

Dynamic Social Interactions: Identification and Characterization ^{*}

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Abstract

Preferences for conformity arguably provide a rationale for several important social phenomena, from smoking and other risky behavior in teens, to e.g., criminal activity and to peer effects in school performance. We study *social interactions* in general, and conformity in particular, in *dynamic* economies with overlapping social groups, heterogenous interaction structures with varying levels of connectivity, local and global interactions, complete and incomplete information. For these economies, we provide existence (Markov-Perfect Equilibrium in pure strategies), ergodicity, and welfare results. Also, we characterize equilibria in terms of spatial equilibrium correlations and social multiplier effects, depending on the nature of interactions. Most importantly, we study formally the issue of the identification of social interactions with population data, with special emphasis on the restrictions imposed by dynamic equilibrium conditions.

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1 Introduction

Agents interact in markets as well as socially, that is, in the various socioeconomic groups they belong to. The integration of models of social interactions within economic theory is an active and interesting area of research.¹ Models of social interactions are designed to capture in a simple abstract way socioeconomic environments in which markets do not mediate all of agents' choices. In such environments agents' choices are determined by their preferences as well as by their ability to interact with others, on their position in a predetermined network of relationships, e.g., a family, a peer group, or more generally any socioeconomic group.

Distinguishing the relative importance of social interactions, as opposed to preferences and attitudes, in explaining adolescent behavior is, for instance of great positive and normative interest. Peer effects, in particular, have been indicated as one of the main empirical determinants of risky behavior in adolescents.² Relatedly, peer effects have been studied in connection with education outcomes,³ obesity,⁴ friendship and sex,⁵ labor market referrals,⁶ neighborhood and employment segregation,⁷ criminal activity,⁸ and several other socioeconomic phenomena.⁹

¹See the recent *Handbook of Social Economics*, Benhabib, Bisin, and Jackson (2010). Main references for the theory of social interaction include: Akerlof (1997), Bala and Goyal (2000), Becker (1991), Becker and Murphy (2000), Bisin et al. (2006), Blume (1993, 1995), Blume and Durlauf (2001, 2003), Brock (1993), Brock and Durlauf (2001a, 2001b, 2002), Durlauf (1997, 2008a), Ely (2002), Follmer (1974), Glaeser et al. (2003), Glaeser and Scheinkman (2000), Horst and Scheinkman (2006a, 2006b), Ioannides (2006), Jackson and Wolinski (1996), Kirman (1997), Özgür (2010), Schelling (1978), Vega Redondo (2007), Young (1993, 1998), Watts (2007).

²On peer effects and smoking behavior of adolescents, see Ali and Dwyer (2009), Axtell et al. (2006), Bauman and Ennett (1996), Chaloupka and Warner (2000), Cutler and Glaeser (2007), DeCicca et al. (2008), Fletcher (2009), Gilleskie and Strumpf (2005), Gilleskie and Zhang (2009), Jones (1994), Kobus (2003), Krauth (2005, 2006), Krosnick and Judd (1982), Lewitt et al. (1981), Nakajima (2007), Norton et al. (1998), Powell et al. (2003), Soetevent and Kooreman (2007), Tyas and Pederson (1998), Wang et al. (1995, 2000). On peer effects and substance abuse, see Bauman and Ennett (1996), Bifulco et al. (2008), Clark and Loheac (2007), Cook and Moore (2000), Eiserberg (2004), Evans et al. (1992), Gaviria and Raphael (2001), Kremer and Levy (2008), Lundborg (2006), Norton et al. (1998), Sacerdote (2001).

³Altonji et al (2005), Ammermuller and Pischke (2006), Benabou (1996), Borjas (1995), Boozer and Cacciola (2001), Carrell et al. (2008), De Giorgi et al. (2009), Evans et al. (1992), Gaviria and Raphael (2001), Hoxby (2000a, 2000b), Soetevent and Kooreman (2007), Zimmerman (2003).

⁴Burke and Heiland (2006), Christakis and Fowler (2007).

⁵Akerlof et al. (1996), Bearman et al. (2004), Bramoulle' et al. (2009), Broadhead et al. (1998), Cipollone and Rosolia (2007), Conti et al. (2009), Currarini et al. (2009), Kandel (1978), Leider et al. (2007), Mihaly (2007), Moody (2001).

⁶Bayer et al. (2008), Bjorn and Vuong (2005), Calvo-Armengol and Jackson (2004), Conley and Topa (2002, 2007), De Giorgi et al. (2009), Goldin and Katz (2002), Granovetter (1973, 1995), Grodner and Kniesner (2007), Ichino and Falk (2006), Ioannides and Datcher Loury (2004), Moro (2003), Topa (2001), Weinberg et al. (2004).

⁷Aizer and Currie (2004), Benabou (1993), Case and Katz (1991), Crane (1991), Durlauf (1996, 2004), Goering and Feins (1997), Hoff and Sen (2005), Ioannides and Topa (2009), Ioannides and Zabel (2008), Ioannides and Zanella (2009), Katz et al. (2001), Ludwig et al. (2001), Mobius (2000), Rosenbaum (1995), Schelling (1971, 1972).

⁸Calvo Armengol et al. (2009), Glaeser et al. (1996), Kling et al. (2005), Ludwig et al. (2001).

⁹ See Bisin et al. (2010), Glaeser and Scheinkman (2001), Moffett (2001) for surveys.

The large majority of the existing models of social interactions are static. Furthermore, when dynamic models of social interactions are studied, it is assumed that agents are myopic and their choices are subject to particular behavioral assumptions.¹⁰ In this paper, we contribute to this literature by studying social interactions in *dynamic* economies. In the class of economies we study agents' choices are determined by *linear* policy (best reply) functions. More specifically, in this class of economies, a symmetric Markov perfect equilibrium is represented by a symmetric policy function, for each agent a , which maps the agent's choice at time t , x_t^a , linearly in each agent's past choices, in each agent's contemporaneous idiosyncratic preference shock, θ_t^b , and in the mean preference shock:

$$x_t^a = \sum_b c^b x_{t-1}^{a+b} + \sum_b d^b \theta_t^{a+b} + e \bar{\theta}$$

The simplicity of linear models allows us to extend our analysis in several directions, including overlapping social groups, heterogenous interaction structures with varying levels of connectivity, local and global interactions, complete and incomplete information. For these economies, we provide existence (symmetric Markov-Perfect Equilibrium in pure strategies), ergodicity, and welfare results (Theorems 1, 2, and 4, respectively). Most importantly, we characterize equilibria in terms of spatial equilibrium correlations and social multiplier effects, depending on the nature of interactions. Also, we are able to solve the model almost in closed form, by reducing the equilibrium conditions to a simple recursive computation (Theorem 3), and hence we produce several interesting comparative dynamics results by simulation.

Finally, we exploit our characterization results of the equilibria to address generally the issue of identification of social interactions in our context, with population data. While the empirical literature has often interpreted a significant high correlation of socioeconomic choices across agents, e.g., peers, as evidence of social interactions, in the form e.g., of preferences for conformity, it is well known at least since the work of Manski (1993) that the empirical study of social interactions is plagued by subtle identification problems. In a class of linear static models of social interactions,¹¹ in fact, Manski (1993) has shown that the observation of a correlation in behavior across socially interacting groups is not sufficient to identify social interactions effects, e.g., due to a propensity of individuals to conform to each other's behavior, from correlated effects, e.g., due to the selection into the same group of individuals sharing similar characteristics or facing similar environments.

Intuitively, in our economy for instance, the spatial correlation of actions at equilibrium can be due to social interactions, as in our model, or to spatial correlation of preference types, which

¹⁰The only exceptions we are aware of are an example on female labor force participation in Glaeser and Scheinkman (2000), Blume (2010) on social stigma, and the theoretical analysis of Bisin, Horst, and Özgür (2006).

¹¹Some of the identification problems outlined by Manski (1993), are avoided in non-linear models, as described by Brock and Durlauf (2001). By restricting ourselves to linear models, in this paper, therefore, we stack the deck against us when studying identification.

we have excluded by assumption in our analysis to this point. More formally, take two agents, e.g., agent a and agent b . A positive correlation between x_t^a and x_t^b could be due to e.g., preference for conformity. But the positive correlation between x_t^a and x_t^b could also be due to a positive correlation between θ_t^a and θ_t^b . In this last case, preferences for conformity and social interactions would play no role in the correlation of actions at equilibrium. Rather, such correlation would be due to the fact that agents have correlated preferences. Correlated preferences are generally due to some sort of assortative matching or positive selection in social interaction, which induces agents with correlated preferences to interact socially.

An interesting literature is developing which addresses this identification issues by means of structural models of social group formation.¹² In this paper, however, we put special emphasis on the restrictions imposed by dynamic equilibrium conditions. In particular, we ask whether the restrictions implied by dynamic equilibrium analysis in our economy help identify social interactions and distinguish them from correlated preferences due to selection or assortative matching? We show that the answer is in fact affirmative, but only if the economy is non-stationary, in a precise sense.

2 Dynamic Economies with Social Interactions

In this section, we introduce our general notion of a dynamic economy with social interactions as well as our equilibrium concept.¹³ Time is discrete and is denoted by $t = 1, \dots, T$. We allow both for infinite economies ($T = \infty$) and economies with an end period ($T < \infty$). A typical economy is populated by a countable number of *agents* $a \in \mathbb{A}$.¹⁴ Each agent lives for the duration of the economy. At the beginning of each period t , agent a 's random *type* θ_t^a is drawn from the set Θ , typically a compact subset of a finite dimensional Euclidean space \mathbb{R}^n . The random variables θ_t^a are independently and identically distributed across time and agents with probability law ν . We assume with no loss of generality that for all t , the random variable $\theta_t := (\theta_t^a)_{a \in \mathbb{A}}$ is defined on the canonical probability space $(\Theta, \mathcal{F}, \mathbb{P})$, where $\Theta := \{(\theta^a)_{a \in \mathbb{A}} : \theta^a \in \Theta\}$. At each period t , agent $a \in \mathbb{A}$ chooses an *action* x_t^a from the set X , a compact subset of a finite dimensional Euclidean space \mathbb{R}^p . Let $\mathbf{X} := \{x = (x^a)_{a \in \mathbb{A}} : x^a \in X\}$ be the space of all configurations of individual actions.

Each agent $a \in \mathbb{A}$ *interacts* with agents in the set $N(a)$, a nonempty subset of the set of agents \mathbb{A} . This is to capture in an abstract way the notion of social proximity, i.e., agent a 's social reference group such as friends, peers, family, etc. The map $\mathbb{A} : N \rightarrow 2^{\mathbb{A}}$ is called a

¹²See Blume, Brock, Durlauf, Ioannides (2010) for a survey.

¹³While we develop most of our analysis in the context of linear models, it is useful to set up the general model first. But the paper can be read also starting from Section 2.1.

¹⁴We study throughout the paper an economy populated by a countably infinite number of agents where $\mathbb{A} := \mathbb{Z}$. The same methodology applies equally well to economies with a finite number of agents.

neighbourhood correspondence and is part of the given parameters of an economy. Agent a 's instantaneous preferences at time t are represented by a continuous utility function

$$\left(x_{t-1}^a, x_t^a, \{x_t^b\}_{b \in N(a)}, \theta_t^a\right) \mapsto u\left(x_{t-1}^a, x_t^a, \{x_t^b\}_{b \in N(a)}, \theta_t^a\right)$$

As seen in its specification above, agent a 's instantaneous preferences depend on the current choices of agents in his reference group, $\{x_t^b\}_{b \in N(a)}$. In this sense, one can think of interactions as direct preference externalities. Apart from the current type realization, θ_t^a and the current own choice x_t^a , the utility function also has the 'own' previous period choice x_{t-1}^a as an argument. This is to allow for preferences that represent an inherent cost to dynamic behavioural changes, as in habits and addiction. Agents discount expected future utilities using the common stationary discount factor $\beta \in (0, 1)$.

The timing of agents' choices and the realization of the type process is as follows. At the very beginning, there is an initial configuration $x_0 \in \mathbf{X}$ for the economy. Let $x^{t-1} = (x_0, x_1, \dots, x_{t-1})$ and $\theta^{t-1} = (\theta_1, \dots, \theta_{t-1})$ be the $t - 1$ -period choices and type realizations. At the beginning of period- t , x^{t-1} is observed by all agents and the current value of the random variable θ_t realizes. Agent $a \in \mathbb{A}$ observes only the part $I_a \theta_t := \{\theta_t^b : b \in I(a)\}$, where $I(a) \subset \mathbb{A}$ is his *information set*. Similarly, let $I_a \theta^{t-1} = (I_a \theta_1, \dots, I_a \theta_{t-1})$. We study both economies with *complete information*, $I(a) = \mathbb{A}$, and economies with *incomplete information*, $I(a) \subsetneq \mathbb{A}$. Endowed with this information, he chooses an action $x_t^a \in X$. At the end of period- t , $x_t = (x_t^b)_{b \in \mathbb{A}} \in \mathbf{X}$ becomes common knowledge. The same scheme of events repeats itself in period $t + 1$, and so on.

A *strategy* for an agent a is a sequence of measurable functions $x^a = (x_t^a)$, where for each t , $x_t^a : \mathbf{X}^t \times (\Theta^{I(a)})^t \rightarrow X$. Agents' strategies along with the probability law for types induce a stochastic process over future configuration paths. Each agent $a \in \mathbb{A}$ chooses x^a to maximize

$$E \left[\sum_{t=1}^T \beta^{t-1} u \left(x_{t-1}^a, x_t^a, \{x_t^b\}_{b \in N(a)}, \theta_t^a \right) \middle| (x_0, I_a \theta_1) \right] \quad (1)$$

We will focus attention on two equilibrium concepts. First one is

Definition 1 A **Perfect Equilibrium** of a dynamic economy with social interactions is a profile of strategies $(x^a)_{a \in \mathbb{A}}$ such that for all $a \in \mathbb{A}$, for all $t = 1, \dots, T$, and for all (x^{t-1}, θ^t)

$$(x_\tau^a)_{\tau \geq t} \in \arg \max_{(\tilde{x}_\tau^a)_{\tau \geq t}} E \left[\sum_{\tau=t}^T \beta^{\tau-1} u \left(x_{\tau-1}^a, x_\tau^a, \{x_\tau^b\}_{b \in N(a)}, \theta_\tau^a \right) \middle| (x^{t-1}, I_a \theta^t) \right] \quad (2)$$

In this paper, we are interested in *symmetric* equilibria. We therefore require that the social and informational structure satisfies the following restrictions.¹⁵

¹⁵ Heterogeneity can be incorporated into the probabilistic structure of the types θ_t^a . Also, we can allow for heterogeneity of the network structure across agents either by dispensing with the symmetry requirement or by augmenting the strategy spaces to incorporate network structure into individual heterogeneity.

Assumption 1 (Symmetry) *The network structure and the individual information sets are translation invariant across agents:*

1. For all $a, b \in \mathbb{A}$, $N(b) = R^{b-a}N(a)$, where R^{b-a} is the canonical shift operator in the direction $b - a$.¹⁶
2. For all $a, b \in \mathbb{A}$, $I(b) = R^{b-a}I(a)$.

Given symmetry, knowing $N(0)$ and $I(0)$ is sufficient to know the structure of the whole economy. We will call a perfect equilibrium $(x^a)_{a \in \mathbb{A}}$ **symmetric** if $x^a = x^* \circ R^a$, \mathbb{P} – a.s. for some sequence of measurable maps $x^* = (x_t^*)$. As we will prove in the sequel, last period configuration and current type realizations will be a sufficient statistic in equilibrium. Hence, our equilibria will belong to a finer class that we call Markov Perfect.

Definition 2 *A symmetric Markov Perfect Equilibrium of a dynamic economy with social interactions is a measurable map $g^* : \mathbf{X} \times \Theta^{I(0)} \times \{1, \dots, T\} \rightarrow X$ such that the strategy profile $(x^a)_{a \in \mathbb{A}}$ defined as*

$$x^a(x^{t-1}, I_a \theta^t) = g^*(R^a x_{t-1}, R^a I_0 \theta_t), \quad \mathbb{P} - a.s.$$

for all $a \in \mathbb{A}$, for all $t = 1, \dots, T$, and for all (x^{t-1}, θ^t) , is a symmetric Perfect Equilibrium.

A symmetric Markov Perfect Equilibrium¹⁷ is a Perfect Equilibrium with the property that the strategy profile at any time t only depends on the previous period configuration and current type realizations.¹⁸

2.1 Linear Economies with Conformity Preferences

We focus our attention on *linear economies with conformity preferences*. These are environments in which each agent's preferences incorporate the desire to conform to the choices of agents in his reference group.

Preferences for conformity arguably provide a rationale for several important social phenomena. The empirical literature has for instance documented preferences for conformity as a motivation for smoking and other risky behaviour in teens. Similarly, the role of conformity is

¹⁶That is, $c \in N(a)$ if and only if $c + (b - a) \in N(b)$. Of course, we let \mathbb{A} be a linear space, typically $\mathbb{A} := \mathbb{Z}^d$ the d -dimensional integer lattice.

¹⁷We shall concentrate our analysis to Markov perfect equilibria. See Bisin, Horst and Özgür (2006) for a discussion of non-Markovian equilibria.

¹⁸In an infinite horizon economy ($T = \infty$), a symmetric Markov Perfect equilibrium is not necessarily stationary. We shall however study stationary symmetric Markov Perfect equilibria (MPE), that is, strategy profiles $g^* : \mathbf{X} \times \Theta^{I(0)} \rightarrow X$ and show that the sequence of unique MPEs for finite horizon economies converges to g^* .

also documented by Glaeser, Sacerdote, and Scheinkman (1996) with regards to criminal activity and by a large literature with regards to peer effects in education outcomes.¹⁹

Conformity represents a natural environment in which to study dynamic equilibrium. In many relevant social phenomena, in fact, the effects of preferences for conformity are amplified by the presence of limits to the reversibility of dynamic choices. This is of course the case for smoking, alcohol abuse and other risky teen behaviour, which are hard to reverse because they might lead to chemical addictions. In other instances, while addiction per se is not at issue, nonetheless behavioural choices are hardly freely reversible because of various social and economic constraints, as is the case, for instance, of engaging in criminal activity. Finally, exogenous and predictable changes in the composition of groups, as e.g., in the case of school peers at the end of a school cycle, introduce important non-stationarities in the agents' choice. These non-stationarity also call for a formal analysis of dynamic social interactions.

With the objective of providing a clean and simple analysis of dynamic social interactions in a conformity economy, we impose strong(er than required) but natural assumptions. In particular (i) we restrict the neighborhood correspondence to represent the minimal interaction structure allowing for overlapping groups, (ii) we restrict preferences to be quadratic, and (iii) we impose enough regularity conditions on the agents' choice problem to render it convex. Formally,

Assumption 2 (Conformity) *A conformity economy satisfies:*

1. Let $\mathbb{A} := \mathbb{Z}$ represent a general social space. Each agent interacts with his immediate neighbors, i.e., for all $a \in \mathbb{A}$, $N(a) := \{a - 1, a + 1\}$.
2. The contemporaneous preferences of an agent $a \in \mathbb{A}$ are represented by the utility function

$$u(x_{t-1}^a, x_t^a, x_t^{a-1}, x_t^{a+1}, \theta_t^a) := -\alpha_1(x_{t-1}^a - x_t^a)^2 - \alpha_2(\theta_t^a - x_t^a)^2 - \alpha_3(x_t^{a-1} - x_t^a)^2 - \alpha_3(x_t^{a+1} - x_t^a)^2 \quad (3)$$

where α_1, α_2 , and α_3 , are positive constants.

3. Let $X = \Theta = [\underline{x}, \bar{x}] \subset \mathbb{R}$, where $\underline{x} < \bar{x}$, $E[\theta] = \int \theta d\nu =: \bar{\theta} \in (\underline{x}, \bar{x})$, and $\text{Var}(\theta) = \int (\theta - \bar{\theta})^2 d\nu < \infty$.

Assumption 2-1 requires that the reference group of each agent $a \in \mathbb{A}$ is composed of his immediate neighbors in some social space, namely the agents $a - 1$ and $a + 1$. The utility function u defined in 2-2 describes the trade-off that agent $a \in \mathbb{A}$ faces between matching his individual

¹⁹ We refer to the Introduction for references. While we model preferences for conformity directly as a preference externality, we intend this as a reduced form of models of behavior in groups which induce indirect preferences for conformity, as e.g., Jones (1984), Cole, Mailath and Postlewaite (1992), Bernheim (1994), Peski (2007).

characteristics (x_{t-1}^a, θ_t^a) and the utility he receives from conforming to the current choices of his peers (x_t^{a-1}, x_t^{a+1}) .²⁰ The different values of α_i represent different levels of intensity of the social interaction motive relative to the own (or intrinsic) motive. Finally, Assumption 2-3 combined with the form of the preferences guarantees that the agents' choice problem is convex and hence optimal choices are almost surely in the interior of the choice set.

2.2 Existence and ergodicity

We provide here the basic theoretical results regarding our dynamic social interaction economy with conformity. The reader only interested in the characterization can skip this section, keeping in mind that equilibria exist (for finite economies they are unique) and they induce an ergodic stochastic process over the configuration of actions. Furthermore, a stationary ergodic distribution also exists for the economy. The proofs of all statements can be found in the Appendices.²¹

Theorem 1 (Existence - Complete Information) *Consider an economy with conformity preferences and complete information.*

1. *If the time horizon is finite ($T < \infty$), then the economy admits a **unique symmetric Markov Perfect Equilibrium** $g^* : \mathbf{X} \times \Theta \times \{1, \dots, T\} \mapsto X$ such that for all t , for all $(x_{t-1}, \theta_t) \in \mathbf{X} \times \Theta$*

$$g_t^*(x_{t-1}, \theta_t) = \sum_{a \in \mathbb{A}} c_t^a x_{t-1}^a + \sum_{a \in \mathbb{A}} d_t^a \theta_t^a + e_t \bar{\theta} \quad \mathbb{P} - a.s.$$

where $c_\tau^a, d_\tau^a, e_\tau \geq 0$, $a \in \mathbb{A}$, and $e_\tau + \sum_{a \in \mathbb{A}} (c_\tau^a + d_\tau^a) = 1$, $0 \leq \tau \leq T$.

2. *If the time horizon is infinite ($T = \infty$), then the economy admits a **symmetric stationary Markov Perfect Equilibrium** $g^* : \mathbf{X} \times \Theta \mapsto X$ such that*

$$g^*(x_{t-1}, \theta_t) = \sum_{a \in \mathbb{A}} c^a x_{t-1}^a + \sum_{a \in \mathbb{A}} d^a \theta_t^a + e \bar{\theta}$$

where $c^a, d^a, e \geq 0$, for $a \in \mathbb{A}$, and $e + \sum_{a \in \mathbb{A}} (c^a + d^a) = 1$.²²

The proof of the existence theorem requires some subtle arguments. While referring to the Appendix for details, a few comments here in this respect will be useful. The main stumbling

²⁰As long as the solution is interior, we can generalize the setup to allow for preferences of agent a to depend on a stock rather than the flow variable x_{t-1}^a ; e.g., agent a 's preferences can depend on $X_t^a = \rho X_{t-1}^a + x_t^a$.

²¹The theorems in this section can be extended with straightforward modifications to the case of incomplete information. We state e.g., without proof the existence theorem for economies with incomplete information in Appendix B.

²²Several assumptions can be relaxed while guaranteeing existence. In particular, the symmetry of the neighborhood structure can be substantially relaxed, adapting the analysis of Horst and Scheinkman (2006a) to our dynamic environment.

block we need to overcome to prove existence of a Markov perfect equilibrium in our environment is the following. Consider the (infinite dimensional) choice problem of each agent $a \in \mathbb{A}$. To be able to apply standard variational arguments to this problem it is necessary to bound the marginal effect of any infinitesimal change dx^a on the agent's value function. To this end, the Envelope theorem (as e.g., in Benveniste and Scheinkman (1979)) is not enough , as dx^a affects agent a 's value function directly and indirectly, through its effects on all agents $b \in \mathbb{A} \setminus a$'s choices, which in turn affect agent a 's value function. The marginal effect of any infinitesimal change dx^a is then an infinite sum. Furthermore, each term in the sum contains endogenous terms from some agent $b \in \mathbb{A} \setminus a$ policy function, which makes it impossible to adopt the methodology used by Santos (1991) to prove the smoothness of the policy function in infinite dimensional recursive choice problems. With quadratic utility, the case we study, policy functions are necessarily linear and, provided we show that equilibria are interior, symmetric Markov perfect equilibria can be represented by a policy function which is obtained as a fixed point of a recursive map which can be directly studied. Extending the existence proof to general preferences would require therefore sufficient conditions on the structural parameters to control the curvature of the policy function of each agent decision problem. We conjecture that this can be done although sufficient conditions do not appear transparently from our proof.

On the other end, however, by exploiting the linearity of policy functions, our method of proof is constructive and the recursive map which induces the symmetric policy function at equilibrium provides a direct and useful computation method which we repeatedly exploit in the next section to characterize equilibria and to produce comparative dynamics exercises. Since a generic agent's problem is symmetric across agents, to economize on notation consider the problem of agent 0 in a 1-period economy with conformity preferences and complete information. First order condition of his problem can be written, as we demonstrate in more details in the Appendix, in the following form

$$0 = -x_1^0 \Delta_1 + \alpha_1 x_0^0 + \alpha_2 \theta_1^0 + \alpha_3 x_1^{-1} + \alpha_3 x_1^1 \quad (4)$$

where Δ_1 is the total effect on agent 0's marginal utility of an infinitesimal change in agent 0's optimal choice, x_1^0 , and is given by $\Delta_1 = \alpha_1 + \alpha_2 + 2\alpha_3$. Let

$$L_c := \{(c, d, e) : e \geq 0, c^a \geq 0, d^a \geq 0, \forall a \text{ and } e + \sum_a (c^a + d^a) = 1\}$$

be the space of nonnegative coefficient sequences whose sum is 1. The existence of an equilibrium policy is then equivalent to the existence of a coefficient sequence (c_1^*, d_1^*, e_1^*) which is the fixed point of a map $L_1 : L_c \rightarrow L_c$ where $(\hat{c}, \hat{d}, \hat{e}) = L_1(c, d, e)$ and for each $a \in \mathbb{A}$

$$\begin{aligned} \hat{c}^a &= \Delta_1^{-1} (\alpha_1 \mathbf{1}_{\{a=0\}} + \alpha_3 c^{a-1} + \alpha_3 c^{a+1}) \\ \hat{d}^a &= \Delta_1^{-1} (\alpha_2 \mathbf{1}_{\{a=0\}} + \alpha_3 d^{a-1} + \alpha_3 d^{a+1}) \\ \hat{e} &= \Delta_1^{-1} (2\alpha_3 e) \end{aligned}$$

Suppose that the claim in Theorem 1 is true up to $T - 1$ -period economies. By the same token, agent 0 believes that all other agents, including his own reincarnations, will use the unique symmetric equilibrium policy coefficients $(c_s^*, d_s^*, e_s^*)_{s=2}^T$ in the $T - 1$ -period continuation from the second period on. Consequently, first order condition of agent 0's problem in a T -period economy with conformity preferences and complete information can be written as

$$0 = -x_1^0 \Delta_T + \alpha_1 x_0^0 + \alpha_2 \theta_1^0 + \sum_{b \neq 0} \gamma_T^b x_1^b + \mu_T \bar{\theta}$$

where Δ_T is as before the total effect on agent 0's marginal utility of an infinitesimal change in agent 0's optimal choice x_1^0 in the first period, evaluated at the equilibrium path.²³ Similarly, γ_T^a is the total effect on agent 0's marginal utility of an infinitesimal change in agent a's optimal choice x_1^a in the first period. Finally, μ_T is the total effect on agent 0's marginal utility of an infinitesimal change in the mean type $\bar{\theta}$. All these are easily computed, given the equilibrium coefficient sequence for the continuation, since the FOC of agent 0 is linear in choice variables. So, once again, the above equation induces a map $L_T : L_c \rightarrow L_c$ where $(\hat{c}, \hat{d}, \hat{e}) = L_T(c, d, e)$ and for each $a \in \mathbb{A}$

$$\begin{aligned} \hat{c}^a &= \Delta_T^{-1} \left(\alpha_1 \mathbf{1}_{\{a=0\}} + \sum_{b \neq 0} \gamma_T^b c^{a-b} \right) \\ \hat{d}^a &= \Delta_T^{-1} \left(\alpha_2 \mathbf{1}_{\{a=0\}} + \sum_{b \neq 0} \gamma_T^b d^{a-b} \right) \\ \hat{e} &= \Delta_T^{-1} \left(\mu_T + e \sum_{b \neq 0} \gamma_T^b \right) \end{aligned}$$

whose fixed point gives the equilibrium policy coefficients for the T -period policy function. The core idea is that each time we combine dynamic network effects into the fixed point map form we showed above, that map is a function only of already computed parameters of the future period policies. Moreover, as we show, for each T , that map is a contraction and the unique fixed point is easily computed.

Finally, the sequence of fixed point maps that we computed at each iteration converges to a policy sequence, which turns out to be the one of the infinite-horizon stationary MPE. Fixed point calculations take less than a few seconds for each period and the sequence of finite-truncation policy sequence also converges very fast, under a few minutes to the infinite-horizon one. All these are summarized in the following

Theorem 2 (Recursive Computability) *Consider a finite-horizon T -period economy with conformity preferences ($\alpha_i > 0$, $i = 1, 2, 3$) and complete information.*

- (i) *The map L_1 for a one-period economy (equivalently the final period of any finite-horizon economy) forms a second-order difference equation whose unique exponential solution is the fixed point of L_1 . Hence, we can compute the coefficient sequence analytically in closed-form.*

²³To be more rigorous, one should write Δ_T as $\Delta_T \left((c_s^*, d_s^*, e_s^*)_{s=2}^{T-1} \right)$ and similarly for $\gamma_T^a \left((c_s^*, d_s^*, e_s^*)_{s=2}^{T-1} \right)$. We use the current syntax to economize on notation.

- (ii) The coefficients $(c_s^*, d_s^*, e_s^*)_{s=1}^T$ of the sequence of Markov polices whose existence is guaranteed by Theorem 1 can be computed recursively as the unique fixed points of the recursive maps $L_s : L_c \rightarrow L_c$, $s = 1, \dots, T$, whose parameters $\Delta_s, (\gamma_s^a), \mu_s$ are in turn computed recursively.
- (iii) Moreover, $\lim_{T \rightarrow \infty} (c_T^*, d_T^*, e_T^*) = (c_\infty^*, d_\infty^*, e_\infty^*)$ exists and is the coefficient sequence of the stationary equilibrium policy function for the infinite horizon economy whose existence is proved in Theorem 1

With such computation power at hand, we are able to characterize, as will be demonstrated, very tightly the spatial (cross-sectional) and intertemporal behavior of the equilibrium process emerging from the class of dynamic models we study. Let π_0 be the initial measure on the configuration space \mathbf{X} , that is, the distribution of

$$x_0 = \left(\frac{e \bar{\theta}}{1 - C} + \sum_{s=1}^{\infty} \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\dots+b_s} \right) \right)_{a \in \mathbb{A}}$$

where $C := \sum_{a \in \mathbb{A}} c^a$ is the sum of coefficients in the stationary policy function that multiply corresponding agents' last period choices. Given the initial distribution π_0 , a Markov perfect equilibrium of the economy with conformity induces an equilibrium process $(x_t \in \mathbf{X})_{t=0}^{\infty}$ (via the policy function $g^*(x_{t-1}, \theta_t)$) and a sequence of distributions $(\pi_t)_{t=0}^{\infty}$ on the configuration space \mathbf{X} . We shall show that, for any initial distribution π_0 and a policy function $g^*(x_{t-1}, \theta_t)$, the limit distribution of x_t^a (for any arbitrary agent a), is independent of π_0 and therefore, the equilibrium process $(x_t \in \mathbf{X})_{t=0}^{\infty}$ is ergodic.²⁴ More specifically,

Theorem 3 (Ergodicity) *Suppose the process $((\theta_t^a)_{t=-\infty}^{\infty})_{a \in \mathbb{A}}$ is i.i.d. with respect to a and t according to ν . The equilibrium process $(x_t \in \mathbf{X})_{t=0}^{\infty}$ induced by a Markov perfect equilibrium of an economy with conformity via the policy function $g^*(x_{t-1}, \theta_t)$ is ergodic. Furthermore, its unique stationary distribution is given by π , the joint distribution of*

$$x_t = \left(\frac{e \bar{\theta}}{1 - C} + \sum_{s=1}^{\infty} \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{t+1-s}^{a+b_1+\dots+b_s} \right) \right)_{a \in \mathbb{A}} . \quad (5)$$

2.3 Inefficiency of Equilibrium

Preferences for conformity introduce an externality in each agent $a \in \mathbb{A}$'s decision problem, which depends directly on the actions of all agents in neighbourhood $N(a)$ and, indirectly, on the actions of all agents in the economy. Not surprisingly, then, equilibria of a conformity economy will not in general be efficient. More precisely,

²⁴ Note however that Theorem 1 does not guarantee that the policy function $g^*(x_{t-1}, \theta_t)$ is unique.

Theorem 4 (Inefficiency of equilibrium) *A symmetric Markov perfect equilibrium of a conformity economy is generically inefficient. Moreover, a social planner who is required to treat agents symmetrically would dictate to the agent to decrease the own-effect weights and increase the weights on other agents (flatten the coefficient sequence) relative to the laissez-faire equilibrium.*

3 Characterization and Simulations

Consider as a benchmark the infinite horizon dynamic social interaction economy in the case of complete information. Theorem 1 provides us with the existence of the following stationary policy function for each agent $a \in \mathbb{A}$,

$$x_t^a = g(R^a x_{t-1}, R^a \theta_t) = \sum_{b \in \mathbb{A}} c^b x_{t-1}^{a+b} + \sum_{b \in \mathbb{A}} d^b \theta_t^{a+b} + e \bar{\theta} \quad (6)$$

where all parameters are non negative and their sum $e + \sum_{a \in \mathbb{A}} (c^a + d^a) = 1$.

The constant c_b may be viewed as a measure for the total impact the action x_{t-1}^{a+b} of agent $a+b$ has on the optimal current choice of agent a . Since the policy function g is well-defined, we have $\lim_{|b| \rightarrow \infty} c^{a+b} = 0$. Thus, the impact of an agent $a+b$ on agent a tends to zero as $|b| \rightarrow \infty$. In this sense, we consider economies with weak social interactions.

The strength of the agents' preferences for conformity depends on the size of α_3 relatively to α_1 and α_2 . A policy function is represented in Figure 1, which compares a case with *mild* preferences for conformity (with parametrization $\frac{\alpha_1}{\alpha_2} = \frac{\alpha_2}{\alpha_3} = 1$)²⁵ with one with *strong* preferences for conformity (with parametrization $\frac{\alpha_1}{\alpha_2} = 1$, $\frac{\alpha_2}{\alpha_3} = \frac{1}{20}$). On the x-axis, we plot agent a and the other agents in his vicinity, on the y-axis, we plot the weights $(c^b)_{b \in \mathbb{A}}$ that the symmetric policy function g associates with the last period actions of agents $(a+b)_{b \in \mathbb{A}}$. While each agent's interaction neighborhood is only composed of two agents, in effect local interactions involve indirectly larger groups. How large are the groups depends endogenously on the strength of the agents' preferences for conformity. Notice e.g., that in Figure 1, local interactions involve effectively a group of about ten neighbors when preferences for conformity are mild and involve a group of about thirty neighbors when preferences for conformity are strong. Furthermore, for the same cases of mild and strong conformity, we compare in Figure 2 the case in which neighborhoods are overlapping, $N(a) = \{a-1, a+1\}$, with the case of non-overlapping one-sided neighborhoods, $N(a) = \{a+1\}$.²⁶ Two effects are in action here. Firstly, as in Figure

²⁵The discount rate is fixed at $\beta = .95$ in all the simulations unless noted otherwise.

²⁶In this case, the policy function is

$$x_t^a = g(R^a x_{t-1}, R^a \theta_t) = \sum_{b \geq 0} c^b x_{t-1}^{a+b} + \sum_{b \geq 0} d^b \theta_t^{a+b} + e \bar{\theta}.$$

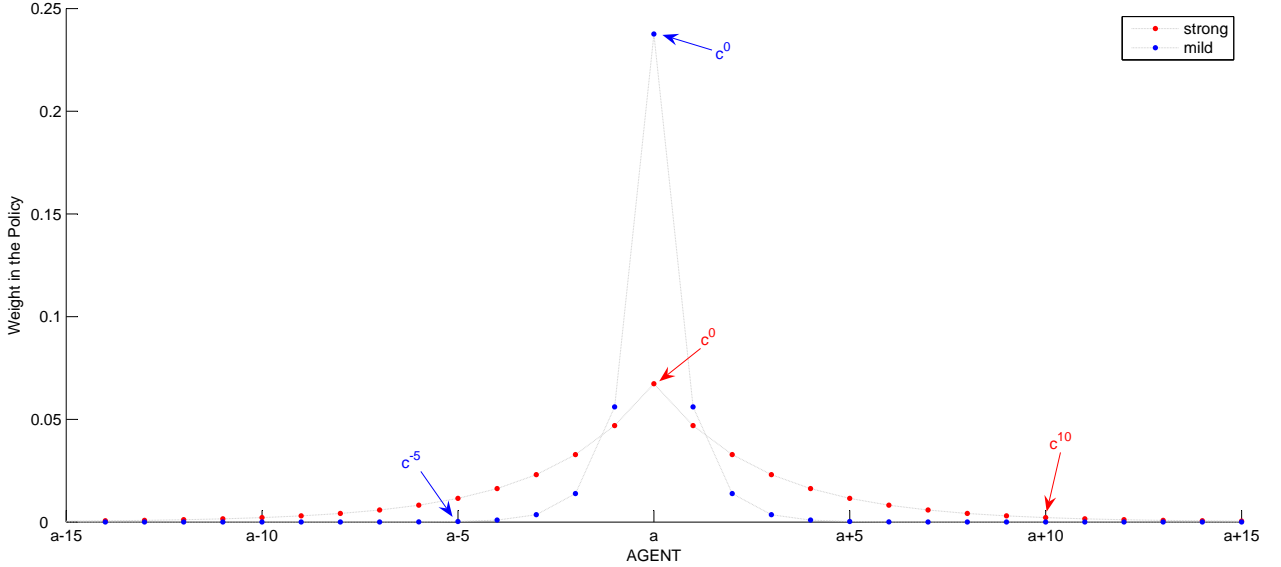


Figure 1: Weights on past history in the stationary policy function.

1, an increase in the strength of the interaction parameter spreads the interaction effects over a larger social geography. Secondly, this spread is observed most significantly in the case of non-overlapping neighborhoods due to the uni-directional nature of the interactions.

In Figure 3, we compare the case in which agents have complete information with the case in which they have incomplete information.²⁷ The exercise here is to keep track of the effect of an expansion of the information set I (individuals whose types are observed by agent a) on best responses. We start with ‘own type’ only and go eventually to complete information by increasing the number of types observed by a , by two at each iteration (due to symmetry of two-sided interactions). The red dots represent the optimal weights in the policy of agent a as a response to the informational structure. The lower left vertex represents (**H**)istory, the total sum of weights assigned to last period’s choices. The lower right vertex represents (**I**)nformation, the sum of weights on current types observed. Finally, the upper vertex represents average information, (**M**)ean type, $\bar{\theta}$. In part (a), we have **mild preferences** for conformity once again. The dots are concentrated near the middle of the triangle (equal weights on history, information, and mean type) and they do not move much as a response to changes in the amount of current information. Part (d) is the counterpart with **strong interactions**. Hence the significant change from almost

²⁷ In this case, the policy function is

$$x_t^a = g(R^a x_{t-1}, R^a \theta_t^I) = \sum_{b \in \mathbb{A}} c^b x_{t-1}^{a+b} + \sum_{b \in I(0)} d^a \theta_t^{a+b} + e \bar{\theta}.$$

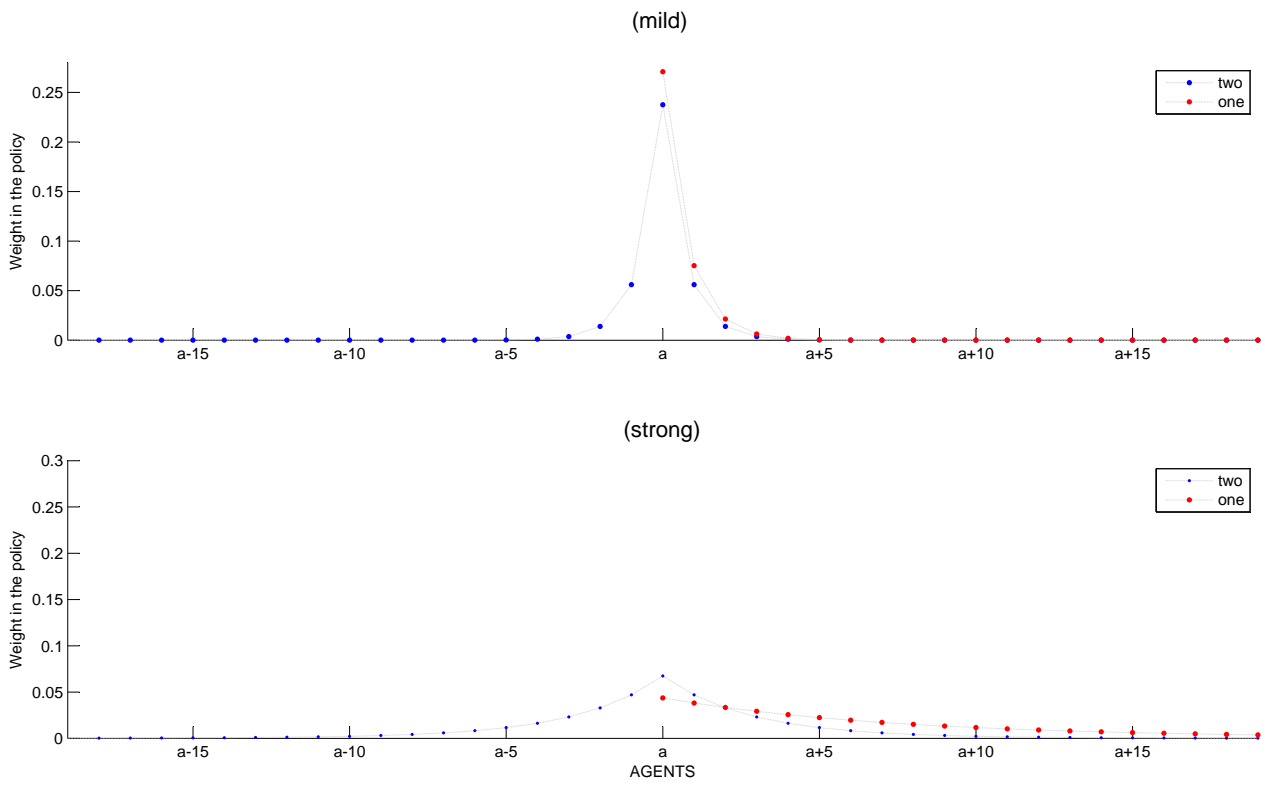


Figure 2: One-sided vs. two-sided interactions.

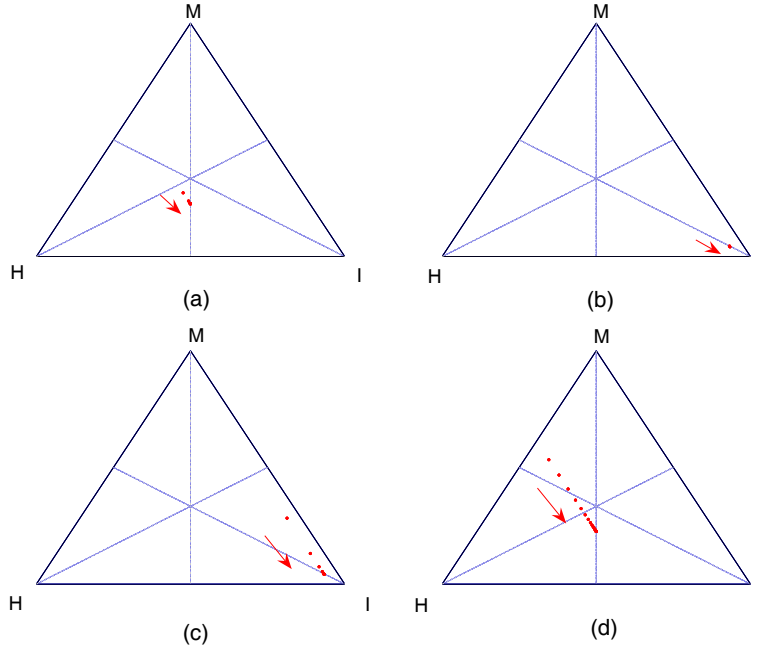


Figure 3: Effect of Information on Interactions.

no weight on current information to almost equal weights. Individuals use the information in the best possible way by putting more weight on it in their policy functions. This is due to the fact that guessing more precisely how neighbors will behave becomes more important due to the increased strength of interactions. Part (c) is **mild interactions** but **strong ‘own-type effect’** ($\frac{\alpha_1}{\alpha_2} = \frac{1}{20}$, $\frac{\alpha_2}{\alpha_3} = 20$) and part (b) is **strong interactions** and **strong own-type effect** ($\frac{\alpha_1}{\alpha_2} = \frac{1}{20}$, $\frac{\alpha_2}{\alpha_3} = 1$). We do not see much change in (b) although most of the total weight is put on information. This is mainly due to the fact that agent a cares so much about his current type that, he neglects the other effects. In (c), although the own-effect is still strong, due to the strength of interactions, agent uses the average information to form the best expectations of other agents behavior. As the amount of information he gets increases, he starts forming better expectations by transferring the policy weight from average information to precise information on close neighbors.

We analyzed, so far, the effects of changes in the information content and the strength of interactions on the equilibrium composition of the weights in the policy function. A third dimension through which we observe interesting effects is the time horizon of interactions. Since agents in this model are forward looking and rational, their equilibrium behavior and this latter’s distributional consequences are sensitive to the length of time that they expect to spend interacting with agents in their reference groups. The observables to an outsider (read econometrician) are

the statistical properties of the choices that the individuals make. One such property of interest is how ‘correlated’ individuals’ choices are with those of their neighbors, their neighbors’ neighbors... and so on; and whether those correlations are stationary across time. As the next Lemma shows, interestingly, those correlations can be ranked according to the *rate at which they decline* with respect to social or geographical distance.

Lemma 1 (Tail Convergence) ²⁸ *Given a T -period finite-horizon economy, define the t -period “rate of tail convergence” of the spatial autocorrelations as*

$$r_t := \lim_{a \rightarrow \infty} \frac{\text{Cov}\left(x_t^0, x_t^{a+1} \mid x_{t-1}\right)}{\text{Cov}\left(x_t^0, x_t^a \mid x_{t-1}\right)}, \quad \text{for all } t = 1, \dots, T$$

- (i) *The rate r_t is monotone decreasing across periods, i.e. $r_{t+1} < r_t$, for $t = 0, \dots, T$.*
- (ii) *For any given $t < T$, the rate r_t for the finite-horizon economies converges to that of the infinite-horizon as the horizon of the economy expands, i.e. $r_\infty = \lim_{T \rightarrow \infty} r_t$ for any given $t > 0$ and $r_\infty < 1$.*

In other words, how strongly individual choices are correlated for a given social distance can be approximated by a correlation function which is exponential in the distance. Moreover, the rate of decline is higher closer the final period of the economy, i.e., rational agents choose to correlate their actions more with their neighbors at early periods and progressively less so as they approach the end of their relationship. We use this intuition to the full extent possible when we talk about identification in Section 4.

Social interactions manifest themselves at the stationary ergodic distribution by means of spatial autocorrelation of actions. From the form of the stationary policy function in (6), the limit covariance between two agents b agents away from each other can be written as

$$\begin{aligned} \lim_{t \rightarrow \infty} \text{Cov}\left(x_t^a, x_t^{a+b}\right) &= \lim_{t \rightarrow \infty} \text{Cov}\left(x_t^0, x_t^b\right) \\ &= \lim_{t \rightarrow \infty} \sum_{a_1 \in \mathbb{A}} \sum_{b_1 \in \mathbb{A}} c^{a_1} c^{b_1} \text{Cov}\left(x_{t-1}^{a_1}, x_{t-1}^{b_1+b_1}\right) + \text{Cov}\left(\sum_{a_1 \in \mathbb{A}} d^{a_1} \theta_t^{a_1}, \sum_{b_1 \in \mathbb{A}} d^{b_1} \theta_t^{b_1+b_1}\right) \\ &= \sum_{a_1 \in \mathbb{A}} \sum_{b_1 \in \mathbb{A}} c^{a_1} c^{b_1} \lim_{t \rightarrow \infty} \text{Cov}\left(x_{t-1}^{a_1}, x_{t-1}^{b_1+b_1}\right) + \text{Var}(\theta) \sum_{a_1 \in \mathbb{A}} d^{a_1} d^{a_1-b} \end{aligned}$$

hence has a nice recursive structure. Let $\rho_{t,b}$ denote the conditional correlation between the actions of agents a and $a + b$, at time t , given x_0 i.e.

$$\rho_{t,b} = \frac{\text{Cov}(x_t^a, x_t^{a+b} \mid x_0)}{\text{Var}(x_t^a \mid x_0)}$$

²⁸The proof is in Appendix E.

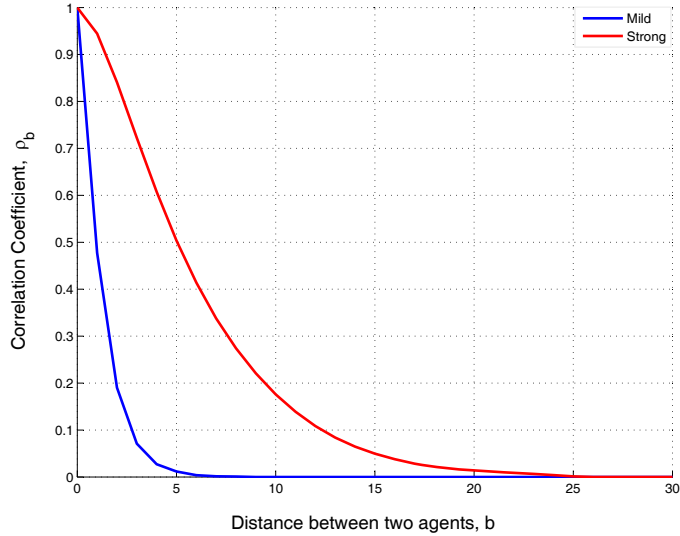


Figure 4: Correlation function at the ergodic distribution for Mild and Strong Interactions.

Because of the symmetry imposed on our economy, it is straightforward to see that such correlation is independent of a (as made clear in the notation). Also, let the correlation at the ergodic distribution between the equilibrium actions of agents a and $a + b$ be represented by ρ_b .

Proposition 1 *The limit correlation ρ_b between the actions of agents a and $a + b$ satisfies $\rho_b = \lim_{t \rightarrow \infty} \rho_{b,t}$. Moreover, ρ_b is *independent* of x_0 and declines *geometrically* with b .*

In Figure 4, we report the correlation functions in both the mild and strong conformity parameterizations as a function of social distance, b . Two effects are worth mentioning here. Firstly, both correlation functions converge to zero as the distance between two agents become arbitrarily large. Secondly, this convergence is much faster in the case of mild interactions than in the case of strong. For example, the correlation between the equilibrium choices of agent a and agent $a + 3$ (or $a - 3$ due to symmetry) is about 7% in the case of mild interactions whereas it is about 75% in the case of strong interactions. As the social distance becomes 6, these correlations are about 0% and 40% respectively. Hence, the strength of the fundamental desire to conform built in individuals' preferences determine endogenously, in the social equilibrium, the size of the neighborhood with which an individual interacts.

In turn, spatial correlations induce correlated actions of agents in endogenously formed groups. At the ergodic stationary distribution, when the dependence of the agents' actions in equilibrium are independent of the initial configuration of actions x_0 , such correlations in endogenously formed groups is manifested in a phenomenon which we refer to as *local norms of behavior* (see Figure

5).²⁹ In Figure 5, we plot 100 neighboring agents on the x-axis and their optimal choices drawn from the limit distribution at the same future date, on the y-axis. In the top panel, clearly the optimal actions are more spread and they do not follow a significant pattern. In the bottom panel though, the optimal choices are more concentrated and follow a clear path. This is due to the fact that, in equilibrium agents conform to the actions of neighboring agents, leading the way to the creation of similar local behavior. If one considers smoking as an example, given an appropriate normalization for the quantity of nicotine intake, we have in the bottom panel of Figure 5, communities (agents near agent 20) where people smoke very few or do not smoke at all, and communities (around agent 70) where people smoke a lot. Two interesting aspects of this phenomenon are firstly that every individual uses the same symmetric policy function to make his choices and all heterogeneity is captured by random types and we still have high spatial correlation and high spatial variation. Secondly, the initial fraction of smokers is irrelevant since the limit smoking distribution of individuals living in this economy is ergodic.

3.1 Rationality vs. Myopia

In this section we compare equilibrium configurations of economies with rational agents with those of economies with myopic agents which have been studied in the theoretical and empirical literature on social interactions. Following the mathematical physics literature in statistical mechanics on interacting particle systems several models of social interactions in economics introduce myopic agents by postulating that agents, when called to make a choice, act as if they expect never to be called to act again.³⁰ Given an initial history x_{t-1} and realization θ_t , each myopic agent $a \in \mathbb{A}$ chooses $x_t^a \in X$ to maximize

$$u(x_{t-1}^a, x_t^a, x_t^{a+1}, \theta_t^a) := -\alpha_1(x_{t-1}^a - x_t^a)^2 - \alpha_2(\theta_t^a - x_t^a)^2 - \alpha_3(x_t^{a+1} - x_t^a)^2 - \alpha_3(x_t^{a-1} - x_t^a)^2$$

It is straightforward to show that there exists a unique symmetric policy function g_m (m for ‘myopic’) where

$$g_m(x_{t-1}, \theta_t) := \sum_{b \in \mathbb{A}} c_m^b x_{t-1}^{a+b} + \sum_{b \in \mathbb{A}} d_m^b \theta_t^{a+b}.^{31}$$

²⁹ See Appendix D for details about how we simulate the ergodic stationary distribution of actions of the economy.

³⁰ See e.g., Blume and Durlauf (1998), Brock and Durlauf (2001); and Glaeser and Scheinkman (2000) for a comprehensive survey. Liggett (2004) is the standard reference for the mathematical literature.

³¹ In some of the literature, myopic agents are modelled not only as assuming that all agents in the economy only interact once, but also that their neighbors are not changing their previous period action. We refer to these agents as *backward looking*. A *backward looking* agent a solves

$$\max_{x_t^a \in X} -\alpha_1(x_{t-1}^a - x_t^a)^2 - \alpha_2(\theta_t^a - x_t^a)^2 - \alpha_3(x_{t-1}^{a+1} - x_t^a)^2 - \alpha_3(x_{t-1}^{a-1} - x_t^a)^2.$$

The policy function of *backward looking* agent a is

$$x_t^a = \beta_1 x_{t-1}^a + \beta_2 \theta_t^0 + \beta_3 x_{t-1}^1 + \beta_3 x_{t-1}^{-1}.$$

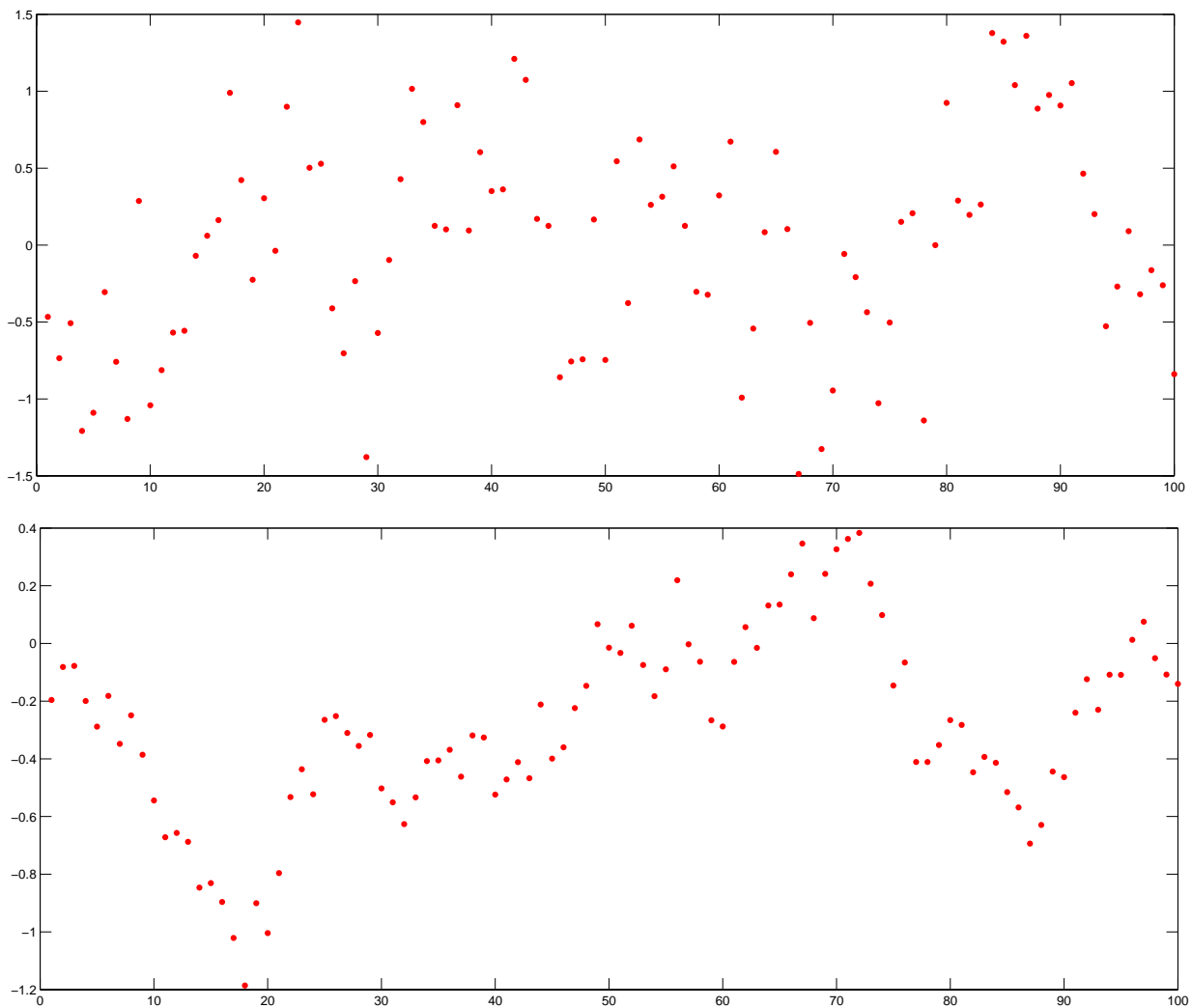


Figure 5: Ergodic Limit of *Mild* (top) and *Strong* (bottom) Interactions for 100 adjacent agents.

Proposition 2 *At the ergodic stationary distribution of actions in the economy with myopic agents, the correlation function ρ_b is indistinguishable from its rational counterpart.*

The spatial correlations obtained from the rational model, in Proposition 1, and those obtained from the myopic model, in Proposition 2 have the exact same geometric form. In other words, data on the stationary distribution of actions does not help us discern the underlying model, rational vs. myopic. Nonetheless we can ask the following question: An econometrician, observing data on the stationary distribution of actions generated by the Markov Perfect Equilibrium of a rational dynamic model but estimating preferences for conformity under the assumptions that agents are myopic, will under-estimate or over-estimate conformity? We do address this question by simulation. We first simulate the ergodic stationary distribution of actions generated by the Markov Perfect Equilibrium of a rational dynamic model (see Appendix D for details). We then fit the induced spatial correlations under the assumption that the distribution is generated by the equilibrium actions of myopic agents (see Appendix D for details) to estimate α_i , for $i = 1, \dots, 3$. Results are reported next. [discuss]

4 Identification in dynamic economies

Manski (1993) studies (lack of) identification of social interactions in static linear model.³² He in fact restricts his analysis to linear economies in which the social interactions operate through the mean action in the population, rather than the actions of neighbors models, that is, linear in means models. In this class of economies, according to our notation, each agent $a \in \mathbb{A}$, chooses an action x^a (we drop the time index as the economy is static) according to the following policy function:

$$g(\bar{x}, \theta^a) = c\bar{x} + d\theta^a$$

where \bar{x} is the population mean of x^a and θ^a is agent a 's preference shock, with mean $\bar{\theta}$. But, in equilibrium, $\bar{x} = \frac{d}{1-c}\bar{\theta}$ and d and c cannot be independently identified.³³

A recent literature has studied identification in this context, static linear in means models, under the condition that data from different non-overlapping population of agents are available. Intuitively, observing populations with different observable mean $\bar{\theta}$ would be enough for identification.³⁴

It can be shown, see Glaeser and Scheinkman (2000), that the ergodic stationary distribution of actions in the economy with *backward looking* agents coincides with that of *myopic* agents.

³² By *identification* we mean *identification in population*, that is, with infinite data. Sometimes *identification in population* is called *identifiability*; see e.g., Chiappori and Ekeland (2009)

³³ Notice that, exploiting the properties of conformity economies, at an equilibrium the policy function will satisfy $c = 1 - d$, but social interactions will remain not identified.

³⁴ See e.g., Graham and Hahn (2005) and Graham (2008), who have identified inclusion/exclusion restriction which allow identification in a general class of static linear in means economies; see also Davezies, D'Haultfoeuille

The economy we study in this paper is distinct from the one studied by Manski (1993), Graham and Hahn (2005) and others in this literature along two fundamental dimensions: i) social interactions operate through overlapping neighborhoods of finite dimension, ii) the economy is dynamic. Both dimensions could in principle help with identification, as long as the composition of the neighborhoods and a panel of population data over time are available. By studying populations composed of an infinite number of overlapping neighborhoods we intend to shed some light on the nature of the identification results in Graham and Hahn (2005) and others, which exploit instead an infinite number of non-overlapping groups. In both cases, in fact at equilibrium we are bound to observe variation in the distribution of actions across neighborhoods or groups. The overlapping structure of our neighborhoods, however, breaks their independence, which is instead required when non-overlapping groups are considered. A lack of identification result in our case would highlight the fundamental importance of independence across groups in the identification results in the literature.³⁵ By studying dynamic models we are able to exploit the theoretical implications deriving from the optimality of the dynamic choices of agents on time series autocorrelations of actions, over and above the implications regarding the cross-sectional (spatial) correlations.

We study the identification of social interactions in our linear dynamic economy with conformity. Identification in the context of our economy requires distinguishing preferences for conformity from correlated preferences. Intuitively, the spatial correlation of actions at equilibrium can be due to social interactions, as in our model, or to spatial correlation of preference types, which we have excluded by assumption in our analysis to this point. More formally, take two agents, e.g., agent a and agent b . A positive correlation between x_t^a and x_t^b could be due to e.g., preference for conformity. But the positive correlation between x_t^a and x_t^b could also be due to a positive correlation between θ_t^a and θ_t^b . In this last case, preferences for conformity and social interactions would play no role in the correlation of actions at equilibrium. Rather, such correlation would be due to the fact that agents have correlated preferences. Correlated preferences could be generally due to some sort of assortative matching or positive selection in social interaction, which induces agents with correlated preferences to interact socially.

By Theorem 1, under the assumption of our economy at a symmetric Markov perfect equilibrium, each agent $a \in \mathbb{A}$ acts according to the policy function

$$g_t^{a,*}(x_{t-1}, \theta_t^a; \alpha) = \sum_{b \in \mathbb{A}} c_t^b(\alpha) x_{t-1}^{a+b} + \sum_{b \in \mathbb{A}} d_t^b(\alpha), \theta_t^{a+b} + e_t(\alpha), \bar{\theta} \quad \mathbb{P} - a.s. \quad (7)$$

where we make once again explicit the dependence of the policy function on the preference parameters $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. If $T = \infty$, the policy function is stationary $g^{a,*}(x_{t-1}, \theta_t^a; \alpha)$. Recall

and Fougere (2006) who extend these results exploiting variation over the size of the populations.

³⁵On the other hand, we work with symmetric models, where each neighborhoods are ex-ante identical. Observable asymmetric neighborhoods are bound to have identification power in an otherwise symmetric economy; see Bramoullé, Djebbari and Fortin (2009), in a static linear in means model.

that the parameter α_3 represents the weight of conformity in each agent's preferences. It follows that $\alpha_3 = 0$ corresponds to an economy with no social interactions. We say that $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$ is a stochastic process induced by α and $(\theta_t^a)_{t \geq 0}^{a \in \mathbb{A}}$ if it satisfies

$$x_t^a = g_t^{a,*}(x_{t-1}, \theta_t^a; \alpha), \text{ for any } a \in \mathbb{A} \text{ and any } t \geq 0$$

We are now ready to construct our definition of identification of social interactions.

Definition 3 Let $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$ denote a stochastic process induced by α and $(\theta_t^a)_{t \geq 0}^{a \in \mathbb{A}}$, where $(\theta_t^a)_{t \geq 0}^{a \in \mathbb{A}}$ is i.i.d. across agents and serially uncorrelated, that is, where $\text{cov}(\theta_t^a, \theta_t^b) = \text{cov}(\theta_t^a, \theta_{t+1}^a)$ for any $a \neq b \in \mathbb{A}$ and any $t \geq 0$. We say that α is **identified** if there does not exist an $\hat{\alpha}$, with $\hat{\alpha}_3 = 0$, such that the process $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$ is also induced by $\hat{\alpha}$ and some stochastic process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$. We say that **social interactions are identified** if some α , with $\alpha_3 > 0$, is identified.

The conditions for identification of social interactions can be weakened by restricting the stochastic process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$. We say that α is (resp. social interactions are) **identified relatively to a set of preference shocks** if $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ in Definition 3 is required to belong to a set of preference shocks which satisfies some specific restriction.

Finally, the conditions for identification of social interactions can be strengthened by limiting the observable properties of the process $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$. We say that α is (resp. social interactions are) **identified by some properties of the process $(x_t^a)_{a \in \mathbb{A}}^{t=0,1,\dots,T}$** if Definition 3 holds when such specific properties of $(x_t^a)_{a \in \mathbb{A}}^{t=0,1,\dots,T}$ are substituted to $(x_t^a)_{a \in \mathbb{A}}^{t=0,1,\dots,T}$.

4.1 Infinite horizon economies

We first look at the case of an infinite horizon economy, with stationary policy functions and an ergodic distribution. In this context, we study first the possibility of obtaining identification by observing the properties of the stationary distribution of actions rather than the whole panel $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$. We then pass on to identification tout court, that is exploiting the whole dynamic restrictions imposed by the model on $(x_t^a)_{t \geq 0}^{a \in \mathbb{A}}$, not just the restrictions on the stationary distribution. We shall see that results are negative in both cases, that is, identification is not obtained in general. Secondly, we study identification relatively to a series of relevant restrictions on the stochastic process for preference shocks $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$. These restrictions are meant to capture natural properties of the selection mechanism which induces agents with correlated preference shocks to interact socially.

Consider first the stationary distribution of actions as identified by its implied spatial correlation function ρ_b .

Proposition 3 *Social interactions are **not identified by the properties of the spatial correlation function ρ_b of the stationary distribution of actions in infinite horizon economies.***

The proof is simple and instructive and hence it is reported following in the text.

Proof: We have shown in Section 2.2 that the stationary distribution of our dynamic economy with social interactions, that is, $\alpha_3 > 0$, and i.i.d. preference shock process $\{\theta_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$, is given by the ergodic measure π in (5), i.e. π is the joint distribution of

$$x_t = \left(\frac{e(\alpha) \bar{\theta}}{1 - C(\alpha)} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_{s-1}} \left((d(\alpha))^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} \right) \right)_{a \in \mathbb{A}}$$

Consider now an alternative specification of our economy with *no interactions* between agents ($\hat{\alpha}_3 = 0$) and *no habits* ($\hat{\alpha}_1 = 0$) but simply a preference shock process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ and *own type effects* with $\hat{\alpha}_2 > 0$. For this economy, equilibrium choice of agent a at time t is given by

$$x_t^a = \hat{\theta}_t^a$$

As long as the process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ is the one where

$$\hat{\theta}_t^a := \frac{e(\alpha) \bar{\theta}}{1 - C(\alpha)} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_{s-1}} \left((d(\alpha))^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} \right)$$

the probability distributions that the two specifications (with and without interactions) generate on the observables of interest, $\{x_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$, are identical. Hence, one cannot identify from the stationary distribution of choices which specification generates the data. \blacksquare

More generally, we investigate if the dynamic equilibrium restrictions of our model are sufficient to identify social interactions.

Proposition 4 *Social interactions are **not identified** in infinite horizon economies.*

This proof is also simple and instructive and hence it is reported following in the text.

Proof: In the case of complete information, the policy function is:

$$x_t^a = \sum_{b \in \mathbb{A}} c_b(\alpha) x_{t-1}^{a+b} + \sum_{b \in \mathbb{A}} d_b(\alpha) \theta_t^{a+b} + e \bar{\theta}$$

As we saw in Lemma 5, one can obtain by iteration the reduced form

$$\begin{aligned} x_t^a &= \sum_{b_1} \cdots \sum_{b_t} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_t} x_0^{a+b_1+\cdots+b_t} \\ &\quad + \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_{s-1}} \left((d(\alpha))^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e(\alpha) \bar{\theta} \right) \end{aligned}$$

Consider now the alternative specification with *no interactions* between agents ($\hat{\alpha}_3 = 0$) and *no habits* ($\hat{\alpha}_1 = 0$), a preference shock process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ and *own type effects* with $\hat{\alpha}_2 > 0$. For this economy, equilibrium choice of agent a at time t is given by

$$x_t^a = \hat{\theta}_t^a$$

Defining the new preference shock process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ as

$$\begin{aligned} \hat{\theta}_t^a : &= \sum_{b_1} \cdots \sum_{b_t} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_t} x_0^{a+b_1+\cdots+b_t} \\ &+ \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_s} \left((d(\alpha))^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e(\alpha) \bar{\theta} \right) \end{aligned}$$

would imply that for an arbitrary initial distribution π_0 for x_0 , the joint probability distributions that the two specifications (with and without interactions) generate on the process $\{x_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$, are identical. Moreover, if one allows for infinite histories, one can define the preference shock process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ as before

$$\hat{\theta}_t^a := \frac{e(\alpha) \bar{\theta}}{1 - C(\alpha)} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} (c(\alpha))^{b_1} \cdots (c(\alpha))^{b_s} \left((d(\alpha))^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} \right)$$

and obtain observational equivalence once again. Hence, we conclude that identification is not possible. \blacksquare

An intuition about this result can be obtained by loosely reducing the identification of social interactions in infinite horizon economies to the well known problem of distinguishing a VAR from an MA(∞) process. Stacking in a vector \mathbf{x}_t (resp. θ_t) the actions x_t^a over the index $a \in \mathbb{A}$ (resp. the preference shocks θ_t^a), policy functions can be loosely written as a VAR:

$$\mathbf{x}_t = \Phi \mathbf{x}_{t-1} + \delta_t, \quad \text{with } \delta_t = \Gamma \theta_t + e \bar{\theta}$$

where $E(\delta_t \delta_{t-\tau}) = 0$ for all $\tau > 0$. Under standard stationarity assumption, the VAR has an MA(∞) representation

$$\mathbf{x}_t = (I_A - \Phi \mathbf{L})^{-1} \delta_t = \delta_t + \Psi_1 \delta_{t-1} + \Psi_2 \delta_{t-2} + \dots$$

for a sequence Ψ_1, Ψ_2, \dots such that $(I_A - \Phi \mathbf{L}) (I_A + \Psi_1 \mathbf{L} + \Psi_2 \mathbf{L}^2 + \dots) = I_A$. The argument in the proof of Proposition 4 therefore amounts to picking

$$\mathbf{x}_t = \hat{\theta}_t = \delta_t + \Psi_1 \delta_{t-1} + \Psi_2 \delta_{t-2} + \dots$$

4.2 Finite horizon

When the economy has a finite horizon, the unique policy function and the distribution of actions are *not stationary*. We show that identification is possible in this case.

Consider a finite horizon economy with social interactions ($T < \infty$). Assume that we restrict the class of admissible preference shock processes $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ to satisfy the following *covariance stationarity* restrictions:

Definition 4 (Covariance Stationarity) ³⁶ A process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ is said to be covariance stationary if $\text{cov}(\hat{\theta}_t^a, \hat{\theta}_t^b) = Z(|a - b|)$, for $a, b \in \mathbb{A}$, all $t = 0, 1, \dots, T$, and $Z(s) \in \mathbb{R}$ for $s = 0, 1, \dots$, i.e., any process that is stationary with respect to the social distance across agents.

The data that are generated by this kind of process would be fundamentally different than the data generated by a social interaction process. Here is why: As we saw in the section on characterization of the policy function, the weight distribution of the policy function gets more spread and more weight is shifted to the mean type as the number of periods of the continuation economy increases. In other words, the equilibrium policy in a finite horizon economy is *non-stationary*. Consequently, the observed distribution that such a policy function generates is non-stationary across time too. The immediate testable implication of this is the variation in the spatial autocorrelation functions. While the spatial autocorrelations between agents' choices (same period, at the cross-section) are the same across periods in the absence of interaction effects, they change in presence of interactions. Moreover, they respect an interesting *monotonicity* property that allows one to order the autocorrelation functions with respect to their spatial rate of tail convergence, which is summarized in the following Lemma.

Next, we will argue that although the spatial correlation functions across periods are the same in the absence of interactions, they are different in the case of interactions. This is a clean testable implication of the existence of interaction effects. This is at the heart of our identification idea which is summarized in the following proposition.

Proposition 5 *Social interactions are identified relatively to processes satisfying the covariance stationarity restrictions.*

Proof: Consider a finite-horizon, T -period economy with $T \geq 2$. In the absence of interactions ($\hat{\alpha}_3 = 0$), agent a 's last period optimal choice is

$$x_T^a = c_T(\hat{\alpha}) x_{T-1}^a + d_T(\hat{\alpha}) \hat{\theta}_T^a$$

from which the a -step covariance between equilibrium choices of agent 0 and a is easily computed to be

$$\text{cov} \left(x_T^0, x_T^a \mid x_{T-1} \right) = d_T(\hat{\alpha})^2 \text{Cov} \left(\hat{\theta}_T^0, \hat{\theta}_T^a \right) \quad (8)$$

In order the specification with no interactions to be observationally indistinguishable from the interactions case, the a -step conditional covariances, for all $a \in \mathbb{A}$, under interactions and in the absence of interactions should match, in period T . Thus, we have the freedom to choose the $\left\{ \text{Cov} \left(\hat{\theta}_T^0, \hat{\theta}_T^a \right) \right\}_{a \in \mathbb{A}}$ accordingly. Similar calculations in period $T - 1$ yield

$$\text{cov} \left(x_{T-1}^0, x_{T-1}^a \mid x_{T-2} \right) = d_{T-1}(\hat{\alpha})^2 \text{Cov} \left(\hat{\theta}_{T-1}^0, \hat{\theta}_{T-1}^a \right) \quad (9)$$

³⁶This is also called *weak stationarity* in the Times Series literature (see Hamilton (1994), p. 45-46 and chapter 10). Thus, we employ a weak notion of spatial stationarity, independent of time and any parametric specification.

Once again, in order the two model to be observationally indistinguishable, the a -step conditional covariances, for all $a \in \mathbb{A}$, under interactions and in the absence of interactions should match, in period $T - 1$. Moreover, under the *covariance stationarity* hypothesis, the $T - 1$ period a -step covariances for the preference shocks, in the absence of interactions $\left\{ Cov \left(\hat{\theta}_{T-1}^0, \hat{\theta}_{T-1}^a \right) \right\}_{a \in \mathbb{A}}$, must equal their T period counterparts $\left\{ Cov \left(\hat{\theta}_T^0, \hat{\theta}_T^a \right) \right\}_{a \in \mathbb{A}}$. This latter implies by substitution from (8) and (9) that for all $a \in \mathbb{A}$

$$d_{T-1}(\hat{\alpha})^{-2} cov \left(x_{T-1}^0, x_{T-1}^a \mid x_{T-2} \right) = d_T(\hat{\alpha})^{-2} cov \left(x_T^0, x_T^a \mid x_{T-1} \right)$$

Writing the last equation for $a + 1$ and dividing it by the equation for a yields

$$\frac{Cov \left(x_{T-1}^0, x_{T-1}^{a+1} \mid x_{T-2} \right)}{Cov \left(x_{T-1}^0, x_{T-1}^a \mid x_{T-2} \right)} = \frac{Cov \left(x_T^0, x_T^{a+1} \mid x_{T-1} \right)}{Cov \left(x_T^0, x_T^a \mid x_{T-1} \right)} \quad (10)$$

Since the choice of a is arbitrary, we can look at the same expression as a becomes progressively larger. So, as $a \rightarrow \infty$, the expression should give

$$\lim_{a \rightarrow \infty} \frac{Cov \left(x_{T-1}^0, x_{T-1}^{a+1} \mid x_{T-2} \right)}{Cov \left(x_{T-1}^0, x_{T-1}^a \mid x_{T-2} \right)} = r_{T-1} = r_T = \lim_{a \rightarrow \infty} \frac{Cov \left(x_T^0, x_T^{a+1} \mid x_{T-1} \right)}{Cov \left(x_T^0, x_T^a \mid x_{T-1} \right)} \quad (11)$$

which is a contradiction to Lemma 1. Therefore, there does not exist a covariance stationary preference shock process $\{\hat{\theta}_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ that generates an equilibrium choice process $\{x_t^a\}_{t \geq 0}^{a \in \mathbb{A}}$ under the no interactions specification ($\hat{\alpha}_3 = 0$) that is observationally equivalent to the process generated by the local interactions ($\alpha_3 \neq 0$) process. This concludes the proof. \blacksquare

5 Discussion and Conclusion

TBW

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7 Appendix A: The existence proof (Theorem 1)

The proof is constructive and works by induction on the length of the continuation economies. Here is a sketch of the steps involved:

Step 1: We first show that in the last period, T , (1-period continuation) of any finite-horizon economy, FOCs induce an operator which is a contraction in the space of bounded measurable functions that have as arguments any T -length history. Hence, there exists a unique symmetric (possibly history-dependent) equilibrium. We then show that the equilibrium policy must be Markovian (in the set G) and it takes the convex combination form in the statement of the theorem (Lemma 2).

Step 2: For any finite horizon ($T < \infty$) economy, we assume that in the continuation from period $t + 1$ on agents choose according to the unique symmetric MPE $g^* : \mathbf{X} \times \Theta \times \{t + 1, \dots, T\} \mapsto X$. Given the linearity of the policy in the continuation, a generic agent's dynamic program stays strictly concave and FOCs are necessary and sufficient for a pure strategy maximum. We then show that FOC can be written as a function only of current period choices, current preference shocks, and expected type and that it induces an operator similar to the one in Step 1, which turns out to be a contraction and maps the set of Markovian strategies into itself. Hence, there exists a unique fixed point $g_t : \mathbf{X} \times \Theta \mapsto X$. We conclude that for the $T - t + 1$ -period continuation economy, the map $(g_t, g^*) : \mathbf{X} \times \Theta \times \{t, t + 1, \dots, T\} \mapsto X$ is the unique symmetric MPE in pure strategies and has the convex combination form as in the statement of the theorem. We use Lemma 3 for the alternative formulation. Lemma 3 uses another fundamental Lemma (5) both of whose proofs are collected at the end of this section to ease the lecture.

Step 3 Final step involves taking a limit. We construct a series of finite economies, approximating the ∞ -horizon economy, given an appropriate topology. We then show that, the finite truncation equilibrium correspondence is u.h.c. with respect to the parametrization. We use Lemma 4 to do that. This is not enough for stationarity. Thanks to the compactness of the behavioral Markovian strategy set (the set G) under the supnorm, the sequence of equilibrium policy functions converges to a policy function (which is an equilibrium policy due to u.h.c of the equilibrium correspondence), hence the same one every period. This gives us stationarity.

Proof: Step 1: Existence, uniqueness and Markovianity in the final period. Let any finite-horizon, T -period economy be given. In this symmetric environment, it is enough to analyze the optimization problem of a single agent, say of agent $0 \in \mathbb{A}$. In the last period ($t = T$), given any history (x^{T-1}, θ^{T-1}) and current type draw θ_T , agent 0 solves

$$\max_{x_T^0 \in X} \left\{ -\alpha_1 (x_{T-1}^0 - x_T^0)^2 - \alpha_2 (\theta_T^0 - x_T^0)^2 - \alpha_3 (x_T^{-1} - x_T^0)^2 - \alpha_3 (x_T^1 - x_T^0)^2 \right\} \quad (12)$$

The first order condition

$$2 [\alpha_1 (x_{T-1}^0 - x_T^0) + \alpha_2 (\theta_T^0 - x_T^0) + \alpha_3 (x_T^{-1} - x_T^0) + \alpha_3 (x_T^1 - x_T^0)] = 0$$

implies that

$$x_T^0 = \Delta_T^{-1} (\alpha_1 x_{T-1}^0 + \alpha_2 \theta_T^0 + \alpha_3 x_T^{-1} + \alpha_3 x_T^1) \quad (13)$$

where $\Delta_T := (\alpha_1 + \alpha_2 + 2\alpha_3) > 0$. This choice is feasible (in X) since it is a convex combination of elements of X , a convex set by assumption. The objective function (12) is strictly concave in x_T^0 , thus x_T^0 in (13) is the unique optimizer. We see from (13) that showing the existence of a symmetric equilibrium in the continuation given (x^{T-1}, θ^T) is equivalent to finding the fixed point of an operator $L_1 : B((\mathbf{X} \times \Theta)^T, X) \rightarrow B((\mathbf{X} \times \Theta)^T, X)$ that acts on the class of bounded measurable functions $x_T : (\mathbf{X} \times \Theta)^T \rightarrow X$ according to

$$(L_1 x_T)(x^{T-1}, \theta^T) = \Delta_T^{-1} (\alpha_1 x_0^0 + \alpha_2 \theta_1^0 + \alpha_3 x_T(R^{-1} x^{T-1}, R^{-1} \theta^T) + \alpha_3 x_T(R x^{T-1}, R \theta^T))$$

Clearly, L_1 is a self-map. We show next that it is a contraction. Endow $B((\mathbf{X} \times \Theta)^T, X)$ with the sup norm which makes $(B((\mathbf{X} \times \Theta)^T, X), \|\cdot\|_\infty)$ a Banach space. Pick $x_T, \hat{x}_T \in B((\mathbf{X} \times \Theta)^T, X)$. We have for all (x^{T-1}, θ^T)

$$\begin{aligned} \left| (L_1 x_T)(x^{T-1}, \theta^T) - (L_1 \hat{x}_T)(x^{T-1}, \theta^T) \right| &= \Delta_T^{-1} \left| \alpha_1 x_0^0 + \alpha_2 \theta_1^0 + \alpha_3 x_T(R^{-1} x^{T-1}, R^{-1} \theta^T) + \alpha_3 x_T(R x^{T-1}, R \theta^T) \right. \\ &\quad \left. - \alpha_1 x_0^0 - \alpha_2 \theta_1^0 - \alpha_3 \hat{x}_T(R^{-1} x^{T-1}, R^{-1} \theta^T) - \alpha_3 \hat{x}_T(R x^{T-1}, R \theta^T) \right| \\ &= \Delta_T^{-1} \left| \alpha_3 (x_T(R^{-1} x^{T-1}, R^{-1} \theta^T) - \hat{x}_T(R^{-1} x^{T-1}, R^{-1} \theta^T)) \right. \\ &\quad \left. + \alpha_3 (x_T(R x^{T-1}, R \theta^T) - \hat{x}_T(R x^{T-1}, R \theta^T)) \right| \\ &\leq \left(\frac{\alpha_3}{\Delta_T} \right) \left| x_T(R^{-1} x^{T-1}, R^{-1} \theta^T) - \hat{x}_T(R^{-1} x^{T-1}, R^{-1} \theta^T) \right| \\ &\quad + \left(\frac{\alpha_3}{\Delta_T} \right) \left| x_T(R x^{T-1}, R \theta^T) - \hat{x}_T(R x^{T-1}, R \theta^T) \right| \\ &\leq \left(\frac{2\alpha_3}{\Delta_T} \right) \|x_T - \hat{x}_T\|_\infty \end{aligned}$$

The coefficient $2\alpha_3 \Delta_T^{-1} < 1$ since $\alpha_i > 0$, for $i = 1, 2, 3$. Hence L_1 is a contraction on $B((\mathbf{X} \times \Theta)^T, X)$. Thus, L_1 has a unique fixed point x_T^* in $B((\mathbf{X} \times \Theta)^T, X)$. Next, we argue that this equilibrium strategy must be Markovian.

Lemma 2 (Markovianity and the Convex Combination Form) *Unique symmetric continuation equilibrium strategy x_T^* is Markovian, i.e., it depends solely on last period equilibrium choices and current preference shock realizations: $x_T^*(x^{T-1}, \theta^T) = g_T(x_{T-1}, \theta_T)$, for some $g_T : \mathbf{X} \times \Theta \rightarrow X$. Moreover, it has the convex combination form as in the statement of the theorem.*

Proof: Let

$$G := \left\{ \begin{array}{l} g : \mathbf{X} \times \Theta \rightarrow X \text{ s.t. } g(x, \theta) = \sum_{a \in \mathbb{A}} c^a x^a + \sum_{a \in \mathbb{A}} d^a \theta^a + e \bar{\theta} \\ \text{with} \\ \text{(i) } c^a, d^a, e \geq 0 \text{ and } e + \sum_{a \in \mathbb{A}} (c^a + d^a) = 1 \\ \text{(ii) } (\frac{1}{2})c^{a+1} + (\frac{1}{2})c^{a-1} \geq c^a, \forall a \neq 0 \\ \text{(iii) } c^b \leq c^a, \forall a, b \in \mathbb{A} \text{ with } |b| > |a|. \\ \text{(iv) } c^a = c^{-a}, \forall a \in \mathbb{A} \\ \text{and properties (ii), (iii), (iv) holding similarly for the } d \text{ sequence.} \end{array} \right\} \quad (14)$$

be the class of functions that are convex combinations (i) of one-period before history, current types and average type, possessing the (ii) ‘convexity’, (iii) ‘monotonicity’, and (iv) ‘symmetry’ properties. Property

(ii) states that the rate of ‘spatial’ (cross-sectional) convergence of the policy weights is non-increasing in both directions, from the center. Monotonicity property has a very natural interpretation: agent b ’s effect on agent 0’s marginal utility is smaller than agent a ’s effect on it, if a is closer to 0 than b is. Finally, (iv) says that the correlation pattern is symmetric around 0. Let $g \in G$ be such that after history (x^{T-1}, θ^T)

$$x_T(x^{T-1}, \theta^T) = g(x_{T-1}, \theta_T)$$

and let (c, d, e) be the coefficient sequence associated with g . Applying L_1 to x_T (hence to g), we get

$$\begin{aligned} (L_1 x_T)(x^{T-1}, \theta^T) &= \Delta_T^{-1} (\alpha_1 x_{T-1}^0 + \alpha_2 \theta_T^0 + \alpha_3 g(R^{-1} x_{T-1}, R^{-1} \theta_T) + \alpha_3 g(R x_{T-1}, R \theta_T)) \\ &= \Delta_T^{-1} \left(\alpha_1 x_0^0 + \alpha_2 \theta_1^0 \right. \\ &\quad \left. + \alpha_3 \left(\sum_{a \in \mathbb{A}} c^a x_{T-1}^{a-1} + \sum_{a \in \mathbb{A}} d^a \theta_T^{a-1} + e \bar{\theta} \right) + \alpha_3 \left(\sum_{a \in \mathbb{A}} c^a x_{T-1}^{a+1} + \sum_{a \in \mathbb{A}} d^a \theta_T^{a+1} + e \bar{\theta} \right) \right) \end{aligned} \quad (15)$$

Reorganizing the terms gives

$$\begin{aligned} &= \Delta_T^{-1} \left(\alpha_1 x_{T-1}^0 + \alpha_2 \theta_T^0 \right. \\ &\quad \left. + \alpha_3 \left(\sum_{a \in \mathbb{A}} c^{a+1} x_{T-1}^a + \sum_{a \in \mathbb{A}} d^{a+1} \theta_T^a + e \bar{\theta} \right) + \alpha_3 \left(\sum_{a \in \mathbb{A}} c^{a-1} x_{T-1}^a + \sum_{a \in \mathbb{A}} d^{a-1} \theta_T^a + e \bar{\theta} \right) \right) \end{aligned}$$

and rearranging gives

$$\begin{aligned} &= \Delta_T^{-1} \left(x_{T-1}^0 \underbrace{(\alpha_1 + \alpha_3 c^{-1} + \alpha_3 c^1)}_{\Delta_1 \hat{c}^0} + \theta_T^0 \underbrace{(\alpha_2 + \alpha_3 d^{-1} + \alpha_3 d^1)}_{\Delta_T \hat{d}^0} + 2\alpha_3 e \bar{\theta} \right. \\ &\quad \left. + \sum_{a \neq 0} \underbrace{(\alpha_3 c^{a-1} + \alpha_3 c^{a+1})}_{\Delta_1 \hat{c}^a} x_{T-1}^a + \sum_{a \neq 0} \underbrace{(\alpha_3 d^{a-1} + \alpha_3 d^{a+1})}_{\Delta_T \hat{d}^a} \theta_T^a \right) \end{aligned} \quad (16)$$

The function after the last equality sign is linear in x_{T-1}, θ_T and $\bar{\theta}$. So, $L_1 x_T$ preserves the same linear form. By definition of the new coefficient sequence $(\hat{c}, \hat{d}, \hat{e})$ in (16), each element of the sequence is nonnegative since each element of the original one was so. New coefficients sum up to 1 since convex combination form of g makes the sum of the coefficients inside the two parentheses on the right hand side of (15) equal to 1. Thus, the total sum of coefficients on the right hand side of (15) is $\Delta_T^{-1}(\alpha_1 + \alpha_2 + 2\alpha_3) = 1$, which proves property (i). The final form in (16) is just a regrouping of elements in (15). Let $(\hat{c}^a)_{a \in \mathbb{A}}$ be the new coefficient sequence associated with $L_1 x_T$ as defined in equation (16). Pick $a \neq 0$ in \mathbb{A} ,

$$\begin{aligned} \hat{c}^{a+1} + \hat{c}^{a-1} &\geq \left(\frac{\alpha_3}{\Delta_T} \right) (c^a + c^{a+2}) + \left(\frac{\alpha_3}{\Delta_T} \right) (c^{a-2} + c^a) \\ &\geq \left(\frac{\alpha_3}{\Delta_T} \right) (2c^{a+1} + 2c^{a-1}) \\ &= 2 \left(\frac{\alpha_3}{\Delta_T} \right) (c^{a+1} + c^{a-1}) \\ &= 2\hat{c}^a \end{aligned}$$

By definition of \hat{c} in (16), first inequality is strict if $|a| = 1$, is an equality otherwise; second inequality is by property (ii) on c ; last equality is once again by definition of \hat{c} in (16). Therefore, for any $a \neq 0$ in \mathbb{A} , $\hat{c}^{a+1} + \hat{c}^{a-1} \geq 2\hat{c}^a$, which is property (ii). Pick any $a, b \in \mathbb{A}$ with $|a| < |b|$.

$$\begin{aligned}
\hat{c}^a &= \left(\frac{\alpha_3}{\Delta_T}\right) c^{a-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{a+1} \\
&= \left(\frac{\alpha_3}{\Delta_T}\right) c^{|a|-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{|a|+1} \\
&\geq \left(\frac{\alpha_3}{\Delta_T}\right) c^{|b|-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{|b|+1} \\
&= \left(\frac{\alpha_3}{\Delta_T}\right) c^{b-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{b+1} \\
&= \hat{c}^b
\end{aligned}$$

First equality is from (16); second by property (iv) in G ; the inequality is property (iii) in G ; next equality is due to property (iv) in G ; and finally the last equality is by (16). Hence, property (iii) holds for the new sequence. We next show that \hat{c} satisfies (iv).

$$\begin{aligned}
\hat{c}^a &= \left(\frac{\alpha_3}{\Delta_T}\right) c^{a-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{a+1} \\
&= \left(\frac{\alpha_3}{\Delta_T}\right) c^{-a-1} + \left(\frac{\alpha_3}{\Delta_T}\right) c^{-a+1} \\
&= \hat{c}^{-a}
\end{aligned}$$

where first equality is by (16); the second is due to (iv) in G ; finally the last is once again by (16). Thus, restriction of L_1 to G maps elements of G into itself. Endowed with the sup norm, the set of bounded measurable functions that agree with an element of G after any history (X^{T-1}, θ^T) is a closed subset of $B((\mathbf{X} \times \Theta)^T, X)$ since it is defined by equality and inequality constraints, hence a complete metric space in its own right. Since L_1 is a contraction on this latter as we just showed, the unique fixed point in $B((\mathbf{X} \times \Theta)^T, X)$ must agree with G too. Therefore, the unique symmetric equilibrium in the final period after a T -length history (x^{T-1}, θ^T) must be Markovian and should assume the convex combination form stated in the theorem. This concludes the proof of the Lemma. \blacksquare

Step 2: Induction. Assume that there exists a unique symmetric MPE for the T -period finite-horizon economy from period $t + 1$ on. Agent 0 believes that all other agents, including his own reincarnations, will use the unique symmetric equilibrium map $g : \mathbf{X} \times \Theta \times \{t + 1, \dots, T\} \mapsto X$ in the continuation. Given the t -length history (x^{t-1}, θ^t) , he solves

$$\begin{aligned}
&\max_{x_t^0 \in X} \left\{ -\alpha_1 (x_{t-1}^0 - x_t^0)^2 - \alpha_2 (\theta_t^0 - x_t^0)^2 - \alpha_3 (x_t^{-1} - x_t^0)^2 - \alpha_3 (x_t^1 - x_t^0)^2 \right. \\
&\quad \left. + E \left[\sum_{\tau=t+1}^T \beta^{\tau-t} \left(-\alpha_1 (x_{\tau-1}^0 - x_\tau^0)^2 - \alpha_2 (\theta_\tau^0 - x_\tau^0)^2 - \alpha_3 (x_\tau^{-1} - x_\tau^0)^2 - \alpha_3 (x_\tau^1 - x_\tau^0)^2 \right) \middle| (x^{t-1}, \theta^t) \right] \right\} \quad (17)
\end{aligned}$$

By hypothesis, agents' future choices on the equilibrium path are composite linear functions after any history, hence agent 0's problem is differentiable and the unconstrained ($x_t^0 \in \mathbb{R}$) first order condition for

(17) is

$$\begin{aligned}
0 &= \alpha_1 (x_{t-1}^0 - x_t^0) + \alpha_2 (\theta_t^0 - x_t^0) + \alpha_3 (x_t^{-1} - x_t^0) + \alpha_3 (x_t^1 - x_t^0) \\
&+ E \left[\sum_{\tau=t+1}^T \beta^{\tau-t} \left(-\alpha_1 (x_{\tau-1}^0 - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_{\tau-1}^0 - x_\tau^0) + \alpha_2 (\theta_\tau^0 - x_\tau^0) \frac{\partial}{\partial x_t^0} x_\tau^0 \right. \right. \\
&\quad \left. \left. - \alpha_3 (x_\tau^{-1} - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_\tau^{-1} - x_\tau^0) - \alpha_3 (x_\tau^1 - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_\tau^1 - x_\tau^0) \right) \middle| (x^{t-1}, \theta^t) \right]
\end{aligned} \tag{18}$$

Agent 0's problem is strictly concave in his choice x_t^0 since the second partial of the objective function in (17) with respect to x_t^0 , $-\Delta_t$ by definition, is negative, or

$$\begin{aligned}
\Delta_t &:= \alpha_1 + \alpha_2 + 2\alpha_3 \\
&+ \sum_{\tau=t+1}^T \beta^{\tau-t} \left(\alpha_1 \left(\frac{\partial}{\partial x_t^0} (x_{\tau-1}^0 - x_\tau^0) \right)^2 + \alpha_2 \left(\frac{\partial}{\partial x_t^0} x_\tau^0 \right)^2 \right. \\
&\quad \left. + \alpha_3 \left(\frac{\partial}{\partial x_t^0} (x_\tau^{-1} - x_\tau^0) \right)^2 + \alpha_3 \left(\frac{\partial}{\partial x_t^0} (x_\tau^1 - x_\tau^0) \right)^2 \right) > 0
\end{aligned} \tag{19}$$

Consequently, the FOC characterizes the *unique maximizer* of the unconstrained problem ($x_1^0 \in \mathbb{R}$). The following Lemma shows that equation (18) has a much simpler representation (whose proof is postponed till the end of the existence proof in order not to interrupt the reading).

Lemma 3 (Interiorness) *Equation (18) can be written in the following alternative form*

$$0 = -x_t^0 \Delta_t + \alpha_1 x_{t-1}^0 + \alpha_2 \theta_t^0 + \sum_{a \neq 0} \gamma_t^a x_t^a + \mu_t \bar{\theta} \tag{20}$$

where $\Delta_t := \alpha_1 + \alpha_2 + \sum_{a \neq 0} \gamma_t^a + \mu_t$. Moreover, the coefficients α_1 , α_2 , $(\gamma_t^a)_{a \neq 0}$, and μ_t are all non-negative.

By isolating the choice x_t^0 on the LHS, we have

$$x_t^0 = \Delta_t^{-1} \left(\alpha_1 x_{t-1}^0 + \alpha_2 \theta_t^0 + \sum_{a \neq 0} \gamma_t^a x_t^a + \mu_t \bar{\theta} \right) \tag{21}$$

which means that the maximizer of the unconstrained problem is a convex combination of x_{t-1}^0 , θ_t^0 , $(x_t^a)_{a \neq 0}$ and $\bar{\theta}$. Each of these are elements of X , a convex set. Hence, the maximizer of the unconstrained problem is in the feasible set of the constrained problem. Therefore, it is the unique maximizer of (17). The form in (21) implies that showing the existence of a symmetric continuation equilibrium policy for the t -th period of a T -period economy is equivalent to finding the fixed point of an operator $L_t : B((\mathbf{X} \times \Theta)^t, X) \rightarrow B((\mathbf{X} \times \Theta)^t, X)$ that acts on the class of bounded measurable functions $x_t : (\mathbf{X} \times \Theta)^t \rightarrow X$ according to

$$(L_t x_t)(x^{t-1}, \theta^t) = \Delta_t^{-1} \left(\alpha_1 x_{t-1}^0 + \alpha_2 \theta_t^0 + \sum_{a \neq 0} \gamma_t^a x_t(R^a x^{t-1}, R^a \theta^t) + \mu_t \bar{\theta} \right)$$

Clearly L_t is a self-map. Using straightforward modifications of the arguments in the proof of **Step 1**, we can show for $x_t, \hat{x}_t \in B((\mathbf{X} \times \Theta)^t, X)$ that

$$\left| (L_t x_t)(x^{t-1}, \theta^t) - (L_t \hat{x}_t)(x^{t-1}, \theta^t) \right| \leq \sum_{a \neq 0} \left(\frac{\gamma_t^a}{\Delta_t} \right) \|x_t - \hat{x}_t\|_\infty$$

The coefficient $\sum_{a \neq 0} \left(\frac{\gamma_t^a}{\Delta_t} \right) < 1$ since $\alpha_i > 0$, $i = 1, 2, 3$. Thus, L_t is a contraction on the Banach space of bounded measurable functions $(B((\mathbf{X} \times \Theta)^t, X), \|\cdot\|_\infty)$, consequently has a unique fixed point x_t^* . Once again, straightforward modifications of the arguments in Lemma 2 imply that we can focus attention on Markovian strategies. As in the proof of Lemma 2, it suffices to show, with a bit of abuse, that $L_t(G) \subset G$. To that effect, pick $g \in G$ and let (c, d, e) be the associated coefficient sequence. Applying L_t to g

$$\begin{aligned} (L_t g)(x_{t-1}, \theta_t) &= \Delta_t^{-1} \left(\alpha_1 x_{t-1}^0 + \alpha_2 \theta_t^0 + \sum_{a_1 \neq 0} \gamma_t^{a_1} g(R^{a_1} x_{t-1}, R^{a_1} \theta_t) + \mu \bar{\theta} \right) \\ &= \Delta_t^{-1} \left(\alpha_1 x_{t-1}^0 + \alpha_2 \theta_t^0 + \mu \bar{\theta} \right) \end{aligned} \quad (22)$$

$$\begin{aligned} &+ \sum_{a_1 \neq 0} \gamma_t^{a_1} \left(\sum_{a_2 \in \mathbb{A}} c^{a_2} x_{t-1}^{a_1+a_2} + \sum_{a_2 \in \mathbb{A}} d^{a_2} \theta_t^{a_1+a_2} + e \bar{\theta} \right) \\ &= \Delta_t^{-1} \left(\underbrace{\left[\alpha_1 + \sum_{a_1 \neq 0} \gamma_t^{a_1} c^{-a_1} \right]}_{\Delta_t \hat{c}^0} x_{t-1}^0 + \underbrace{\left[\alpha_2 + \sum_{a_1 \neq 0} \gamma_{K-t+1}^{a_1} d^{-a_1} \right]}_{\Delta_t \hat{d}^0} \theta_t^0 \right. \\ &\quad \left. + \underbrace{\left[\sum_{a_1 \neq 0} \gamma_t^{a_1} c^{a-a_1} \right]}_{\Delta_t \hat{c}^a} x_{t-1}^a + \underbrace{\left[\sum_{a_1 \neq 0} \gamma_t^{a_1} d^{a-a_1} \right]}_{\Delta_t \hat{d}^a} \theta_t^a + \underbrace{[\mu + e \sum_{a_1 \neq 0} \gamma_t^{a_1}]}_{\Delta_t \hat{e}} \bar{\theta} \right) \end{aligned} \quad (23)$$

The function in (23) is linear in x_{t-1} , θ_t and $\bar{\theta}$. So, $L_t g$ is linear. By definition of the new coefficient sequence $(\hat{c}, \hat{d}, \hat{e})$ in (23), each element of the new sequence is nonnegative since each element of the original one was so and the new elements are positive weighted sums of the original ones. The sum of the coefficients inside the parentheses on the right hand side of (22) is 1 since g has the convex combination form. But then, the total sum of coefficients on the right hand side of (22) is $\Delta_t^{-1}(\alpha_1 + \alpha_2 + \sum_{a \neq 0} \gamma_t^a + \mu) = 1$, which proves property (i). The final form in (23) is just a regrouping of elements in (22). Let $(\hat{c}^a)_{a \in \mathbb{A}}$ be the new coefficient sequence associated with $L_t g$ as defined in equation (23). The proof of the properties (ii), (iii), and (iv) follows straightforward modifications of the arguments in Lemma 2 hence is left to the reader.

We just proved that if the symmetric continuation equilibrium policies are Markovian, i.e., $g : \mathbf{X} \times \Theta \times \{t+1, \dots, T\} \mapsto X$, after any history (x^{t-1}, θ^t) , the unique symmetric equilibrium policy x_t^* is Markovian too. Since the choice of t was arbitrary, this must be true for all periods. Therefore, the T -period economy has a unique symmetric equilibrium which is Markovian. Consider now a $T+1$ -period economy. This latter can be *separated* into a first period and a T -period continuation economy. Clearly, same arguments as above imply that a unique symmetric Markovian equilibrium exists and the T -period continuation policy is the unique symmetric Markovian equilibrium of the T -period economy. Therefore, the claim in Theorem 1 is true for any finite horizon economy.

Step 3: Convergence and stationarity. We would like to show that the sequence of unique finite horizon symmetric Markovian equilibria tends to a stationary symmetric Markov Perfect equilibrium. To do that, we treat finite-horizon economies as finite truncations of the infinite-horizon economy. Let $G^\infty := \prod_{t=1}^\infty G$ be the infinite-horizon Markovian strategy set. For a fixed discount factor $\beta \in (0, 1)$, let $L_\beta := \{\beta^T \in [0, 1]^\infty \mid \beta_t^T = \beta^t, \text{ for } t \leq T, \text{ and } \beta_t^T = 0, \text{ for } t > T, \text{ where } T \in \{1, 2, \dots\} \cup \{\infty\}\}$ be the

space of exponentially declining sequences (at the rate of β) that are equal to zero after the T -th element. Endow L_β with the sup norm.

Given $g \in G^\infty$, let $x^a(g)$ be agent a 's strategy induced by g , i.e., $x^a(g)(x^{t-1}, \theta^t) = g_t(R^a x_{t-1}, R^a \theta_t)$, for all $a \in \mathbb{A}$ and all (x^{t-1}, θ^t) . Define the objective function U for agent 0 in the class of truncated economies as $U : G^\infty \times L_\beta \times G^\infty$ as

$$U(g^0 ; \beta^T, g) := E \left[\sum_{t=1}^{\infty} \beta_t^T u(x_{t-1}^0(g^0), x_t^0(g^0), \{x_t^b(g)\}_{b \in N(0)}, \theta_t^0) \right]$$

given x_0 , where u takes the form in Assumption 2. Let the feasibility correspondence $\Gamma : L_\beta \times G^\infty \rightarrow G^\infty$ be defined by $\Gamma(\beta^T, g) = \{g^0 \in G^\infty \mid g_t^0(x, \theta) = \bar{\theta}, \forall t > T, \forall (x, \theta) \in \mathbf{X} \times \Theta\}$, for $T < \infty$, and $\Gamma(\beta^\infty, g) = G^\infty$. It is easy to check that Γ is a compact-valued and continuous correspondence. Moreover, as the next Lemma shows, the parameterized objective function U is continuous in g^0 , the choice variable.

Lemma 4 *For any given $(\beta^T, g) \in L_\beta \times G^\infty$, $U(\cdot; \beta^T, g)$ is continuous on $\Gamma(\beta^T, g)$ with respect to the product topology.*

Proof: Since G endowed with the sup norm is a compact metric space, the metric $d(g, g') := \sum_{t=1}^{\infty} 2^{-t} \|g_t - g'_t\|_\infty$ induces the product topology (see Aliprantis and Border (1999, p. 85)) on G^∞ , where $\|\cdot\|_\infty$ is the supnorm as before. Let $(\beta^T, g) \in L_\beta \times G^\infty$ and $\epsilon > 0$ be given. Set $\epsilon' := (\frac{1-\beta}{1-\beta^{T+1}})\epsilon$. The period utility u is uniformly continuous since X is compact. Thus, one can choose a $\delta' > 0$ such that $|x_t^0 - y_t^0| < \delta'$, $\forall t$ implies

$$|u(x_{t-1}^0, x_t^0, \{x_t^b(g)\}_{b \in N(0)}, \theta_t^0) - u(y_{t-1}^0, y_t^0, \{x_t^b(g)\}_{b \in N(0)}, \theta_t^0)| < \epsilon'.$$

Set $\delta = 2^{-T} \delta'$. Pick $g^0, g'^0 \in \Gamma(\beta^T, g)$ such that $d(g^0, g'^0) < \delta$. This implies that for all $t \leq T$, $\|g_t^0 - g_t'^0\|_\infty < 2^T \delta = \delta'$ hence $|x_t^0(g^0) - x_t^0(g'^0)| < \delta'$. Uniform continuity of u then implies that the period utility levels are uniformly bounded above by ϵ' for all periods $t \leq T$. The claim therefore follows from

$$|U(g^0 ; \beta^T, g) - U(g'^0 ; \beta^T, g)| < \frac{1 - \beta^{T+1}}{1 - \beta} \epsilon' = \epsilon$$

■

For every T -period symmetric Markovian equilibrium policy sequence g^{*T} , define $g^{**T} \in G^\infty$ as

$$\forall t, \forall (x, \theta) \in \mathbf{X} \times \Theta, g_t^{**T}(x, \theta) := \begin{cases} g_t^{*T}(x, \theta), & \text{if } t \leq T \\ \bar{\theta}, & \text{if } t > T \end{cases}$$

G^∞ endowed with the product topology is compact since each G endowed with the supnorm is compact. Since it is also metrizable the sequence $(g^{**T})_T$ has a convergent subsequence $(g^{**T_n})_{T_n}$ in G^∞ that converges say to $g^* \in G^\infty$. Let $M : L_\beta \times G^\infty \rightarrow G^\infty$ be the correspondence of maximizers of U given the value of the parameters. Also, let $\mathcal{E} : L_\beta \rightarrow G^\infty$ be the symmetric equilibrium correspondence for the sequence of finite economies. Since g^{*T_n} is a symmetric Markovian equilibrium for any T_n , for all $g^{T_n} \in G^\infty$ we

have

$$\begin{aligned}
U(g_{T_n}^* ; \beta^{T_n}, g_{T_n}^*) &= E \left[\sum_{t=1}^{\infty} \beta_t^{T_n} u(x_{t-1}^0(g^{*T_n}), x_t^0(g^{*T_n}), \{x_t^b(g^{*T_n})\}_{b \in N(0)}, \theta_t^0) \right] \\
&= E \left[\sum_{t=1}^{T_n+1} \beta^{t-1} u(x_{t-1}^0(g^{*T_n}), x_t^0(g^{*T_n}), \{x_t^b(g^{*T_n})\}_{b \in N(0)}, \theta_t^0) \right] \\
&\geq E \left[\sum_{t=1}^{T_n+1} \beta^{t-1} u(x_{t-1}^0(g^{T_n}), x_t^0(g^{T_n}), \{x_t^b(g^{*T_n})\}_{b \in N(0)}, \theta_t^0) \right] \\
&= E \left[\sum_{t=1}^{\infty} \beta_t^{T_n} u(x_{t-1}^0(g^{T_n}), x_t^0(g^{T_n}), \{x_t^b(g^{*T_n})\}_{b \in N(0)}, \theta_t^0) \right] \\
&= U(g_{T_n} ; \beta^{T_n}, g_{T_n}^*)
\end{aligned}$$

Thus, $g_{T_n}^* \in M(\beta^{T_n}, g_{T_n}^*)$ for all T_n . Since U is continuous in the choice dimension and the feasibility correspondence Γ is continuous, by the Maximum Theorem (see Berge (1963), p. 115), the correspondence of maximizers, M , is upper hemi-continuous. This implies that that if $(\beta^{T_n}, g_{T_n}^*) \rightarrow (\beta^\infty, g^*)$, then $g^* \in M(\beta^\infty, g^*)$. Hence, g^* is a symmetric Markovian equilibrium of the infinite-horizon economy. The immediate implication of this is that the equilibrium correspondence \mathcal{E} is upper hemi-continuous too. Since, each finite-horizon T -period economy has a unique symmetric MPE, \mathcal{E} is single-valued, hence continuous for $T < \infty$. Define $\mathcal{F}(\beta^T) := \mathcal{E}(\beta^T)$, for $T < \infty$ and let $\mathcal{F}(\beta^\infty) = g^*$. With this definition, \mathcal{F} is continuous on the space L_β , which is compact under the supnorm. This makes \mathcal{F} uniformly continuous. So, for a given $\epsilon > 0$, we can pick $\delta > 0$ small enough so that $\|\beta^T - \beta^{T'}\|_\infty < \delta$ implies that $d(\mathcal{F}(\beta^T), \mathcal{F}(\beta^{T'})) < \frac{\epsilon}{2}$. We know from the previous approximation that for $\beta^T \rightarrow \beta^\infty$ there is a subsequence $g^{*T_n} \rightarrow g^*$. Since $(\beta^T)_T$ is convergent, it is Cauchy. So, choose $T(\delta)$ large enough such that $\forall T, T' \geq T(\delta)$, $\|\beta^T - \beta^{T'}\| < \delta$ and $\forall T_n \geq T(\delta)$, $\|g^{*T_n} - g^*\|_\infty < \frac{\epsilon}{2}$. Pick an element, T_n , of the subsequence and any other element, T' , such that $T_n, T' \geq T(\delta)$. We have

$$\begin{aligned}
d(g^{*T'}, g^*) &= d(\mathcal{F}(\beta^{T'}), \mathcal{F}(\beta^\infty)) \\
&\leq d(\mathcal{F}(\beta^{T'}), \mathcal{F}(\beta^{T_n})) + d(\mathcal{F}(\beta^{T_n}), \mathcal{F}(\beta^\infty)) \\
&< \frac{\epsilon}{2} + d(g^{*T_n}, g^*) \\
&< \epsilon
\end{aligned}$$

The first is triangle inequality; the second is due to the uniform continuity of \mathcal{F} and the third is by the fact that $g^{*T_n} \rightarrow g^*$. This proves that the whole sequence $g^{*T} \rightarrow g^*$. The implication of this latter is that, as we near the limit, every two consecutive period, we make choices approximately with respect to the same MPE policy, hence g^* is stationary. This concludes the proof.

In the rest of the proof, we provide the Proof of Lemma 3 which we omitted before. Before doing that, we state another Lemma and its proof that we use often in the rest of the paper and also in the proof of Lemma 3.

Lemma 5 (Convexity and Monotonicity) *For any $a, b \in \mathbb{A}$, and any period $t \geq 2$, equilibrium choices satisfy the following properties*

(i) *Equilibrium choices can be written as a non-negative weighted some of initial conditions and the realized paths of type shocks and their expected value, i.e.,*

$$x_t^a = \sum_{b_1} \cdots \sum_{b_t} c^{b_1} \cdots c^{b_t} x_0^{a+b_1+\cdots+b_t} \\ + \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e \bar{\theta} \right)$$

(ii) $|a| \leq |b|$ implies $\frac{\partial x_t^a}{\partial x_1^a} \leq \frac{\partial x_t^b}{\partial x_1^b}$.

(iii) $\frac{\partial}{\partial x_1^a} (2x_t^0 - x_t^1 - x_t^{-1}) \leq 0$.

(iv) $\frac{\partial}{\partial x_1^0} x_t^0 \leq \left(\frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \frac{\partial}{\partial x_1^0} x_{t-1}^0$

Proof: (i) For $t=2$, $\frac{\partial}{\partial x_1^a} x_2^0 = c_2^a \geq c_2^b = \frac{\partial}{\partial x_1^b} x_2^0$ by 14 (iii). Suppose the claim is true for $t \leq k$ and let $t = k + 1$. Assume w.l.o.g that $a < b$. Let $\underline{s} := \max\{s \in \mathbb{A} : s \leq \frac{a+b}{2}\}$ and $\bar{s} := \min\{s \in \mathbb{A} : s \geq \frac{a+b}{2}\}$. This implies that $\frac{\partial}{\partial x_1^a} x_k^s - \frac{\partial}{\partial x_1^b} x_k^s \geq 0$ (≤ 0) for $s \leq \underline{s}$ ($s \geq \bar{s}$). Due to the assumed symmetry, $\left[\frac{\partial}{\partial x_1^a} x_k^{s-s} - \frac{\partial}{\partial x_1^b} x_k^{s-s} \right] = \left[\frac{\partial}{\partial x_1^{a-\underline{s}+s}} x_k^0 - \frac{\partial}{\partial x_1^{b-\underline{s}+s}} x_k^0 \right]$ and $\left[\frac{\partial}{\partial x_1^{a-\bar{s}-s}} x_k^0 - \frac{\partial}{\partial x_1^{b-\bar{s}-s}} x_k^0 \right] = \left[\frac{\partial}{\partial x_1^a} x_k^{\bar{s}} - \frac{\partial}{\partial x_1^b} x_k^{\bar{s}} \right]$. This implies that for any $s > 0$

$$\left[\frac{\partial}{\partial x_1^{a-\underline{s}+\tau} x_k^0 - \frac{\partial}{\partial x_1^{b-\underline{s}+\tau} x_k^0} \right] = - \left[\frac{\partial}{\partial x_1^{a-\bar{s}-\tau} x_k^0 - \frac{\partial}{\partial x_1^{b-\bar{s}-\tau} x_k^0} \right]$$

Thus, we can use this to separate \mathbb{A} into $\{s \in \mathbb{A} : s \leq \underline{s}\}$ $\{s \in \mathbb{A} : s \geq \bar{s}\}$ and rearrange the sum

$$\frac{\partial}{\partial x_1^a} x_t^0 - \frac{\partial}{\partial x_1^b} x_t^0 = \sum_{s \in \mathbb{A}} c_{k+1}^s \left[\frac{\partial}{\partial x_1^a} x_k^s - \frac{\partial}{\partial x_1^b} x_k^s \right] \\ = \sum_{s \in \mathbb{A}} c_{k+1}^s \left[\frac{\partial}{\partial x_1^{a-s}} x_k^0 - \frac{\partial}{\partial x_1^{b-s}} x_k^0 \right] \\ = \sum_{\tau \geq 0} \left(c_{k+1}^{\underline{s}-s} - c_{k+1}^{\bar{s}+s} \right) \left[\frac{\partial x_k^0}{\partial x_1^{a-\underline{s}+s}} - \frac{\partial x_k^0}{\partial x_1^{b-\underline{s}+s}} \right] \\ \geq 0$$

The term in the brackets is nonnegative by hypothesis. Since $a < b$, $\underline{s} \geq 0$ which implies that $c_{k+1}^{\underline{s}} \geq c_{k+1}^{\bar{s}}$. But this implies that $c_{k+1}^{\underline{s}-s} \geq c_{k+1}^{\bar{s}+s}$ for any $s \geq 0$ which means that the argument in the parenthesis is nonnegative too. So, the claim is true. The analysis for the case $a > b$ is a straightforward modification of the same argument.

(ii) Using the t -th period equilibrium policy

$$\begin{aligned}
\frac{\partial}{\partial x_1^a} (2x_t^0 - x_t^1 - x_t^{-1}) &= \frac{\partial}{\partial x_1^a} \left[2 \sum_{b \in \mathbb{A}} c_t^b x_{t-1}^b + \sum_{b \in \mathbb{A}} d_t^b \theta_t^b + e_t \bar{\theta} \right. \\
&\quad - \sum_{b \in \mathbb{A}} c_t^{b-1} x_{t-1}^b + \sum_{b \in \mathbb{A}} d_t^{b-1} \theta_t^b + e_t \bar{\theta} \\
&\quad \left. - \sum_{b \in \mathbb{A}} c_t^{b+1} x_{t-1}^b + \sum_{b \in \mathbb{A}} d_t^{b+1} \theta_t^b + e_t \bar{\theta} \right] \\
&= \sum_{b \in \mathbb{A}} (2c_t^b - c_t^{b-1} - c_t^{b+1}) \frac{\partial}{\partial x_1^a} x_{t-1}^b \leq 0
\end{aligned} \tag{24}$$

The weights in the last parenthesis are negative by property (ii) in (14). By iteratively applying the policy functions from period t backwards, at each iteration the weights on one-period before choices would all be positive and one preserves the convex combination form. This process ends after $t - 1$ iteration, the end result being a convex combination of $(x_1^b)_{b \in \mathbb{A}}$, θ^t and $\bar{\theta}$. Thus, the weight on x_1^a is positive, which makes the last term in the last line positive. Therefore the claim is true.

(iii) Let $t \geq 2$.

$$\begin{aligned}
\frac{\partial}{\partial x_1^0} x_t^0 &= \sum_{a \in \mathbb{A}} c_t^a \frac{\partial x_{t-1}^a}{\partial x_1^0} = \sum_{a \in \mathbb{A}} c_t^a \frac{\partial x_{t-1}^0}{\partial x_1^a} \\
&\leq \sum_{a \in \mathbb{A}} c_t^a \frac{\partial x_{t-1}^0}{\partial x_1^0} = C_t \frac{\partial x_{t-1}^0}{\partial x_1^0}
\end{aligned}$$

First and second equalities and the first inequality are by the definition of the policy mapping and (i) of the current Lemma; C_t is the sum of coefficients on the past history in the t -th period policy. Since g_t satisfies (20), coefficients should match and we should have

$$\begin{aligned}
0 &= c_t^a \Delta_t - \alpha_1 I_{\{a=0\}} - \sum_{b \neq 0} \gamma_t^b c_1^{a-b} \\
\text{summing over } a, \quad 0 &= C_t \Delta_t - \alpha_1 - \sum_{b \neq 0} \gamma_t^b C_t
\end{aligned}$$

But $\Delta_t = \alpha_1 + \alpha_2 + \sum_{b \neq 0} \gamma_t^b + \mu_t$ by definition. So,

$$C_t = \frac{\alpha_1}{\Delta_t - \sum_{b \neq 0} \gamma_t^b} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \mu_t} \leq \frac{\alpha_1}{\alpha_1 + \alpha_2}$$

Thus,

$$\begin{aligned}
\frac{\partial}{\partial x_1^0} x_t^0 &\leq C_t \frac{\partial}{\partial x_1^0} x_{t-1}^0 \\
&\leq \left(\frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \frac{\partial}{\partial x_1^0} x_{t-1}^0
\end{aligned}$$

which proves the claim.

■

Finally, we give the proof of Lemma 3 that we omitted before.

Proof:(Lemma 3) For each $a \neq 0$, γ_t^a is the coefficient of x_1^a in (20) which is the total effect of a change in x_t^a on the marginal utility of agent 0 (the right hand side of (18)), i.e.,

$$\begin{aligned} \gamma_t^a &:= \alpha_3 I_{\{a \in \{-1, 1\}\}} \\ &- \sum_{\tau=t+1}^T \beta^{\tau-t} \left(\alpha_1 \frac{\partial}{\partial x_t^a} (x_{\tau-1}^0 - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_{\tau-1}^0 - x_\tau^0) + \alpha_2 \frac{\partial}{\partial x_t^a} x_\tau^0 \frac{\partial}{\partial x_t^0} x_\tau^0 \right. \\ &\left. + \alpha_3 \frac{\partial}{\partial x_t^a} (x_\tau^{-1} - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_\tau^{-1} - x_\tau^0) + \alpha_3 \frac{\partial}{\partial x_t^a} (x_\tau^1 - x_\tau^0) \frac{\partial}{\partial x_t^0} (x_\tau^1 - x_\tau^0) \right) \end{aligned} \quad (25)$$

For any $\tau \geq t$, the last two term in the summand for each period in equation (25) can be written as

$$\begin{aligned} &\frac{\partial}{\partial x_t^0} (x_\tau^1 - x_\tau^0) \left[\alpha_3 \frac{\partial}{\partial x_t^a} (x_\tau^{-1} - x_\tau^0) + \alpha_3 \frac{\partial}{\partial x_t^a} (x_\tau^1 - x_\tau^0) \right] \\ &= \frac{\partial}{\partial x_t^0} (x_\tau^1 - x_\tau^0) \left[\alpha_3 \frac{\partial x_\tau^0}{\partial x_t^{a+1}} + \alpha_3 \frac{\partial x_\tau^0}{\partial x_t^{a-1}} - 2\alpha_3 \frac{\partial x_\tau^0}{\partial x_t^a} \right] \\ &\leq 0 \end{aligned} \quad (26)$$

The equality is due to the symmetry of the policy function across agents; The following Lemma 5 (i) and (ii) imply that the terms in the parentheses are non-positive and the terms in the brackets are non-negative, respectively. Similarly, the first terms in the summand in (25) can be written as

$$\begin{aligned} &\alpha_1 \frac{\partial}{\partial x_t^a} (x_\tau^0 - x_{\tau-1}^0) \frac{\partial}{\partial x_t^0} (x_\tau^0 - x_{\tau-1}^0) + \alpha_2 \frac{\partial}{\partial x_t^a} x_\tau^0 \frac{\partial}{\partial x_t^0} x_\tau^0 \\ &\leq \alpha_1 \frac{\partial}{\partial x_t^a} x_\tau^0 \frac{\partial}{\partial x_t^0} (x_\tau^0 - x_{\tau-1}^0) + \alpha_2 \frac{\partial}{\partial x_t^a} x_\tau^0 \frac{\partial}{\partial x_t^0} x_\tau^0 \\ &= \frac{\partial x_\tau^0}{\partial x_t^a} \left[(\alpha_1 + \alpha_2) \frac{\partial}{\partial x_t^0} x_\tau^0 - \alpha_1 \frac{\partial}{\partial x_t^0} x_{\tau-1}^0 \right] \\ &\leq 0 \end{aligned}$$

which is nonpositive since for any $\tau \geq t$

$$\frac{\partial}{\partial x_t^0} x_\tau^0 \leq \frac{\alpha_1}{(\alpha_1 + \alpha_2)} \frac{\partial}{\partial x_t^0} x_{\tau-1}^0$$

due to Lemma 5 (iii). Thus, we established the non-positiveness of each term of the summand for any period $\tau \geq t$ in (25). Since, the latter is basically a finite weighted some of such terms with a negative sign in front, for any $a \in \mathbb{A}$, $\gamma_t^a \geq 0$. Finally we account for the coefficients multiplying $\bar{\theta}$ in equation (18) and show that

$$\begin{aligned} \mu_K &= \frac{\partial}{\partial \bar{\theta}} E \left[\sum_{t=2}^K \beta^{t-1} \left(-\alpha_1 (x_{t-1}^0 - x_t^0) \frac{\partial}{\partial x_1^0} (x_{t-1}^0 - x_t^0) + \alpha_2 (\theta_t^0 - x_t^0) \frac{\partial}{\partial x_1^0} x_t^0 \right. \right. \\ &\quad \left. \left. - \alpha_3 (x_t^{-1} - x_t^0) \frac{\partial}{\partial x_1^0} (x_t^{-1} - x_t^0) - \alpha_3 (x_t^1 - x_t^0) \frac{\partial}{\partial x_1^0} (x_t^1 - x_t^0) \right) \right] \\ &\geq 0 \end{aligned} \quad (27)$$

Expectation washes out all individual θ_t^a 's and we have only $\bar{\theta}$ apart from $(x_1^a)_{a \in \mathbb{A}}$ in each t -th period expression in (27). By symmetry of the form in Lemma (5) (i) across agents, the weight on $\bar{\theta}$ in x_t^0 , is equal to the weight on $\bar{\theta}$ in x_t^1 and on x_t^{-1} . Thus, $\frac{\partial}{\partial \bar{\theta}} E[(x_t^0 - x_t^1)] = \frac{\partial}{\partial \bar{\theta}} E[(x_t^0 - x_t^{-1})] = 0$. This makes the second line of (27) equal to zero. By Lemma (5) (iv) the weight on $\bar{\theta}$ in x_t^0 , $1 - C_t$ is bigger than that in x_{t-1}^0 , $1 - C_{t-1}$; hence the term $\frac{\partial}{\partial \bar{\theta}} E[(x_{t-1}^0 - x_t^0)] \leq 0$. Moreover, by Lemma (5) (i), $\frac{\partial}{\partial \bar{\theta}} E[x_t^0] \geq 0$ and we also know that $\frac{\partial}{\partial x_1^0} (x_t^0 - x_{t-1}^0) \leq 0$ due to (iii) of Lemma (5). All these put together makes the expression in (27) non-negative.

So far, we have showed that the coefficients $\Delta_t, (\gamma_t^a), \mu_t, \alpha_1, \alpha_2$, on the right hand side of equation (20) are non-negative. It is very easy to show that each $E[x_\tau^b]$ in (18) can be written as a convex combination of $(x_t^a)_{a \in \mathbb{A}}, \bar{\theta}, x_{t-1}^0, \theta_t^0$, by iteratively substituting the relevant policy functions for the equilibrium choice variables and taking expectations, i.e.,

$$\begin{aligned} x_\tau^b &= \sum_{b_1 \in \mathbb{A}} \cdots \sum_{b_{\tau-t} \in \mathbb{A}} c_{t+1}^{b_1} \cdots c_\tau^{b_{\tau-t}} x_t^{a+b_1+\cdots+b_{\tau-t}} \\ &+ \sum_{\tau=0}^{t-2} \sum_{b_1 \in \mathbb{A}} \cdots \sum_{b_{\tau+1} \in \mathbb{A}} c_2^{b_1} \cdots c_{\tau+2}^{b_{\tau+1}} (d_{\tau+2}^{b_{\tau+1}} \theta_{t-\tau}^{a+b_1+\cdots+b_\tau} + e_{T-t+\tau+1} \bar{\theta}) \end{aligned} \quad (28)$$

Since at each iteration, convex combination structure is preserved, it is at the end too. Then, the sum of coefficients in each of the differences involving those variables in the parentheses is zero. This in turn implies that the total sum of coefficients in (18) is zero. Consequently the alternative formulation in (20) is true. ■

8 Appendix B: Existence and Characterization with Incomplete Information

We state here without proof, the theorem for the existence and characterization of equilibrium for economies with incomplete information. The proof for the theorem uses straightforward modifications of the arguments used for the complete information case. Let the map $\mathbb{A} : I \rightarrow 2^{\mathbb{A}}$ be an *information correspondence*. Agent $a \in \mathbb{A}$ observes the period- t realization of the random variable $\theta_t^{I(a)}$ with support $\Theta^{I(a)} := \{(\theta^b)_{b \in I(a)} : \theta^b \in \Theta\}$ before making his period- t choice. The economy is one of incomplete information if $I(a) \subsetneq \mathbb{A}$.

Theorem 5 (Existence - Incomplete Information) *Consider an economy with conformity preferences and with incomplete information.*

1. If $K < \infty$, then S admits a unique (up to a set of measure zero) symmetric Markov perfect equilibrium $g^* : \mathbf{X} \times \Theta^I \times \{1, \dots, K\} \mapsto X$ such that for all t ,

$$g_{K-(t-1)}^*(x_{t-1}, \theta_t^I) = \sum_{a \in \mathbb{A}} c_{K-t+1}^a x_{t-1}^a + \sum_{a \in I} d_{K-(t-1)}^a \theta_t^a + e_{K-t+1} \bar{\theta} \quad \mathbb{P} - a.s.$$

where for $0 \leq \tau \leq K$, for $a \in \mathbb{A}$, $c_\tau^a, d_\tau^a, e_\tau \geq 0$ and $e_\tau + \sum_{a \in \mathbb{A}} c_\tau^a + \sum_{a \in I} d_\tau^a = 1$.

2. If $K = \infty$, then S admits a symmetric Markov perfect equilibrium $g^* : \mathbf{X} \times \Theta^I \mapsto X$ such that

$$g^*(x_{t-1}, \theta_t) = \sum_{a \in \mathbb{A}} c^a x_{t-1}^a + \sum_{a \in I} d^a \theta_t^a + e \bar{\theta}$$

where for $a \in \mathbb{A}$, $c^a, d^a, e \geq 0$ and $e + \sum_{a \in \mathbb{A}} c^a + \sum_{a \in I} d^a = 1$.

9 Appendix C: Proof of Inefficiency (Theorem 4)

We will prove the theorem for an incomplete information economy where each agent observes his own type only before making a choice. The proof, as it will be clear below, extends straightforwardly to more general incomplete and complete information scenarios, which we leave to the reader. In the final period of a finite period economy, with initial condition x_{t-1} , the planner solves (for agent 0)

$$\max_{\{x : X^\infty \times \Theta \rightarrow X\}} \int u(x_{t-1}^0, x(x_{t-1}, \theta_t^0), x(Tx_{t-1}, \theta_t^1), x(T^{-1}x_{t-1}, \theta_t^{-1}), \theta_t^0) \nu(d\theta_t^0) \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \pi(dx_{t-1})$$

FOC: $x(\hat{x}, \hat{\theta}) \rightarrow$

$$\begin{aligned} 0 &= \int u_2(\hat{x}^0, x(\hat{x}, \hat{\theta}), x(T\hat{x}, \theta_t^1), x(T^{-1}\hat{x}, \theta_t^{-1}), \hat{\theta}) \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \\ &+ \int u_3(\hat{x}^{-1}, x(T^{-1}\hat{x}, \theta^0), x(\hat{x}, \hat{\theta}), x(T^{-2}\hat{x}, \theta_t^{-1}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^{-1}) \\ &+ \int u_4(\hat{x}^1, x(T\hat{x}, \theta^0), x(T^2\hat{x}, \theta_t^1), x(\hat{x}, \hat{\theta}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^1) \end{aligned}$$

For the quadratic specification, the FOC becomes

$$\begin{aligned} 0 &= \int \left[\alpha_1(\hat{x}^0 - x(\hat{x}, \hat{\theta})) + \alpha_2(\hat{\theta} - x(\hat{x}, \hat{\theta})) \right. \\ &+ \alpha_3(x(T\hat{x}, \theta_t^1) - x(\hat{x}, \hat{\theta})) + \alpha_3(x(T^{-1}\hat{x}, \theta_t^{-1}) - x(\hat{x}, \hat{\theta})) \left. \right] \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \\ &- \int \alpha_3(x(\hat{x}, \hat{\theta}) - x(T^{-1}\hat{x}, \theta_t^0)) \nu(d\theta_t^0) \\ &- \int \alpha_3(x(\hat{x}, \hat{\theta}) - x(T\hat{x}, \theta_t^0)) \nu(d\theta_t^0) \end{aligned}$$

But this implies that

$$x(\hat{x}, \hat{\theta}) [\alpha_1 + \alpha_2 + 4\alpha_3] = \alpha_1 \hat{x}^0 + \alpha_2 \hat{\theta} + 2\alpha_3 \int x(T\hat{x}, \theta) \nu(d\theta) + 2\alpha_3 \int x(T^{-1}\hat{x}, \theta) \nu(d\theta)$$

As in the proof of existence, due to this being a contraction, there is a unique solution in the space of linear maps, hence one can fit the following solution

$$x(\hat{x}, \hat{\theta}) = \sum_a c^a \hat{x}^a + d\hat{\theta} + e\bar{\theta}$$

substituting, we get

$$\begin{aligned} \sum_a c^a \hat{x}^a + d\hat{\theta} + e\bar{\theta} &= \beta_1 \hat{x}^0 + \beta_2 \hat{\theta} + 2\beta_3 \left[\sum_a c^a \hat{x}^{a+1} + (d+e)\bar{\theta} \right] \\ &+ 2\beta_3 \left[\sum_a c^a \hat{x}^{a-1} + (d+e)\bar{\theta} \right] \end{aligned}$$

where $\beta_i := \alpha_i / (\alpha_1 + \alpha_2 + 4\alpha_3)$. Thus,

$$d = \beta_2 = \frac{\alpha_2}{\alpha_1 + \alpha_2 + 4\alpha_3}$$

and

$$e = \left(\frac{2\alpha_2}{\alpha_1 + \alpha_2} \right) \left(\frac{2\alpha_3}{\alpha_1 + \alpha_2 + 4\alpha_3} \right)$$

and

$$c^a = 2\beta_3 [c^{a-1} + c^{a+1}] + \beta_1 \mathbf{1}_{\{a=0\}}, \quad \forall a \in \mathbb{A}$$

whereas the FOC for the “laissez-faire” equilibrium gives

$$\begin{aligned} 0 = & \int \left[\alpha_1(\hat{x}^0 - x(\hat{x}, \hat{\theta})) + \alpha_2(\hat{\theta} - x(\hat{x}, \hat{\theta})) \right. \\ & \left. + \alpha_3(x(T\hat{x}, \theta^1) - x(\hat{x}, \hat{\theta})) + \alpha_3(x(T^{-1}\hat{x}, \theta^{-1}) - x(\hat{x}, \hat{\theta})) \right] \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \end{aligned}$$

At the symmetric equilibrium, $x^{a+1} = x^{a-1} = x$, this implies that

$$x(\hat{x}, \hat{\theta}) = \beta_1 \hat{x}^0 + \beta_2 \hat{\theta} + \beta_3 \int x(T\hat{x}, \theta) \nu(d\theta) + \beta_3 \int x(T^{-1}\hat{x}, \theta) \nu(d\theta)$$

Similarly, there exists a unique solution whose form is

$$x(\hat{x}, \hat{\theta}) = \sum_a c_{LF}^a \hat{x}^a + d_{LF} \hat{\theta} + e_{LF} \bar{\theta}$$

Solving for it, we obtain

$$d_{LF} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + 2\alpha_3} > d$$

and

$$e_{LF} = \left(\frac{2\alpha_2}{\alpha_1 + \alpha_2} \right) \left(\frac{2\alpha_3}{\alpha_1 + \alpha_2 + 2\alpha_3} \right) > e$$

Thus, the laissez-faire equilibrium and the planner’s solution do not agree in the final period of any finite period economy. Hence, the equilibrium is inefficient for finite period economies.

Infinite-Horizon Incomplete

Here is the strategy of attack here. We know that the equilibrium whose existence we know has the following structure

$$x(\hat{x}, \hat{\theta}) = \sum_a c^a \hat{x}^a + d\hat{\theta} + e\bar{\theta}$$

We are going to argue that this solution cannot satisfy the central planner’s problem’s optimality condition. For the central planner’s problem, we will restrict ourselves to linear policy functions from the same class as the laissez-faire equilibrium. Let $\lambda(\cdot) := \prod_a \nu(\cdot)$ and $\theta_t := (\theta_t^a)_a$. So, the planner’s problem is (for agent 0)

$$\begin{aligned} \max_{\{x : X^\infty \times \Theta \rightarrow X\}} & \int u(x_{t-1}^0, x(x_{t-1}, \theta_t^0), x(Tx_{t-1}, \theta_t^1), x(T^{-1}x_{t-1}, \theta_t^{-1}), \theta_t^0) \nu(d\theta_t^0) \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \pi(dx_{t-1}) \\ & + \beta \int V((x(T^a \hat{x}, \theta^a))_a, \theta_t^0) \pi(dx_{t-1}) \lambda(d\theta_t) \end{aligned}$$

Policy is linear, utility is quadratic and action space is compact; so V_a exists for any a and FOC: $x(\hat{x}, \hat{\theta}) \rightarrow$

$$\begin{aligned}
0 &= \int u_2(\hat{x}^0, x(\hat{x}, \hat{\theta}), x(T\hat{x}, \theta_t^1), x(T^{-1}\hat{x}, \theta_t^{-1}), \hat{\theta}) \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \\
&+ \int u_3(\hat{x}^{-1}, x(T^{-1}\hat{x}, \theta^0), x(\hat{x}, \hat{\theta}), x(T^{-2}\hat{x}, \theta_t^{-1}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^{-1}) \\
&+ \int u_4(\hat{x}^1, x(T\hat{x}, \theta^0), x(T^2\hat{x}, \theta_t^1), x(\hat{x}, \hat{\theta}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^1) \\
&+ \beta \int \sum_a V_a((x(T^a \hat{x}, \theta^a))_a, \theta_t^0) \pi(dx_{t-1}) \lambda(d\theta_t)
\end{aligned}$$

But the FOC for the laissez-faire equilibrium is

$$\begin{aligned}
0 &= \int u_2(\hat{x}^0, x(\hat{x}, \hat{\theta}), x(T\hat{x}, \theta_t^1), x(T^{-1}\hat{x}, \theta_t^{-1}), \hat{\theta}) \nu(d\theta_t^1) \nu(d\theta_t^{-1}) \\
&+ \beta \int \sum_a V_a((x(T^a \hat{x}, \theta^a))_a, \theta_t^0) \pi(dx_{t-1}) \lambda(d\theta_t)
\end{aligned}$$

For the same solution to satisfy both FOCs, it has to be that for all \hat{x} , for all $\hat{\theta}$, the difference of the two FOCs

$$\begin{aligned}
0 &= \int u_3(\hat{x}^{-1}, x(T^{-1}\hat{x}, \theta^0), x(\hat{x}, \hat{\theta}), x(T^{-2}\hat{x}, \theta_t^{-1}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^{-1}) \\
&+ \int u_4(\hat{x}^1, x(T\hat{x}, \theta^0), x(T^2\hat{x}, \theta_t^1), x(\hat{x}, \hat{\theta}), \theta^0) \nu(d\theta_t^0) \nu(d\theta_t^1)
\end{aligned}$$

For the quadratic specification, this entails

$$\begin{aligned}
0 &= \alpha_3 \int (x(\hat{x}, \hat{\theta}) - x(T^{-1}\hat{x}, \theta_t^0)) \nu(d\theta_t^0) \\
&+ \alpha_3 \int (x(\hat{x}, \hat{\theta}) - x(T\hat{x}, \theta_t^0)) \nu(d\theta_t^0)
\end{aligned}$$

Substituting the linear policy and algebra gives

$$\alpha_3 \left[\sum_a (2c^a - c^{a+1} - c^{a-1}) \hat{x}^a + 2d(\hat{\theta} - \bar{\theta}) \right] = 0$$

Clearly for $\hat{x}^a = \bar{x}$, a constant sequence and $\hat{\theta} \neq \bar{\theta}$, this equation would not be satisfied. This concludes the proof. ■

10 Appendix D: Proof of Ergodicity (Theorem 3)

Let $((\theta_t^a)_{t=-\infty}^0)_{a \in \mathbb{A}}$ be an \mathcal{F} -measurable random variable where each θ_t^a is distributed i.i.d. according to ν as before. Let π be the initial measure on the configuration space \mathbf{X} which is the distribution of

$$x_0 = \left(\frac{e^{\bar{\theta}}}{1-C} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\cdots+b_s} \right) \right)_{a \in \mathbb{A}} \quad (29)$$

Given the initial distribution π for x_0 ,

$$\begin{aligned} x_1^a &= \sum_{b_1 \in \mathbb{A}} c^{b_1} x_0^{a+b_1} + \sum_{b_1 \in \mathbb{A}} d^{b_1} \theta_1^{a+b_1} + e^{\bar{\theta}} \\ &= \sum_{b_1 \in \mathbb{A}} c^{b_1} \left(\frac{e^{\bar{\theta}}}{1-C} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\cdots+b_s} \right) \right) \\ &\quad + \sum_{b_1 \in \mathbb{A}} d^{b_1} \theta_1^{a+b_1} + e^{\bar{\theta}} \\ &= \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_{s+1}} c^{b_1} \cdots c^{b_s} \left(d^{b_{s+1}} \theta_{1-s}^{a+b_1+\cdots+b_{s+1}} \right) + \sum_{b_1 \in \mathbb{A}} d^{b_1} \theta_1^{a+b_1} \\ &\quad + C \frac{e^{\bar{\theta}}}{1-C} + e^{\bar{\theta}} \\ &= \frac{e^{\bar{\theta}}}{1-C} + \sum_{s=1}^{\infty} \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{2-s}^{a+b_1+\cdots+b_s} \right) \end{aligned}$$

which has exactly the same form as in (29), with the convention that for $s = 1$. Hence, x_0^a and x_1^a are distributed identically when the initial measure is π . Since the choice of a was arbitrary, π is a stationary distribution of the Markov process $(x_t)_{t=0}^{\infty}$. Moreover, from Lemma 5 for a stationary policy function

$$\begin{aligned} x_t^a &= \sum_{b_1} \cdots \sum_{b_t} c^{b_1} \cdots c^{b_t} x_0^{a+b_1+\cdots+b_t} \\ &\quad + \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e^{\bar{\theta}} \right) \end{aligned} \quad (30)$$

$$\begin{aligned} &= C^t \sum_{b_1} \cdots \sum_{b_t} \left(\frac{c^{b_1} \cdots c^{b_t}}{C^t} \right) x_0^{a+b_1+\cdots+b_t} \\ &\quad + \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e^{\bar{\theta}} \right) \end{aligned} \quad (31)$$

For any initial value for x_0 , x_t^a converges to

$$\begin{aligned} \lim_{t \rightarrow \infty} x_t^a &= \lim_{t \rightarrow \infty} \left[C^t \sum_{b_1} \cdots \sum_{b_t} \left(\frac{c^{b_1} \cdots c^{b_t}}{C^t} \right) x_0^{a+b_1+\cdots+b_t} \right. \\ &\quad \left. + \sum_{s=1}^t \sum_{b_1} \cdots \sum_{b_s} c^{b_1} \cdots c^{b_{s-1}} \left(d^{b_s} \theta_{t+1-s}^{a+b_1+\cdots+b_s} + e^{\bar{\theta}} \right) \right] \end{aligned}$$

The first term of the previous expression $C^t \rightarrow 0$ since $C < 1$ due to $\alpha_i > 0$, for all i . The first term in the parentheses in the summand is a convex combination of uniformly bounded terms. Hence, the first part of the above expression goes to 0 as $t \rightarrow \infty$. Due to i.i.d. assumption, for any t , the distribution of the second part is identical to the distribution of

$$\sum_{s=1}^t \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\dots+b_s} + e^{\bar{\theta}} \right)$$

which is the ‘ t -translated-into-the-past’ version of the former. This implies that for any given initial value x_0 , the distribution of $\lim_{t \rightarrow \infty} x_t^a$ is the same as the distribution of

$$\frac{e^{\bar{\theta}}}{1-C} + \sum_{s=1}^{\infty} \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\dots+b_s} \right)$$

But then, for any initial distribution π_0 , the limit distribution of x_t^a is independent of π_0 . Since a was arbitrary, by the same token the limit distribution of x_t is independent of π_0 and is identical to the distribution of

$$\left(\frac{e^{\bar{\theta}}}{1-C} + \sum_{s=1}^{\infty} \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{1-s}^{a+b_1+\dots+b_s} \right) \right)_{a \in \mathbb{A}}$$

which is distributed according to π . Therefore, the equilibrium process (x_t) is ergodic and its unique stationary distribution is given by π .

Fitting spatial correlations: Can we fit the myopic b -step autocorrelations ρ_b , induced by the myopic policy g_m , that we computed above into the stationary autocorrelations generated by the stationary policy g and back up the deep parameters α_i , $i = 1, 2, 3$, from the nonlinear regression? In other words, let y be a 10×1 column vector of b -step sample autocorrelations generated by the simulation s.t. y_b is the b -step autocorrelation for each $b = 1, \dots, 10$. Our nonlinear regression equation system is the following

$$y_b = (\beta_3)^b + \epsilon_b, \quad b = 1, \dots, 10$$

After computing the coefficient $\hat{\beta}_3$ of the best fit, we compare it with the ratio $\frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3}$ of the parameters of the rational model that we used to run the simulations to see if there is any under- or over-estimation of the relative interaction parameter.

11 Appendix E: Omitted proofs

Proof of Lemma 1 :

■

Proof of Proposition 1: From the stationary policy function in (6) and using translation invariance across agents, the conditional covariance between a and $a + b$ given x_0 is

$$\begin{aligned}
& Cov(x_1^a, x_1^{a+b}; x_0) = Cov(x_1^0, x_1^b; x_0) \\
&= Cov\left(\sum_{s \in \mathbb{A}} c^s x_0^s + \sum_{s \in \mathbb{A}} d^s \theta_1^s + e \bar{\theta}, \sum_{s \in \mathbb{A}} c^s x_0^{b+s} + \sum_{s \in \mathbb{A}} d^s \theta_1^{b+s} + e \bar{\theta}; x_0\right) \\
&= Cov\left(\sum_{s \in \mathbb{A}} d^s \theta_1^s, \sum_{s \in \mathbb{A}} d^s \theta_1^{b+s}\right) \\
&= Var(\theta) \sum_{s \in \mathbb{A}} d^s d^{s-b}
\end{aligned}$$

Clearly for $b = 0$, this expression gives the conditional variance of agent a 's period one equilibrium action $Var(x_1^a; x_0) = \sum_{s \in \mathbb{A}} (d^s)^2$. Hence, one-period ahead conditional correlation between equilibrium actions of agents a and $a + b$ is

$$\rho_{1,b} = \frac{\sum_{s \in \mathbb{A}} d^s d^{s-b}}{\sum_{s \in \mathbb{A}} (d^s)^2}$$

For the same agents, using the expression in Lemma 5 (i), the conditional covariance between their period- t equilibrium limit actions x_t^a, x_t^{a+b} given x_0 is

$$\begin{aligned}
& Cov(x_t^a, x_t^{a+b}; x_0) = Cov(x_t^0, x_t^b; x_0) \\
&= Cov\left(\sum_{s=1}^t \sum_{a_1} \dots \sum_{a_s} c^{a_1} \dots c^{a_{s-1}} \left(d^{b_s} \theta_{t-s+1}^{a_1+\dots+a_s}\right), \sum_{s=1}^t \sum_{b_1} \dots \sum_{b_s} c^{b_1} \dots c^{b_{s-1}} \left(d^{b_s} \theta_{t-s+1}^{b_1+\dots+b_s}\right)\right) \\
&= \sum_{s=1}^t \sum_{a_1} \dots \sum_{a_s} \sum_{b_1} \dots \sum_{b_s} c^{a_1} \dots c^{a_{s-1}} c^{b_1} \dots c^{b_{s-1}} d^{a_s} d^{b_s} Cov\left(\theta_{t-s+1}^{a_1+\dots+a_s}, \theta_{t-s+1}^{b_1+\dots+b_s}\right) \\
&= Var(\theta) \sum_{s=1}^t \sum_{a \in \mathbb{A}} \left(\sum_{a_1} \dots \sum_{a_{s-1}} c^{a_1} \dots c^{a_{s-1}}\right) \left(\sum_{b_1} \dots \sum_{b_{s-1}} c^{b_1} \dots c^{b_{s-1}}\right) d^{a-a_1-\dots-a_{s-1}} d^{(a-b)-b_1-\dots-b_{s-1}}
\end{aligned}$$

Similarly, the t -period ahead conditional correlation between equilibrium actions of agents a and $a + b$ is given by

$$\rho_{t,b} = \frac{\sum_{s=1}^t \sum_{a \in \mathbb{A}} \left(\sum_{a_1} \dots \sum_{a_{s-1}} c^{a_1} \dots c^{a_{s-1}}\right) \left(\sum_{b_1} \dots \sum_{b_{s-1}} c^{b_1} \dots c^{b_{s-1}}\right) d^{a-a_1-\dots-a_{s-1}} d^{(a-b)-b_1-\dots-b_{s-1}}}{\sum_{s=1}^t \sum_{a \in \mathbb{A}} \left(\sum_{a_1} \dots \sum_{a_{s-1}} c^{a_1} \dots c^{a_{s-1}}\right) \left(\sum_{b_1} \dots \sum_{b_{s-1}} c^{b_1} \dots c^{b_{s-1}}\right) (d^{a-a_1-\dots-a_{s-1}})^2}$$

Using the same sort of analysis for equilibrium actions of a and $a + b$ at the limit distribution (which is given by (5)), we obtain

$$\rho_b = \frac{\sum_{s=1}^{\infty} \sum_{a \in \mathbb{A}} \left(\sum_{a_1} \cdots \sum_{a_{s-1}} c^{a_1} \cdots c^{a_{s-1}} \right) \left(\sum_{b_1} \cdots \sum_{b_{s-1}} c^{b_1} \cdots c^{b_{s-1}} \right) d^{a-a_1-\cdots-a_{s-1}} d^{(a-b)-b_1-\cdots-b_{s-1}}}{\sum_{s=1}^{\infty} \sum_{a \in \mathbb{A}} \left(\sum_{a_1} \cdots \sum_{a_{s-1}} c^{a_1} \cdots c^{a_{s-1}} \right) \left(\sum_{b_1} \cdots \sum_{b_{s-1}} c^{b_1} \cdots c^{b_{s-1}} \right) (d^{a-a_1-\cdots-a_{s-1}})^2}$$

Clearly, $\lim_{t \rightarrow \infty} \rho_{t,b} = \rho_b$.

12 Appendix F: Details about the simulations

Simulating ergodic distributions: We build an artificial economy that consists of a large number (2,500) of agents distributed on the one-dimensional integer lattice, one next to another. We start the economy with an initial history of choices for all agents; in particular, we do it for the case where all agents choose initially the highest action D in the interval $[-D, D]$. At each period t , we let the computer draw $(\theta_t^a)_{a=1}^{2,500}$ from the interval $[-D, D]$ according to the uniform distribution. We also append a buffer block of agents at the end that choose randomly according to their θ draws to make sure that the very last agents of interest can apply their policy functions to histories and θ s of agents (buffer) that are after them on the line. So, given $(x_{t-1}^a, \theta_t^a)_{a=1}^{2,500}$, current choices $(x_t^a)_{a=1}^{2,500}$ are determined by a truncated version of the rational policy function g (we ask the computer to run a recursive argument to compute the first 100 c and d weights of the policy function beforehand). We repeat the same routine 1,000 periods. We know theoretically that the Markov process $(x_t)_{t=1}^{\infty}$ induced by the law of θ s and the policy function g converges to a unique invariant distribution. We let the computer start picking realizations from the empirical ergodic distribution after the 1000'th period and we compute b -step sample correlations using these realizations for $b = 1, \dots, 10$. We continue doing that for 100 period and take a sample average of them. This way, we obtain an empirical approximation to b -step population autocorrelations of the ergodic distribution.