = \max_x \left(\sum_{i \in C} \lambda_i \int_\Omega u_i(x_i(\omega))\pi(d\omega) \right) \tag{18}$$

subject to

(i) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$

(ii) $e_i - x_i$ *is* $\mathcal{F}_i$*-measurable for all* i.

Definition 6.12 (Private Value Allocation) *An allocation x is said to be a* private value allocation *(PVA) of the differential information economy $\mathcal{E}$, if the following conditions hold:*

(i) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$

(ii) $e_i - x_i$ *is* $\mathcal{F}_i$*-measurable for all* i.

(iii) There exist $\lambda_i \geq 0$, for every $i = 1, ..., n$, which are not all equal to zero, such that for all i,

$$\int_\Omega \lambda_i u_i(x_i(\omega))\pi(d\omega) = Sh_i(V_\lambda),$$

where $Sh_i(V_\lambda)$ is the Shapley value of Agent i derived from the game (I, V_λ), defined in (18) above.[45]

[45] Since e_i and $e_i - x_i$ are both measurable, x_i is measurable and the integrals above are well defined.

As it is the case with the relationship of the maximin core and the private core, that is, they are different and they may have an empty intersection, this is also the case between the maximin value allocation and the private value allocation. However, in the case of allocations which are private information measurable, we have the following proposition:

Proposition 6.13 *If $x = (x_i)_{i \in I}$ is a maximin value allocation and x_i is $\mathcal{F}_i$-measurable, then x is also a private value allocation.*

Proof: Let $x = (x_i)_{i \in I}$ be a measurable maximin value allocation and let us prove that it is also a private value allocation. It is clear that (i) and (ii) in definition 6.12 are satisfied because both e_i and x_i are $\mathcal{F}_i$-measurable. Now, since x_i is $\mathcal{F}_i$-measurable then, as in the proof of Lemma 2.4,

$$\min_{\pi \in \mathcal{P}_i} \int_\Omega u_i(x_i(\omega))\pi(d\omega) = \int_\Omega u_i(x_i(\omega))\pi(d\omega).$$

Then (iii) of definition 6.12 follows from (ii) of definition 6.9. ■

As a consequence of Theorem 4.1, we obtain the following corollary:

Corollary 6.14 *Any maximin value allocation is maximin incentive compatible.*

The advantage of the maximin value allocation over the private value allocation is that the outcomes that the maximin value allocation yields are superior in terms of efficiency than the ones that the private value allocation generates. Futhermore, the existence of the maximin value allocation follows directly from the standard existence of value allocation results, without the need to appeal to non-trivial measurable theoretical arguments, as it is the case for the private value allocation.

6.3 Maximin Walrasian Expectations Equilibrium

In this section, we introduce the notion of maximin Walrasian expectations. Before that, however, it is convenient to recall the standard definitions. For this discussion, it is necessary to assume that the measure μ_i is well defined for all subsets of Ω (or in the finest σ-algebra in the infinite case). Thus, for the moment let us assume that μ_i is defined for all subsets of Ω, although the individuals still have partitions defining the private information. For any $x_i \in \mathcal{L}_i$,[46] the *ex ante expected utility* of agent i is given by

$$V_i(x_i) \equiv \int_\Omega u_i(x_i(\omega))\mu_i(d\omega) = \sum_\Omega u_i(\omega, x_i(\omega))\mu_i(\omega).$$

[46]Recall that $\mathcal{L}_i$ is the set of all $\mathcal{F}_i$-measurable functions $f : \Omega \to \mathbb{R}^\ell_+$.

We define a *price system* to be a non-zero function $p : \Omega \rightarrow \mathbb{R}^\ell$ and the *budget set* of agent i is given by

$$B_i(p) = \{x_i \in \mathcal{L}_i : \sum_{\omega\in\Omega} p(\omega)x_i(\omega) \leq \sum_{\omega\in\Omega} p(\omega)e_i(\omega)\}.$$

We now define an ex ante equilibrium concept, which is due to Radner (1968).

Definition 6.15 (WEE) *A pair (p, x), where p is a price system and $x = (x_i)_{i\in I} \in \prod_{i\in I} \mathcal{L}_i$ is an allocation, is a Walrasian Expectation Equilibrium (WEE) if*

(i) $V_i(x_i) \geq V_i(y_i)$*, for all* $y_i \in B_i(p)$*;*

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$*, for all* $\omega \in \Omega$*;*

Just for comparison, it is useful to define the maximin Walrasian expectation equilibrium. For this, define:

$$\underline{V}_i(x_i) \equiv \min_{\pi\in\mathcal{P}_i} \int_\Omega u_i(x_i(\omega))\pi(d\omega) = \int_\Omega \underline{u}_i(x_i(\omega))\mu_i(d\omega).$$

Definition 6.16 (maximin WEE) *A pair (p, x), where p is a price system and $x = (x_i)_{i\in I}$ is an allocation, is a* maximin Walrasian expectation equilibrium (MWE) *if*

(i) $\underline{V}_i(x_i) \geq \underline{V}_i(y_i)$*, for all y_i satisfying* $\sum_{\omega\in\Omega} p(\omega)y_i(\omega) \leq \sum_{\omega\in\Omega} p(\omega)e_i(\omega)$*;*

(ii) $\sum_{i=1}^n x_i(\omega) = \sum_{i=1}^n e_i(\omega)$*, for all* $\omega \in \Omega$*;*

We have the following:

Proposition 6.17 *If $x = (x_i)_{i\in I} \in \prod_{i\in I} \mathcal{L}_i$ and (p, x) is maximin WEE then (p, x) is also a WEE, but the reverse is not true.*

Proof: For the implication, repeat the argument in the proof of Proposition 6.4. For the counterexample of the converse, the unique WEE in Example 3.1 is no trade, but the maximin WEE is $x_1 = (5, 4, 1)$ and $x_2 = (5, 1, 4)$. ■

It should be noted that in the example 3.1, the free disposal WEE is $x_1 = (4, 4, 1)$ and $x_2 = (4, 1, 4)$ and $p_a = 0$ and $p_b = p_c = 1$. Notice that the free disposal WEE is not incentive compatible because if a is the realized state of nature, agent 1 has an incentive to report c and become better off. It is also important to notice that the free disposal WEE yields lower efficiency than the maximin WEE $x_1 = (5, 4, 1)$ and $x_2 = (5, 1, 4)$.

Notice that the existence of a maximin WEE follows from the existence of standard deterministic Walrasian equilibrium simply by increasing the dimensionality of the commodity space (by the number of states). To the contrary, the existence of a no free disposal WEE is quite complicated because the private information measurability consumption set constraints does not allow one to include initial endowments on the interior of the consumption set, as noted in Podczeck and Yannelis (2008).[47]

Finally, as a consequence of Theorem 4.1, we obtain the following corollary:

Corollary 6.18 *Any maximin Walrasian expectation equilibrium is maximin incentive compatible.*

As already alluded above, the maximin WEE yields superior outcomes in terms of efficiency than standard WEE, which is based on the private information measurability and as consequence is only private information efficient.

7 Results in the type model

The main results of this paper can be adapted to Harsanyi's type model. de Castro and Yannelis (2009) describe the relationship between type and partition model for the Bayesian case. The adaptation for the MEU case discussed here is easy, as this section shows. A more lengthy discussion of how to do this translation can be found in de Castro and Yannelis (2009).

For the statement of Theorem 4.1 in type model, we need some definitions and assumptions. Let us consider a finite set of payoff-relevant states S.[48] Each

[47] Correia-da Silva and Hervés-Beloso (2009) have used a maximin expected utility for a general equilibrium model with uncertain deliveries, and proved the existence of a new equilibrium concept, which they call prudent equilibrium. Although the focus of their paper is different from ours, their paper seems to contain the first concrete application of the MEU to equilibrium analysis with asymmetric information.

[48] In most papers with type models, all payoff relevant characteristics are captured only by the types, which corresponds in our model to a case without S. Of course, it is always possible to include the information given by S directly in the types. We consider explicitly S following Morris (1994). For applications where S is not considered, it is sufficient to assume that S is a singleton in all the developments below.

agent $i \in I$ observes a signal in some finite set of possible signals, $t_i \in T_i$. Write $T = T_1 \times \cdots \times T_n$. A vector $t = (t_1, \ldots, t_i, \ldots, t_n)$ represents the vector of all types. T_{-i} denotes $\Pi^n_{i \neq j} T_j$ and, similarly, t_{-i} denotes $(t_1, \ldots, t_{i-1}, t_{t+1}, \ldots, t_n)$. The set of states of the world is $T \times S$.

We will denote the partitions in the type model by Π_i, defined in the following way: for any (t, s) and (t', s'), $(t, s) \in \Pi_i(t', s')$ if and only if $t'_i = t_i$. For example, if there are two agents with two possible types, $T_1 = \{t'_1, t^*_1\}$ and $T_2 = \{t'_2, t^*_2\}$, agent 1 cannot distinguish state (t'_1, t'_2) from state (t'_1, t^*_2), that is, $\Pi_1(t'_1, t'_2) = \Pi_1(t'_1, t'^*_2) = \{(t'_1, t'_2), (t'_1, t^*_2)\}$.

Let $\Delta(T \times S)$ denote the set of probability distributions over $T \times S$. Each agent has a prior probability distribution $\pi_i \in \Delta(T \times S)$. For simplicity, we will assume that π_i has always full support and, for the moment, that it is defined in all subsets of $T \times S$:

Assumption 7.1 (Full support) *For each $i \in I$ and each element A of the partition Π_i, we have $\pi_i(A) > 0$.*

Each individual has a *utility function* $v_i : \mathbb{R}^\ell_+ \to \mathbb{R}$. We denote this type model by $(T \times S, (v_i, \pi_i)_{i \in I})$. For each i, define the *initial endowment* of agent i by $\epsilon_i : T_i \to \mathbb{R}^\ell_+$.[49] It will be convenient to write ϵ_i as a function of $T \times S$, that is, $\epsilon_i : T \times S \to \mathbb{R}^\ell_+$, although it depends only on T_i. Notice that by definition the initial endowment is T_i-measurable (private information measurable).

To sum up, the type economy is formally described by the profile $\mathcal{E}^T = \{T \times S, (v_i, T_i, \Pi_i, \pi_i, \epsilon_i)_{i \in I}\}$, where the superscript T (for types) will be omitted whenever there is no peril of confusion.

If agent i has EU preferences, has type t_i but reports type t'_i, his interim utility function will be:

$$V_i^{x_i}(t'_i | t_i) \equiv \int_{T_{-i}} v_i \left[x_i(t_{-i}, t'_i, s) - e_i(t'_i) + e_i(t_i) \right] \; \pi_i(dt_{-i} | t_i), \qquad (19)$$

Note that the interim expected utility requires an integration with respect to the types of the others. On the other hand, there is no integration in the maximin case, because the decision maker is, instead, taking the minimum over all possible types. That is, in the case of If agent i is of type t_i but reports type t'_i, his interim utility function will be:

$$\underline{V}_i^{x_i}(t'_i | t_i) \equiv \underline{v}_i(x_i(t_{-i}, t'_i, s) - e_i(t'_i) + e_i(t_i)), \qquad (20)$$

[49] We will use ϵ for the endowment in the types model only in this section and in section 7. In the rest of the paper we will e for endowment in both the partition and the type model. This should cause no confusion, since equivalence will require the equality of the two (see definition **??**).

where $\underline{v}_i\left(y_i\left(t_i, t_{-i}, s\right)\right) = \min_{t_{-i}\in T_{-i}(t_i)} v_i\left(y_i\left(t_i, t_{-i}, s\right)\right)$ and $T_{-i}(t_i)$ is the set of types $t_{-i} \in T_{-i}$ that happen with positive probability given t_i, that is, $T_{-i}(t_i) \equiv \{t_{-i} \in T_{-i} : \Pr(\{(t_i, t_{-i})\}) > 0\}$. If x is clear from the context, we will write just $\underline{V}_i(t_i'|t_i)$ instead of $\underline{V}_i^{x_i}(t_i'|t_i)$.

Definition 7.2 (Maximin IC in the type model) *An allocation x is said to be maximin incentive compatible with respect to type model if*

$$\underline{V}_i(t_i|t_i) \geq \underline{V}_i(t_i'|t_i), \text{ for all } i \in I, t_i, t_i' \in T_i.$$

The following assumption is the key to translate results from partition to type models. de Castro and Yannelis (2009) provide sufficient conditions (essentially technical) for this assumption to be satisfied.

Assumption 7.3 *Given the type economy $\mathcal{E}^T = \{T \times S, (v_i,\ T_i,\ \Pi_i,\ \pi_i,\ \epsilon_i)_{i\in I}\}$, there is a economy $\mathcal{E}^P = \{\Omega, (v_i,\ \mathcal{F}_i,\ e_i, \mu_i)_{i\in I}\}$ and an equivalence mapping $f : \Omega \to T \times S$ satisfying the following:*

1. $\mathcal{F}_i(\omega) = \Pi_i(f(\omega))$;
2. $e_i(\omega) = \epsilon_i(f(\omega))$; *and*
3. $\mu_i(\mathcal{F}_i(\omega)) = \pi_i(\Pi_i(f(\omega)))$.

We now translate example 3.1 of the partition model into the type model and show that for the standard expected utility efficient allocations may not be incentive compatible.[50]

Example 7.4 (Example 3.1 in type model.) *We need to enlarge the state space of example 3.1 from three to four states by adding a zero probability state d.*[51] *That is, now $\Omega = \{a, b, c, d\}$; and $\mu\left(\{\omega\}\right) = \frac{1}{3}$, for all $\omega \in \Omega \setminus \{d\}$ and $\mu(\{d\}) = 0$. Agent i's utility function is again $u_i : \mathbb{R}_+ \to \mathbb{R}_+$ given by $u_i\left(x_i\right) = \sqrt{x_i}$. The types correspond to:*

$$t_1 = H_1 = \{a, b\}; \quad t_1 = L_1 = \{c, d\};$$
$$t_2 = H_2 = \{a, c\}; \quad t_2 = L_2 = \{b, d\}.$$

[50] A related example to this was presented by Holmstrom and Myerson (1983), but in their model there are no initial endowments, and all the private information comes from the payoff functions. The example below takes into account the initial endowments.

[51] The introduction of a new state is the procedure described in de Castro and Yannelis (2009) to obtain the equivalence of type and partition models. However the development here is self-contained.

Therefore, the private information $\mathcal{F}_1 = \{\{a,b\},\{c,d\}\}$ *and* $\mathcal{F}_2 = \{\{a,c\},\{b,d\}\}$ *means that each individual knows her own type. Initial endowments* $e_1(a,b,c,d) = (5,5,0,0)$ *and* $e_2(a,b,c,d) = (5,0,5,0)$ *can be written, for* $i = 1,2$ *as*

$$e_i(t_i, t_{-i}) = \begin{cases} 5, & \textit{if } t_i = H \\ 0, & \textit{if } t_i = L \end{cases}$$

The probabilities of the states are the same as before, that is, $\Pr(\{(t_1,t_2)\}) = 0$ *if* $(t_1,t_2) = (L,L)$ *and* $1/3$ *otherwise.*[52] *Let* $U_i(t_i', t_{-i}|t_i)$ *denote the utility of player* i *who is of type* t_i *but reports type* t_i'*. Let* $t = (t_1,t_2)$ *and consider the allocation:*

$$x_1(t) = \begin{cases} 5, & \textit{if } t = (H_1,H_2) \\ 4, & \textit{if } t = (H_1,L_2) \\ 1, & \textit{if } t = (L_1,H_2) \\ 0, & \textit{if } t = (L_1,L_2) \end{cases} ; \; x_2(t) = \begin{cases} 5, & \textit{if } t = (H_1,H_2) \\ 1, & \textit{if } t = (H_1,L_2) \\ 4, & \textit{if } t = (L_1,H_2) \\ 0, & \textit{if } t = (L_1,L_2) \end{cases}$$

This is not incentive compatible for EU decision makers, because if $t_1 = H_1$, then individual 1 has an incentive to misreport $t_1' = L_1$, since:

$$\begin{aligned} V_1(t_1'|t_1) &= \int_{T_2} v_1\left[x_1(t_1',t_2) - e_1(t_1') + e_1(t_1)\right] \; \pi_1(dt_2|t_1), \\ &= v_1\left[x_1(L_1,H_2) - e_1(L_1) + e_1(H_1)\right] \Pr(t_2 = H_2|t_1 = H_1) \\ &\quad + v_1\left[x_1(L_1,L_2) - e_1(L_1) + e_1(H_1)\right] \Pr(t_2 = L_2|t_1 = H_1) \\ &= \sqrt{1-0+5}\,\Pr(t_2 = H_2|t_1 = H_1) + \sqrt{0-0+5}\,\Pr(t_2 = L_2|t_1 = H_1) \\ &> \sqrt{5}\,\Pr(t_2 = H_2|t_1 = H_1) + \sqrt{5}\,\Pr(t_2 = L_2|t_1 = H_1) \\ &= V_1(t_1|t_1). \end{aligned}$$

Therefore, the above example is not incentive compatible in the type model as well. Now we are able to state our result for type model.

Theorem 7.5 *If* x *is maximin Pareto optimal in the type model, it is maximin incentive compatible.*

Proof: From assumption 7.3, there is a partition economy which is equivalent to the type economy. If x is maximin Pareto optimal in the type model, its equivalent allocation is maximin Pareto optimal in the partition model. Theorem 4.1 implies that it is coalitional maximin incentive compatible (definition 3.6), which implies individual maximin incentive compatible (definition 7.2). This concludes the proof. ■

The results in section 5 can be similarly translated to type models.

[52] Therefore, the types are statistically dependent. This is just to maintain the consistency with the previous example, but the dependence of types plays no role here.

8 Discussion

8.1 Related Literature

It is well known that in a finite economy with asymmetric information once people exhibit standard expected utility, then it is not possible to construct allocations which are Pareto optimal and also incentive compatible. Section 3.2 (example 3.3) of this paper made this point clear. The key issue is that in a finite economy each agent's private information has an impact and therefore an agent will take advantage of this private informational effect to influence the equilibrium allocation to favor herself. This is what creates the incentive compatibilitry problem. Several ways to get around this problem have been proposed. Yannelis (1991) imposes the private information measurability condition, and in this case indeed any private information Pareto optimal allocation is incentive compatible (see Koutsougeras and Yannelis (1993)). However, as we showed in Section 4 there is a big loss of efficiency due to the private information measurabiliry condition.

Different solutions have been proposed by Gul and Postlewaite (1992) and McLean and Postlewaite (2002). Those authors impose an "informational smallness" condition and show the existence of incentive compatible and Pareto optima allocations in an approximate sense for a replica economy. The informational smallness can be viewed as an approximation of the idea of perfect competition and as a consequence only approximate results can be obtained in this replica economy framework. Sun and Yannelis (2007) and Sun and Yannelis (2008) formulate the idea of perfect competition in an asymmetric information economy. In this case each individual's private information has negligible influence and as consequence of the negligibility of the private information, they are able to show that any ex ante Pareto optimal allocation is incentive compatible.

The solution proposed in this paper differs from the above modeling as it is based on the adaptation of the maxmin expected utility. Our approach appears not only to solve the problem in a satisfactory way, but also to provide new insights and new ways of looking an equilibrium outcomes. It seems to us that the standard expected utility formulation is not adequate to deal with the asymmetric information problems for two main reasons: First as we showed in section 2.1 there is an incompleteness problem (the Ellsberg paradox makes this clear, as we discussed in section 2.1). This incompleteness problem restrict the choices of the agents and as a consequence effects negatively their expected utility as our examples in section 3 indicated. The second issue is that the standard expected utility does not allow agents to secure themselves against the risk of being cheated. To the contrary the maximin expected utility formulation forces the choices to be incentive compatible by taking into account the "worse possible state scenario" and this guarantees incentive compatibility. In other words incentive compatibility is in a way inherent on the maximin utility formulation.

The maximin criterion has a long history in Economics. It has been analyzed by Milnor (1954). Rawls (1971) argues for its adoption. Barbera and Jackson (1988) provide an axiomatization of the principle. Binmore (2008, Chapter 9) presents an interesting discussion of the principle, making the connection of the large worlds of Savage (1972). Gilboa and Schmeidler (1989) generalize the maximin criterion (see footone 25).

Bewley (2002): introduces a model of decision under incomplete information. His model also includes our preference $\succcurlyeq_i^\circ$, as a special case. However, we do not use Bewley's representation for $\succcurlyeq_i^\circ$, because our case allows the simplification of describing it just as an incomplete expected utility. Interestingly, Bewley (2002) does not think in his model as a possible model for explaining the Ellsberg (1961) thought experiment, since he comments that such experiments "neither confirm nor contradict the theory" (see quote in section 2.1).

The approach taken in this paper is very much related to Gilboa, Maccheroni, Marinacci, and Schmeidler (2008). They consider decision makers who have two preferences. One of these preferences is incomplete and corresponds to the part of her preference that she can justify for third persons. They call this preference objective and model it as a Bewley incomplete preference. The other preference corresponds to a subjective preference, where the decision maker cannot be proven wrong and this is modeled as a maximin expected utility preference. It is clear that our preference models are related and, from a mathematical point of view, ours is a particular case of Gilboa, Maccheroni, Marinacci, and Schmeidler (2008). However, our models have different motivations and also, subtle conceptual differences. First, the incompleteness arises in our paper from the asymmetric information that an individual faces in an economy. Also, the incomplete information part of our preference is modeled only as an expected utility, rather than as a Bewley preference (which requires dealing with a set of priors). Second, their subjective preference is a maximin expected utility in the sense of Gilboa and Schmeidler (1989), which is not exactly the maximin criterion that we describe here (see footnote 25).[53] Third, while their paper is devoted to the axiomatization of these preferences, we just observe that these preferences arise from a new way of looking at

[53]We do not view the fact that it is a *particular case* as a shortcoming. Of course, to prove things in more general settings is preferable, but our point is that our cases are exactly the simplest and more familiar instances of those models. Thus the main justification of the particularization is simplicity. Moreover, these models are well founded in the normative aspect. One can view our contribution as a saying that groups of individuals that have these kind of preferences will have the possibility of trade and, therefore, will become better off. Thus, our results can be seen as a "natural selection" justification for these preferences to appear and flourish.

the old problem of asymmetric information in general economies, and do not attempt to axiomatize them.[54] Our ultimate justification for the preferences considered here is the theorems obtained, which show a considerable advance in relation with previous ones in the asymmetric information literature. In other words, the new equilibrium concepts seem to have appealing properties as we indicated.

In a recent paper, Gul and Pesendorfer (2009) propose an axiomatization of preferences that allows them to deal with unmeasurable acts. They call this preference subjective expected uncertain utility theory (SEUU). The decision maker uses an inner probability to assess the likelihood that the price will be in intervals. In their representation, the decision maker is subjective expected utility maximizer in the class of unambiguous events, which, in our model, correspond to the evens which are measurable with respect to the private information partition. Our models of behavior differ, however, out of the unambiguous events. Note also, that their focus is not the asymmetric information, as it is in this paper.

Rigotti, Shannon, and Strzalecki (2008) characterize conditions for ex ante efficiency for convex preferences (the first) and MEU preferences (the second). Both cases include the preferences in this paper, but we are interested in the normative properties of the asymmetric information equilibrium.

Another related paper is Morris (1994). He works with the Hasanyi's type model, which we discuss in section 7. He departures from the Milgrom and Stokey (1982) no-trade theorem, which requires the common prior assumption, and shows that the incentive compatibility requirement allows for obtaining equivalent no-trade theorems under assumptions weaker than the common prior assumption.

Lehrer (2008) axiomatizes a model with partial probabilities. Our preferences are again a particular case of his, although our presentation and motivation is quite different from his. Mukerji (1998) used a model with ambiguity to analyze the problem of investment holdup and incomplete contracts in a model with moral hazard. Interestingly, he obtains results that go in the opposite direction than those obtained here: in the moral hazard model that he considers, ambiguity makes harder to obtain incentive compatibility, not easier as we proved for our general equilibrium with asymmetric information model.[55] The connection between ambiguity and information has been addressed before by Mukerji (1997) and Ghirardato (2001).

[54]The preferences that we consider have been previously axiomatized by Savage (1972), Milnor (1954) and Barbera and Jackson (1988).

[55]We are grateful to Sujoy Mukerji for bringing this paper to our attention.

8.2 Incompleteness and Bewley preferences

It seems to be a general understanding that Bewley's preferences are the canonical model of incomplete preferences. As we discussed in section 2.1, we have a simpler model using only the measurability condition of expected utility. To see that this is not equivalent to Bewley's preference (although it is a particular case), consider the following example: $\Omega = \{0, 1\}$ and the set of probabilities over Ω is represented by $[0, 1]$ where $p \in [0, 1]$ corresponds to the probability of the value 1. Let

$$\mathcal{B} \equiv \{p \in [0, 1] : p \leq \frac{1}{2}\}.$$

It is clear that $\mathcal{B}$ is compact and convex. Thus the following defines an *incomplete* Bewley's preference:

$$f \succcurlyeq^B g \iff \int_\Omega f \, dp \geq \int_\Omega g \, dp, \forall p \in \mathcal{B}.$$

However, this preference cannot be represented by a preference as $\succcurlyeq_i^\circ$ given by (1). The reason is that there are only two possible partitions, the trivial one and $\{\{0\}, \{1\}\}$. If we define a probability in the last partition, then the preference would be complete and if we define only in the trivial partition, only constant functions could be compared, which is not the case in the above preference. Therefore, the preferences defined by (1) is a *simpler* model of incomplete preferences, strictly distinct from Bewley's preferences, which allow arbitrary set of probabilities.[56] The advantages of simplicity are discussed below.

8.3 Simplicity in the description of preferences

Ellsberg's (1961) influential paper stimulated a huge literature on ambiguity aversion, originated mainly by Schmeidler (1989) and Gilboa and Schmeidler (1989). Although we take no critical position with respect to this literature, it is important to observe that a very simple combination of old ideas can explain Ellsberg's choices. We would like to add two comments on this.

First, why is simplicity important? Many reasons can be given for this, beginning with cognitive reasons and passing by the Occam's razor principle. However, we would like to emphasize that both parts of our description—the expected utility preferences and the maximin criterion—are well-known concepts of classical economics. The *normative* aspects of these notions were also well discussed. The Ellsberg paradox can be understood as an evidence that the composition that we

[56] See also footnote 25.

offer here is a good *description* of actual behavior by people. Moreover, our results suggest advantages for groups of individuals that develop these preferences. If these familiar notions can be so advantageously put together to explain the ambiguity aversion phenomena, why not just using them?

Second, as we observed in section 2.1, the concepts that we use here were already well-known when Ellsberg wrote his criticism of expected utility. Thus, a natural question is: why the two ideas were not combined by Ellsberg himself? In fact, Ellsberg did consider this explanation. It is interesting to see the following quote from his paper:

> It might now occur to him to ask: "What might happen to me if my best estimates of likelihood don't apply? What is the *worst* of the reasonable distributions of pay-off that I might associate with action I? With action II?" He might find that he could answer this question about the lower limit of the reasonable expectations for a give action much more confidently than he could arrive at a single, "best guess" expectation; the latter estimate, he might suspect, might vary almost hourly with his mood, whereas the former might look much more solid, almost a "fact," a piece of evidence definitely worth considering in making his choice.[57]

As we see, Ellsberg (1961) considered the criterion mentioned in section 2.1. Also, in section III of his paper (from where the above was extracted) he explicitly consider the maximin criterion with a combination with a standard expected utility evaluation and shows how this could explain the choices in his examples. His model of preferences, however, is not exactly the one considered in this paper.

A way to interpret Ellsberg (1961) contribution is that he was in fact criticizing the Bayesian doctrine, which prescribes attaching probabilities whenever the decision maker does not know an outcome for sure. This doctrine seemed to be unnecessarily mixed with expected utility theory. As we discuss in our footnote 21, the Bayesian doctrine and the expected utility theory are independent and we surely can disregard the first while maintaining the first (restricted to the convenient events).

8.4 Extreme pessimism

Some authors (including Ellsberg himself) find the maximin criterion too pessimistic to be adopted or suggested as a normative prescription. The results of

[57] Ellsberg (1961, p.662).

this paper suggests that, contrary to the intuition, societies with individuals following the maximin criterion may be better than the societies where agents adopt the standard Bayesian criterion. It is an open question whether other criteria could deliver similar results.

Remember, however, that this extreme pessimism is mostly criticized when the decisions are made "against" an impersonal nature, that has no reason to act strategically to harm the decision maker. Thus, this extreme pessimism seems unjustifiable in a game against an interested nature.[58] However, the situation studied in this paper is that of an economy with asymmetric information, where the exchange of goods depend on private information, which can be manipulated by strategic agents. In this situation, pessimism is a much more reasonable position.

On the other hand, one can ask: if this is a strategic situation, why not model it as a game? Ideally, this would be the right thing to do. However, scientific theories need to make simplifications that allow to obtain a tractable model. The simplification done here allows to obtain such a tractable model, while retaining some important insights. It should be also remembered that most game theoretic models, even of incomplete information, require assumptions as common knowledge of the structure of the game that may be unrealistic in generic economic situations as those treated here. Finally, it should be remembered that the elegant Arrow-Debreu theory of general equilibrium begins with the assumption that individuals are price takers, exactly for saving the theory from the difficulties inherent to games.

8.5 Changing the information used for decision

It may be helpful to consider the theory so far developed in cases where the original information of the agents change. A potential application of this change occurs in models with rational expectation equilibria, in which consumers combine the information revealed by prices with their own private information. Another example occurs when individuals who participate in a coalition decide to pool their own private information.

For this, it is enough to make just a small change in notation. From now, let $\mathcal{F}_i^0$ (and not $\mathcal{F}_i$) denote the initial partition of individual i. However, for reasons specific to each model considered, instead of using the initial partition $\mathcal{F}_i^0$, she makes her decisions considering another partition (which represents her information), as follows:

[58] Although the word "Nature" suggest an impersonal agent, there is nothing in the formalism that requires this to be the case. It is interesting to see how Milnor, in his paper "Games Against Nature" describe such a player: "a fictitious player having no known objective and no known strategy." Milnor (1954, p.49).

- *Rational expectation information*: Let $\mathcal{F}^p$ denote the partition generated by observing the equilibrium prices p. Then, $\mathcal{F}_i^{re}(\omega) \equiv \mathcal{F}_i^0(\omega) \cap \mathcal{F}^p(\omega)$.
- *Common knowledge information*: Let $\wedge_{i\in I}\mathcal{F}_i^0$ denote the common knowledge partition, that is, the finest common coarsening of the partitions of all individuals. Then, $\mathcal{F}_i^{ck}(\omega)$ or $\mathcal{F}_i^{\wedge}(\omega) = \mathcal{F}^{\wedge}(\omega) = \wedge_{i\in I}\mathcal{F}_i^0(\omega)$.
- *Total knowledge information*: Let $\vee_{i\in I}\mathcal{F}_i^0$ denote the total knowledge partition, that is, the coarset refinement of the partitions of all individuals. Then, $\mathcal{F}_i^{tk}(\omega)$ or $\mathcal{F}_i^{\vee}(\omega) = \mathcal{F}^{\vee}(\omega) = \vee_{i\in I}\mathcal{F}_i^0(\omega)$.
- *Coalition information*: this is the partition that refines the partition of all individuals in a given coalition, that is, $\mathcal{F}^C(\omega) = \vee_{i\in C}\mathcal{F}_i^0(\omega)$.

Below, we will adapt previous notations for the rational expectation case. The other cases are analogous. We can change the definition of set of measures in equation (2) to the following:

$$\mathcal{P}_i^{re} \equiv \{\pi \in \Delta : \pi(A) = \mu_i(A), \forall A \in \mathcal{F}_i^{re}\}. \tag{21}$$

Thus, $\mathcal{P}_i$ is the set of all extensions of μ_i to from $\mathcal{F}_i^{re}$ to $\mathcal{F}$, that is, the set of all probability measures defined in $\mathcal{F}$ that agree with μ_i in the events that individual i is informed about. Then, we consider the preference $\succcurlyeq_i^{re}$ which extends $\succcurlyeq_i^{\circ}$ from $\mathcal{L}_i$ to the set of all acts, $\mathcal{L}$:

$$f \succcurlyeq_i^{re} g \iff \min_{\pi\in\mathcal{P}_i^{re}} \int_\Omega u_i(f(\omega))\pi(d\omega) \geqslant \min_{\pi\in\mathcal{P}_i^{re}} \int_\Omega u_i(g(\omega))\pi(d\omega), \forall f, g \in \mathcal{L}. \tag{22}$$

Again as before, we can define:

$$\underline{u}_i^{re}(x_i(\omega)) \equiv \min_{\omega'\in\mathcal{F}_i^{re}(\omega)} u_i(x_i((\omega'))). \tag{23}$$

Whenever the context is clear, we will write $\underline{u}_i$ instead of $\underline{u}_i^{re}$. The preference $\succcurlyeq_i^{re}$ given by (22) is equivalently characterized by:

$$f \succcurlyeq_i^{re} g \iff \int_\Omega \underline{u}_i(f(\omega))\mu_i(d\omega) \geqslant \int_\Omega \underline{u}_i(g(\omega))\mu_i(d\omega), \forall f, g \in \mathcal{L}. \tag{24}$$

All the previous results then have easy generalizations for the corresponding set of different informations and, therefore, additional concepts (e.g. fine maximin value or core, coarse maximin value or core) can be defined as in section 6.

9 Concluding remarks and open questions

By introducing the maximin expected utility into the asymmetric information equilibrium theory we were able to solve the conflict between efficiency and incentive compatibility. We introduced new equilibrium notions, i.e., maximin core, maximin value and maximin WEE, which are now simultaneusly efficient and incentive compatible. This new way of looking at equilibrium concepts seems quite promising because in addition to resolving the conflict of efficiency and incentive compatibility, the MEU provides more effient outcomes . These facts may stimulate applications of our results in other fields and in particular to experimental and neuroeconomics as the passage from the standard expected utility to the maximin one in designing contracts, provides higher efficiency. The need of testing such results with subjects is rather obvious.

We have not pursued the issue of the implementation of the new equlibrium concepts. It is our conjecture that in view of the inherent efficiency and incentive compatibility of the new equilibrium notions, one should be able to show that they are implementable as a maximin perfect equilibrium and thus provide non cooperative foundations for the maximin core and maximin value. At the moment this seems to be an open question. With a finite number of states, the existence and equivalence theorems for most of our new notions is rather straightforward, as the classical finite dimensional results are directly applicable. Indeed, by simply increasing the dimensionality of the commodity to incorporate the states of nature, existing finite dimensional deterministic theorems can be applied. However, with a continuum of states the commodity space becomes infinite dimensional and the appropriate topologies need to introduced for the compatibility of compactness on the space of allocations and of the continuity of the maxmin expected utility. The arguments in this case are more delicate, and can be addressed in future work.

A Lies and its consequences

In a model of asymmetric information, in which the agents are supposed to report their private information, one has to choose what misreports are allowed and what are their consequences. In the types model, an agent may misreport her type and, *provided that all type profiles* $(t_1, ..., t_n)$ *occur with positive probability*, no misreport will ever be detected. Therefore, the question of what misreports are allowed seems trivial for the type model.

In the partition model, there is more freedom for misreports and this question is not so trivial. Rationality of the players and the common knowledge of the partitions of all players (as we implicitly assume) require that each agent reports only elements of the partition and can choose only strategies for misreports which are measurable with respect to her own partition. To formalize this restriction, we need the concept of deceptions, as in Palfrey and Srivastava (1989). A deception for individual i is a function $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$, that is, a function between the elements of the partition. This restriction is completely equivalent to the one in the type model.

However, partition models allow the possibility that some other agent is able to detect a lie. For instance, in example 3.2, where $\mathcal{F}_1 = \{\{a, b, c\}\}$; $\mathcal{F}_2 = \{\{a, b\}, \{c\}\}$; and $\mathcal{F}_3 = \{\{a\}, \{b\}, \{c\}\}$, agent 1 can detect no lies while agent 3 is able to detect any lie.[59] It is then important to specify what happens if an agent or group of agents is able to detect the lie (in all or some states). The simplest choices are: 1) the cheating agent is infinitely punished; or 2) nothing happens. The first choice is equivalent to forbid that kind of lie, while the second is equivalent to allow agents to use it. Therefore, the initial question is translated in a specification of allowed lies. To discuss the possible specifications, let us introduce more notation.

Since we want to consider many different specifications, we allow that a lie is told not only by just one individual, but by a group of individuals, which will be called a coalition. Given a coalition of agents $C \subset I$ (of course C can be formed by only one agent) and two states ω and ω', we will denote by (C, ω, ω') the lie told by coalition C that the state of the world is ω' when is in fact ω. For lying in this way, the individuals in the coalition will use deceptions $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$, for $i \in C$, such that $\alpha_i(\mathcal{F}_i(\omega)) = \mathcal{F}_i(\omega')$. This means that the coalition C try to implement the trades supposed to happen under state ω' when the true state of the world is ω.

Let us list some of the most common specifications of allowed lies by individuals or coalitions of individuals:

1. The lie (C, ω, ω') cannot be detected by the individuals out of C at the state ω. In formal terms, $\mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$ for all $j \notin C$.

   Note that this condition is very weak. If it is not satisfied, some individual not in the coalition will know the lie when the state ω occurs. This individual will not believe the lie (C, ω, ω') and, therefore, can prevent the implementation of the ω'-trades. On the other hand, since this specification does not require that the coalition knows that the true state of the nature is ω, it is possible the existence of another state ω'', where the lie will be detected. That is, this condition does not rule out the existence of $\omega'' \in \cap_{i \in C} \mathcal{F}_i(\omega)$ and $j \notin C$

[59] Since types and partition models are equivalent (see section 7), this difference between the models may seem strange. In fact, in types model with profile types occurring with zero probability, there is also the possibility of detection of lies. As we discussed in section 7, the occurrence of some combinations of types with zero probability may be necessary for the equivalence of the two models. Our analysis encompasses the more general case.

such that $\mathcal{F}_j(\omega'') \neq \mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$. In this case, individual j out of the coalition can detect the lie at a state ω'' that is considered possible by all members of the coalition. This possibility is ruled out by the following:

2. The lie cannot be detected by any agent out of the coalition at any state that the coalition considers possible. In formal terms, if $\omega'' \in \cap_{i \in C} \mathcal{F}_i(\omega)$ then $\mathcal{F}_j(\omega'') = \mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$ for all $j \notin C$.

   This condition is of course more restrictive, but it still allows lies in which the agents in the coalition are not sure that they are free from being detected. That is, although there is no $\omega'' \in \cap_{i \in C} \mathcal{F}_i(\omega)$ and $j \notin C$ such that $\mathcal{F}_j(\omega'') \neq \mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$, it is possible that there is $i \in C$, $j \notin C$ and $\omega'' \in \mathcal{F}_i(\omega)$ such that $\mathcal{F}_j(\omega'') \neq \mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$. In this case, individual i may be afraid of being cheated by other members of the coalition. This leads to another requirement:

3. It is common knowledge among the participants of the coalition that the lie cannot be detected by any agent out of the coalition. In formal terms, $\cup_{i \in C} \mathcal{F}_i(\omega) \subset \mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$.

   This already seems a sufficiently restrictive condition, but it is still possible to require that the member of the coalition actually know that the true state is ω, in order to be sure that the lie will be implemented. This is the content of the following:

4. the agents in the coalition know that the true state is ω and the agents out of the coalition cannot detect the lie. Formally, $\cap_{i \in C} \mathcal{F}_i(\omega) = \{\omega\}$ and $\mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$ for all $j \notin C$. More restrictively yet, we can require that $\mathcal{F}_i(\omega) = \{\omega\}, \forall i \in C$, instead of only $\cap_{i \in C} \mathcal{F}_i(\omega) = \{\omega\}$.

It is clear that we presented the above conditions in the order of increasing restrictiveness. That is, condition 1 allows more lies than condition 2, and so on.[60] Recall that incentive compatibility is the condition that no lie is better than the truth. Thus, the more lies are allowed, the stronger the definition of incentive compatibility, because truth-telling has to be optimal against more alternative lies. This paper will consider only the weaker restriction above (1) and will, therefore, obtain the strongest possible definition of incentive compatibility. In particular, we will allow coalitional lies instead of just individual lies. Since our main result is to prove the incentive compatibility of efficient allocations, the fact that we choose the

[60] It is worth mentioning that this list does not exhaust all possibilities. For instance, we can require that the lies are implemented only by individuals and not coalitions. That is, we could require that the lies are of the form $(\{i\}, \omega, \omega')$ satisfying $\mathcal{F}_j(\omega) = \mathcal{F}_j(\omega')$ for all $j \neq i$. From this, new conditions can be given by adding requirements similar to those described in 3 and 4 above.

strongest notion of incentive compatibility only makes our result stronger and delivers, as immediate corollaries, analogous results for weaker notions of incentive compatibility.

When coalition C tells the lie (C, ω, ω'), the final allocation will be different not only in state ω. The following example illustrates why:

Example A.1 *Let* $\Omega = \{a, b, c, d\}$, $\Pr(\{\omega\}) = \frac{1}{4}$, $\forall \omega \in \Omega$, $\mathcal{F}_1 = \{\{a, b\}, \{c, d\}\}$ *and* $\mathcal{F}_2 = \{\{a, c\}, \{b, d\}\}$. *Let us say that 1 wants to tell the lie* $(\{1\}, a, c)$*, while 2 reports truthfully. This means that if agent 1 observes* $\{a, b\}$*, he reports* $\{c, d\}$*. If the true state is* a*, agent's 2 report will be* $\{a, c\}$ *and* $\{c\} = \{c, d\} \cap \{a, c\}$ *is wrongly considered the true state of the world. However, if the state is* b*, which agent 1 considers possible, agent 2 reports* $\{b, d\}$ *and* $\{d\} = \{c, d\} \cap \{b, d\}$ *is mistakenly taken as the true state of the world.*

More generally, given deceptions $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$, if the true state of the world is ω, the perceived state of the world will be obtained from the intersection of reports $\cap_{i \in I} \alpha_i(\mathcal{F}_i(\omega))$. Recall that Assumption **??** requires that $\cap_{i \in I} \mathcal{F}_i(\omega) = \{\omega\}$, for all $\omega \in \Omega$. This implies that $\cap_{i \in I} \alpha_i(\mathcal{F}_i(\omega))$ is empty or singleton.[61] Palfrey and Srivastava (1989) rules out the case in which this intersection is empty, which is equivalent to impose the restriction that the lies are undetectable. As we discussed above, we do need to restrict the lies in this way: our result is valid even with the stronger incentive compatibility notion that allows detectable lies.[62] If this intersection is empty, that is, the lie is detectable, we assume that the true state is implemented. That is, we will define the perceived state as:

$$\alpha(\omega) \equiv \begin{cases} \tilde{\omega}, & \text{if } \tilde{\omega} \in \cap_{i \in I} \alpha_i(\mathcal{F}_i(\omega)) \\ \omega, & \text{if } \cap_{i \in I} \alpha_i(\mathcal{F}_i(\omega)) = \emptyset \end{cases}$$

Note that this definition is general, that is, this allows any deception $\alpha_i : \mathcal{F}_i \to \mathcal{F}_i$ and not only those coming from a single lie (C, ω, ω'). However, we will be more interested in deceptions that express only the deviations by members of the coalition trying to implement ω' if the true state is ω. In this kind of deceptions, the reports are always truthful outside of the coalition or outside the states in $\mathcal{F}_i(\omega)$. The restriction to these deceptions is without loss of generality for our purposes.

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[61] Let $\omega', \omega'' \in \cap_{i \in I} \alpha_i(\mathcal{F}_i(\omega))$, with $\omega' \neq \omega''$. Since the elements of the partition are disjoint, this means that $\mathcal{F}_i(\omega') = \mathcal{F}_i(\omega'')$ for all i, but this would imply $\omega', \omega'' \in \cap_{i \in I} \mathcal{F}_i(\omega')$, which contradicts Assumption **??**.

[62] Observe that to allow detectable lies is not only more general but also more realistic, for cheating happens in real life even if there is some possibility of being caught.

Finally, we have to specify the allocation that follows from the lies. There are two ways to do this. We can follow Holmstrom and Myerson (1983) and assume that there is a decision rule $\delta : \Omega \to D$, where D is the set of feasible allocations, such that for each perceived state $\omega \in \Omega$, the allocation $\delta(\omega)$ is implemented. Thus, given the deceptions α_i, it will be implemented, for each $\omega \in \Omega$, the allocation $\delta(\alpha(\omega))$. In particular, we can write $\delta_i : \Omega \to \mathbb{R}^{\ell}_{+}$ to represent the i-th coordinate of δ.

Another way to specify the allocation after a lie is to assume that there is an initial endowment and that there is a contracted allocation $x(\omega) = (x_1(\omega), ..., x_n(\omega))$ and say that the lie will lead to the implementation of the allocation $x^{\alpha} = (x^{\alpha}_1, ..., x^{\alpha}_n)$, where:

$$x^{\alpha}_i(\omega) \equiv e_i(\omega) + x_i(\alpha(\omega)) - e_i(\alpha(\omega)) + \tau_i, \tag{25}$$

and $\tau_i \in \mathbb{R}^{\ell}_{+}$ is a transfer among member of the coalition ($\tau_i = 0$ if $i \notin C$). We say that a decision rule δ is equivalent to a pair endowment-allocation (e, x) if[63]

$$\delta_i(\alpha(\omega)) = x^{\alpha}_i(\omega). \tag{26}$$

To summarize, we will assume the specification 1 above for the set of allowed lies and will use the allocation given in (25) as the allocation that will be implemented by a lie (C, ω, ω'). The definition of incentive compatibility is formalized in the subsection 3.3.

References

BARBERA, S., AND M. JACKSON (1988): "Maximin, leximin and the protective criterion: characterizations and comparisons," *Journal of Economic Theory*, 46(1), 34–44.

BEWLEY, T. (1986): "Knightian Decision Theory: Part I," Cowles Foundation Discussion Paper no. 807.

——— (2002): "Knightian decision theory. Part I," *Decisions in Economics and Finance*, 25(2), 79–110.

[63]In the type model, in which the decision rule is $\delta : T \times S \to D$, if agent i reports t'_i instead of the true type t'_i, the allocation will be: $\delta_i(t'_i, t_{-i}, s) = e_i(t_i, t_{-i}, s) + x_i(t'_i, t_{-i}, s) - e_i(t'_i, t_{-i}, s)$. Morris (1994) restricts attention to allocation that do not depend on the types, which he calls payoff-relevant. In this case, equation (26) gives $e_i(t_i, t_{-i}, s) + x_i(t'_i, t_{-i}, s) - e_i(t'_i, t_{-i}, s) = x_i(t'_i, t_{-i}, s)$, and it is sufficient to consider $\delta_i(t'_i, t_{-i}, s) = x_i(t'_i, t_{-i}, s)$, that is, $x_i(t'_i, t_{-i}, s)$ is the implemented allocation with the report t'_i, as Morris (1994) assumes. Note that this does not include transfers.

BINMORE, K. (2008): *Rational Decisions*. Princeton University Press.

CERREIA, S., F. MACCHERONI, M. MARINACCI, AND L. MONTRUCCHIO (2008): "Uncertainty Averse Preferences," Carlo Alberto.

CORREIA-DA SILVA, J., AND C. HERVÉS-BELOSO (2009): "Prudent expectations equilibrium in economies with uncertain delivery," *Economic Theory*, 39(1), 67–92.

DE CASTRO, L. I., AND N. C. YANNELIS (2009): "Relationship between type and partition models in general equilibrium with asymmetric information," Discussion paper, University of Illinois at Urbana-Champaign.

DEBREU, G. (1959): *Theory of value*. Yale University Press.

ELLSBERG, D. (1961): "Risk, Ambiguity, and the Savage Axioms," *The Quarterly Journal of Economics*, 75(4), 643–669.

GHIRARDATO, P. (2001): "Coping with ignorance: unforeseen contingencies and non-additive uncertainty," *Economic Theory*, 17(2), 247–276.

GILBOA, I., F. MACCHERONI, M. MARINACCI, AND D. SCHMEIDLER (2008): "Objective and Subjective Rationality in a Multiple Prior Model," Collegio Carlo Alberto, Universita di Torino.

GILBOA, I., AND D. SCHMEIDLER (1989): "Maxmin expected utility with nonunique prior," *Journal of Mathematical Economics*, 18(2), 141–153.

——— (1994): "Additive representations of non-additive measures and the choquet integral," *Annals of Operations Research*, 52(1), 43–65.

GLYCOPANTIS, D., A. MUIR, AND N. YANNELIS (2001): "An extensive form interpretation of the private core," *Economic Theory*, 18(2), 293–319.

——— (2003): "On extensive form implementation of contracts in differential information economies," *Economic Theory*, 21(2), 495–526.

GUL, F., AND W. PESENDORFER (2009): "Measurable Ambiguity," Discussion paper, Princeton University.

GUL, F., AND A. POSTLEWAITE (1992): "Asymptotic Efficiency in Large Exchange Economies with Asymmetric Information," *Econometrica*, 60, 1273–92.

HARSANYI, J. C. (1967-8): "Games with incomplete information played by "Bayesian" players, Parts I, II, III," *Management Sci.*, 14, 159–182, 320–334, 486–502.

HOLMSTROM, B., AND R. B. MYERSON (1983): "Efficient and Durable Decision Rules with Incomplete Information," *Econometrica*, 51(6), 1799–1819.

KOPYLOV, I. (2007): "Subjective probabilities on 'small' domains," *Journal of Economic Theory*, 133(1), 236 – 265.

KOUTSOUGERAS, L., AND N. YANNELIS (1993): "Incentive compatibility and information superiority of the core of an economy with differential information," *Economic Theory*, 3(2), 195–216.

KRASA, S., AND N. YANNELIS (1994): "The value allocation of an economy with differential information," *Econometrica*, 62, 881–900.

LEHRER, E. (2008): "Partially-specified Probabilities: Decisions and Games," Tel-Aviv University.

LUCE, R., AND H. RAIFFA (1989): *Games and Decisions: Introduction and Critical Survey*. Dover Publications.

MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006): "Ambiguity Aversion, Malevolent Nature, and the Variational Representation of Preferences," *Econometrica*, 74, 1447–98.

MCLEAN, R., AND A. POSTLEWAITE (2002): "Informational Size and Incentive Compatibility," *Econometrica*, 70, 2421–54.

MILGROM, P., AND N. STOKEY (1982): "Information, trade and common knowledge," *Journal of Economic Theory*, 26(1), 17 – 27.

MILNOR, J. (1954): "Games against nature," *Decision Processes*, pp. 49–59.

MORRIS, S. (1994): "Trade with Heterogeneous Prior Beliefs and Asymmetric Information," *Econometrica*, 62(6), 1327–1347.

MUKERJI, S. (1997): "Understanding the nonadditive probability decision model," *Econom. Theory*, 9(1), 23–46.

——— (1998): "Ambiguity Aversion and Incompleteness of Contractual Form," *The American Economic Review*, 88(5), 1207–1231.

MYERSON, R. B. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47(1), 61–73.

MYERSON, R. B., AND M. A. SATTERTHWAITE (1983): "Efficient mechanisms for bilateral trading," *Journal of Economic Theory*, 29(2), 265 – 281.

PALFREY, T. R., AND S. SRIVASTAVA (1989): "Implementation with Incomplete Information in Exchange Economies," *Econometrica*, 57(1), 115–134.

PODCZECK, K., AND N. C. YANNELIS (2008): "Equilibrium theory with asymmetric information and with infinitely many commodities," *Journal of Economic Theory*, 141(1), 152 – 183.

PRESCOTT, E. C., AND R. M. TOWNSEND (1984): "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica*, 52(1), 21–45.

RADNER, R. (1968): "Competitive Equilibrium Under Uncertainty," *Econometrica*, 36(1), 31–58.

RAWLS, J. (1971): *A Theory of justice* . Harvard University Press, Cambridge.

RIGOTTI, L., C. SHANNON, AND T. STRZALECKI (2008): "Subjective Beliefs and ex ante Trade," *Econometrica*, 76(5), 1167–1190.

SAVAGE, L. J. (1972): *The foundations of statistics*. Dover Publications Inc., New York, revised edn.

SCARF, H. (1973): *The Computation of Economic Equilibria*. Cowles Foundation.

SCARF, H. E. (1967): "The Core of an N Person Game," *Econometrica*, 35(1), 50–69.

SCHMEIDLER, D. (1989): "Subjective Probability and Expected Utility Without Additivity," *Econometrica*, 57(3), 571–587.

SHAPLEY, L. (1953): "A Value for n-Person Games," *In: Kuhn, H. W. and Tucker, A. W. (eds.) Contributions to the theory of games. Princeton University Press*, II, 307–317.

SUN, Y., AND N. C. YANNELIS (2007): "Perfect competition in asymmetric information economies: compatibility of efficiency and incentives," *Journal of Economic Theory*, 134(1), 175 – 194.

——— (2008): "Ex Ante Efficiency Implies Incentive Compatibility," *Economic Theory*, 36(1), 35–55.

WALD, A. (1950): *Statistical Decision Functions*. John Wiley & Sons, New York.

WILSON, R. (1978): "Information, Efficiency, and the Core of an Economy," *Econometrica*, 46(4), 807–816.

YANNELIS, N. (1991): "The core of an economy with differential information," *Economic Theory*, 1(2), 183–197.